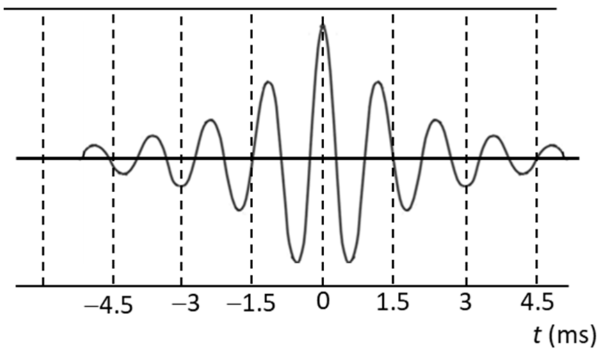
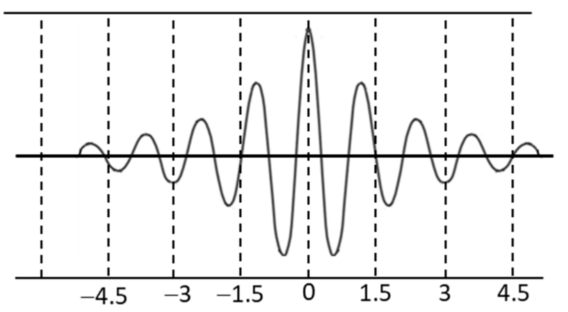
Problem 1 Total 9 points

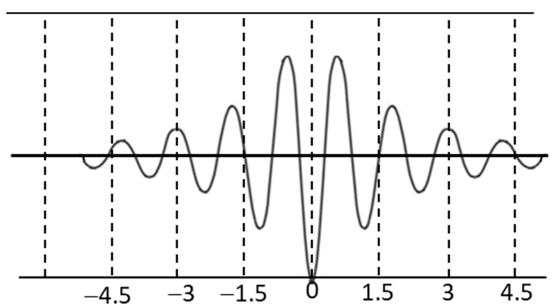
An audio signal, consisting of a periodic train of wave packets, is fed to a speaker and one channel of an oscilloscope. The oscilloscope is adjusted to display one wave packet of this signal, as sketched in the figure below. A microphone is 1 m away from the speaker, and its output is fed to another channel of the oscilloscope. Both signals are fed to oscilloscope channels via coaxial cables. The speed of sound is (1000/3) m/s.



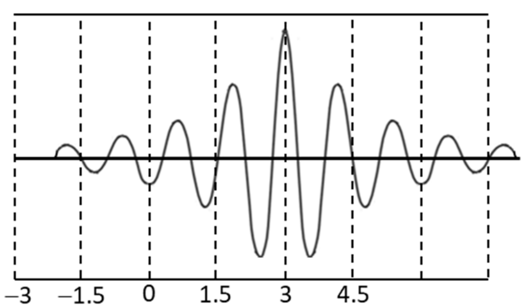
(1) Which of the following sketches best depicts the waveform of the microphone output? Explain your choice.



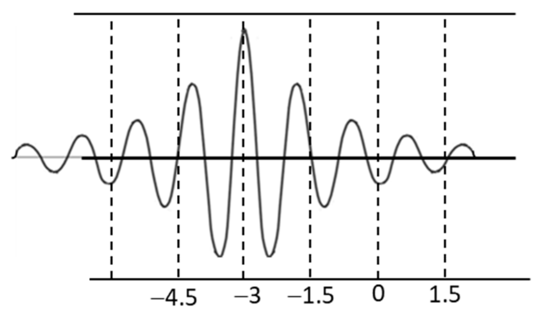
(A)



(B)



(C)



(D)

Solution:

4 The microphone output is delayed with regard to the speaker signal by the time it takes for the sound wave to propagate from the speaker to the microphone, Δt = 1 m / (1000/3 m/s) = 3 ms. Therefore, (C) is the correct waveform.

(2) A train of wave packets in the above contains frequency components within a bandwidth. Each single frequency component of the input signal $V_i\cos\omega t$ to the speaker, t being time, corresponds to the Fourier component of the same frequency, $V_o\cos(\omega t - \phi)$, of the output of the microphone. Find the function $\phi(\omega)$.

