Z-to-Y Transformation

From experience with circuits, you know that sometimes it is more convenient to talk about admittance than impedance.

For example, if two impedances are in parallel, 

\[ Z = \frac{Z_1 Z_2}{Z_1 + Z_2} \]

When \( Z_1 \) and \( Z_2 \) are complex, the calculation is quite tedious.

But, if we use admittance,

\[ Y_1 = \frac{1}{Z_1}, \quad Y_2 = \frac{1}{Z_2} \]

then

\[ Y = Y_1 + Y_2 \]

A transmission line can be “branched”.

![Diagram of transmission line branched at two points, labeled with \( Z_{\text{in}1} \) and \( Z_{\text{in}2} \).]
Use the Smith Chart to handle admittance

First, let’s define the normalized quantities.

\[
Y = \frac{1}{Z}, \quad Y_0 = \frac{1}{Z_0}
\]

Define the normalized conductance and normalized susceptance:

\[
Y = G + jB
\]

Then, the normalized admittance

\[
y = \frac{Y}{Y_0} = \frac{Z}{Z_0} = \frac{1}{3}
\]

Recall that

\[
\beta = \frac{1 + \Gamma}{1 - \Gamma}, \quad \therefore \quad y = \frac{1}{\beta} = \frac{1 - \Gamma}{1 + \Gamma} = \frac{1 + (-\Gamma)}{1 - (-\Gamma)} = \frac{1 + \Gamma e^{-j\pi}}{1 - \Gamma e^{j\pi}}
\]

Now, do you see how to quickly find \( y \) from a given \( z \) using the Smith Chart?
The reflection coefficient at this point is, of course, $-\Gamma = e^{-j\pi}$.

On the chart, locate $z = 2 - j$ again.

Extend the line diametrically and draw a circle (or use the alternative method).

Locate the point diametrically opposite

The reflection coefficient at this point is, of course, $-\Gamma = e^{-j\pi}$.

The “z” corresponding to $-\Gamma$ is $y$ because

$$y = \frac{1}{3} = \frac{1 - \Gamma}{1 + \Gamma} = \frac{1 + (-\Gamma)}{1 - (-\Gamma)}$$

Read $y$ from the chart
The “\(z\)” corresponding to \(-\Gamma\) is \(y\) because

\[
y = \frac{1}{3} = \frac{1-\Gamma}{1+\Gamma} = \frac{1+ (-\Gamma)}{1- (-\Gamma)}
\]

Quick check:

\[
yz = (0.4 + 0.2j)(2 - j) = 0.8 - 0.4j + 0.4j + 0.2 = 1
\]

So the chart is also a tool to find \(y\) from \(z\).
You may think about this from another perspective

Recall the quarter wavelength transformer

\[ Z_{in} = \frac{Z_b^2}{Z_L} \]

\[ Z_{in} Z_L = Z_b^2 \]

\[ \frac{Z_{in}}{Z_0} \frac{Z_L}{Z_0} = 1 \]

\[ Z_{in} Z_L = 1 \]

\[ Z_{in} = Z \left( d = \frac{1}{4} \lambda \right) = \frac{1}{Z_L} \]

If \( z_L \) is real, \( z_{in} = z(\lambda/4) \) is real, too.

Where are they on the Smith chart?

Review textbook Section 2-10. Finish Homework 4.
Quarter wavelength transformer

\[ Z_{in} = Z\left(d=\frac{\lambda}{4}\right) = \frac{1}{Z_L}. \]

If \( Z_L \) is real, \( Z_{in} = z(\lambda/4) \) is real, too.

Where are they on the Smith chart?

Half a round is \( \pi = 180^\circ \), or \( \lambda/4 \).

The maximum is a quarter wavelength away from the adjacent minimum.

At a minimum or a maximum, \( x(d_{min}) = 0 \) or \( x(d_{max}) = 0 \)

Purely resistive.

If you have a purely resistive \( z(d) \), you must have either a maximum or minimum,

At a minimum, the most destructive, \( \Gamma_d < 0 \)
At a maximum, the most constructive, \( \Gamma_d > 0 \)
The quarter wavelength transformer is among the many ways to do impedance matching.

In general, we use a “matching network” to achieve matching.

If \( Z_L \) is real, i.e., \( Z_L = R_L \) and \( X_L = 0 \), the matching network is simply quarter-wavelength transformer with

\[
Z_{0z} = \sqrt{Z_{01} R_L}
\]

If \( Z_L = R_L + jX_L, X_L \neq 0 \),

Smith chart as the “z-chart”
Let’s compare these two cases.

If \( Z_L = R_L \) and \( X_L = 0 \), you only need to match \( R_L \) to \( Z_{01} \). You only need to “turn one knob,” which is \( Z_{02} \). The matching network can be just a quarter-wavelength transformer.

In the more general case, \( Z_L = R_L + jX_L, X_L \neq 0 \), you need to first “tune out” the reactive part, and then match the resistive part. You need to “turn two knobs.”

Here, one knob is the distance from the load, and the other is the \( Z_{02} \) of the quarter-wavelength transformer.

In both cases, multiple reflection inside matching network, as in anti-refection coating.

You can find two other knobs to turn.

Review textbook Section 2-11 prelude (before 2-11.1).