Solution:

5

The microphone output is delayed with regard to the speaker signal by Δt, therefore its Fourier component corresponding to $V_i\cos\omega t$ is

$V_o\cos[\omega(t - \Delta t)] = V_o\cos(\omega t - \omega\Delta t) = V_o\cos(\omega t - \phi)$.

Therefore $\phi(\omega) = \omega\Delta t = \omega(0.003\text{ s})$.

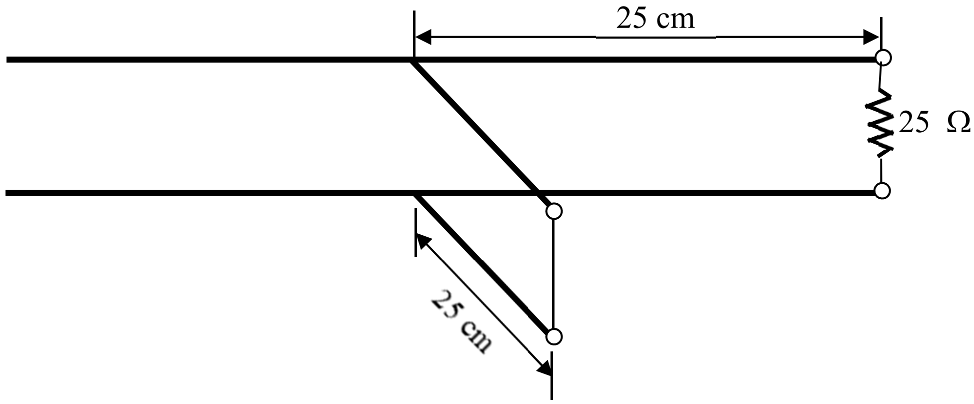
Notice that ϕ depends on ω, and is proportional to ω.

Problem 2 Total 46 points

(1) A transmission line of characteristic impedance $Z_0 = 50\ \Omega$ is terminated in a purely resistive load 25 Ω. For frequency $f = 300$ MHz, find the equivalent impedance $Z_1(d)$ at distance $d = 25$ cm from the load towards the generator.

(2) A transmission line of characteristic impedance $Z_0 = 50\ \Omega$ is terminated in a short circuit. For frequency $f = 300$ MHz, find the equivalent impedance $Z_2(d)$ at distance $d = 25$ cm from the short-circuit termination towards the generator.

(3) The figure below shows a transmission line system with the same characteristic impedance $Z_0 = 50\ \Omega$ for the main line and both branches, operating at frequency $f = 300$ MHz. What is the total impedance at the junction of the two branches? What is the magnitude of the voltage reflection coefficient $|\Gamma_1|$ of the branch terminated in the 25 Ω load? What is the magnitude of the voltage reflection coefficient $|\Gamma_2|$ of the branch terminated in a short circuit? What is the magnitude of the voltage reflection coefficient $|\Gamma|$ of the main line before the junction? The generator (not shown in figure) sends out an incident power $P_{inc} = 9$ mW. What is the power P_{ref} reflected back to the generator? What are the incident and reflected power, respectively, on the branch terminated in the 25 Ω load? What is the power dissipated by the load?



Solution:

$f = 300$ MHz ➔ $\lambda = c/f = (3\times10^8\ \text{m/s}) / (3\times10^8\ \text{/s}) = 1$ m.← 2 points (student automatically receive these points if answers the following correctly)

$d = 25\ \text{cm} = \lambda/4$.

(1) The normalized load is 0.5. The normalized equivalent impedance $\lambda/4$ away is 2. Therefore, $Z_1(d) = 100\ \Omega$. 4

(2) The equivalent impedance $\lambda/4$ away from a short circuit is an open circuit. Therefore, $Z_2(d) = \infty$. 4

(3) The total impedance of 100 Ω in parallel with an open circuit is 100 Ω. 2

$|\Gamma_1| = 1/3$. 3 $|\Gamma_2| = 1$. 3 $|\Gamma| = 1/3$. 2

$P_{ref} = |\Gamma|^2 P_{inc} = (1/9) \times 9\ \text{mW} = 1$ mW on the main line. 3

1 1

On the branch of the 25 Ω load, the incident and reflected power are 9 mW and 1 mW, respectively. The power dissipated by the load is therefore 8 mW.

2

Problem 2 cont'd

(4) For frequency f = 600 MHz, repeat (1).

(5) For frequency f = 600 MHz, repeat (2).

(6) For the transmission line system shown in the above figure with frequency f = 600 MHz, what is the total impedance at the junction of the two branches? What is the magnitude of the voltage reflection coefficient $|\Gamma|$ of the main line before the junction? The generator (not shown in figure) sends out an incident power P_{inc} = 9 mW. What is the power P_{ref} reflected back to the generator? What is the power dissipated by the load?

Solution:

$f = 600$ MHz ➔ $\lambda = c/f = (3\times10^8 \text{ m/s}) / (6\times10^8 \text{ /s}) = 0.5$ m. ← 2 points (student automatically receive these points if answers the following correctly)

$d = 25$ cm $= \lambda/2$.

(4) The equivalent impedance $\lambda/2$ away is the same as the load. Therefore, $Z_1(d) = 25\ \Omega$. 4

(5) The equivalent impedance $\lambda/2$ away from a short circuit is also a short circuit. Therefore, $Z_2(d) = 0$. 4

(6) The total impedance of 25 Ω in parallel with a short circuit is 0. 2

$|\Gamma| = 1$. 3

$P_{ref} = |\Gamma|^2 P_{inc} = 9$ mW. 2

The power dissipated by the load is 0. 2

Problem 3 total 31 + 14 points (14 bonus points for the 2nd solution)

A load $Z_L = (90.5 + 25j)\ \Omega$ is to be matched to a $Z_0 = 50\ \Omega$ lossless transmission line by using a shorted stub. Determine

(1) How far should the stub be away from the load in terms of the wavelength (d/λ);

(2) The required stub **admittance and impedance** (give actual values in S and Ω, respectively), and whether it is inductive or capacitive;

(3) The length of the stub in terms of the wavelength (l_s/λ), and

(4) The standing wave ratios of the transmission line between the stub and the load, that of the stub, and that of the transmission line between the generator and the stub. **Caution**: you need to find 3 SWRs for 3 segments.

(5) What is the voltage reflection coefficient Γ_L at the load? What is the voltage reflection coefficient Γ_S at the short-circuit termination of the stub?

Note: There are two independent solutions to each single-stub matching problem. You only need to get one to get full points for (1) through (3). If you get both solutions right, **you will receive bonus points**. In other words, you may get > 100 points on this test. If you mix up the d or l_s values for the two solutions, however, you lose both the bonus and basic points.

Solution:

$z_L = (90.5 + 25j)/50 = 1.81 + 0.5j$ ← 2 points (student automatically receive these points if answers the following correctly)

Using the Smith chart or simple calculation for this resistive load, we find $y_L = 0.51 - 0.14j$. 3

(1) Solution 1: $d/\lambda = 0.181$, 3 where $y(d) = 1 + 0.71j$ 3

Solution 2: $d/\lambda = 0.377$, where $y(d) = 1 - 0.71j$ ← 2nd solutions earns 3+3=6 bonus points

(2) Solution 1: To cancel $y(d)$, we need $y_{stub} = -0.71j$. ← 1: automatically received if below right

$Y_{stub} = -0.71j / (50\ \Omega) = -j14$ mS 1

$Z_{stub} = j71\ \Omega$ 1

The required stub impedance is inductive. 1

Solution 2: To cancel $y(d)$, we need $y_{stub} = +0.71j$.

$Y_{stub} = +0.71j / (50\ \Omega) = +j14$ mS

$Z_{stub} = -j71\ \Omega$

The required stub impedance is capacitive.

2nd solutions earns 1+1+1+1=4 bonus points

(3) Solution 1: $y_{stub} = -0.71j$ 2

$l_s/\lambda = 0.152$ 2

Solution 2: $y_{stub} = +0.71j$

$l_s/\lambda = 0.348$

2nd solutions earns 2+2=4 bonus points

(4) For both solutions, $|\Gamma| = 1/3$ and $S = 2$ between the load and the stub. 2

not required

$S = 1$ between the stub and the generator. Impedance matched! 4

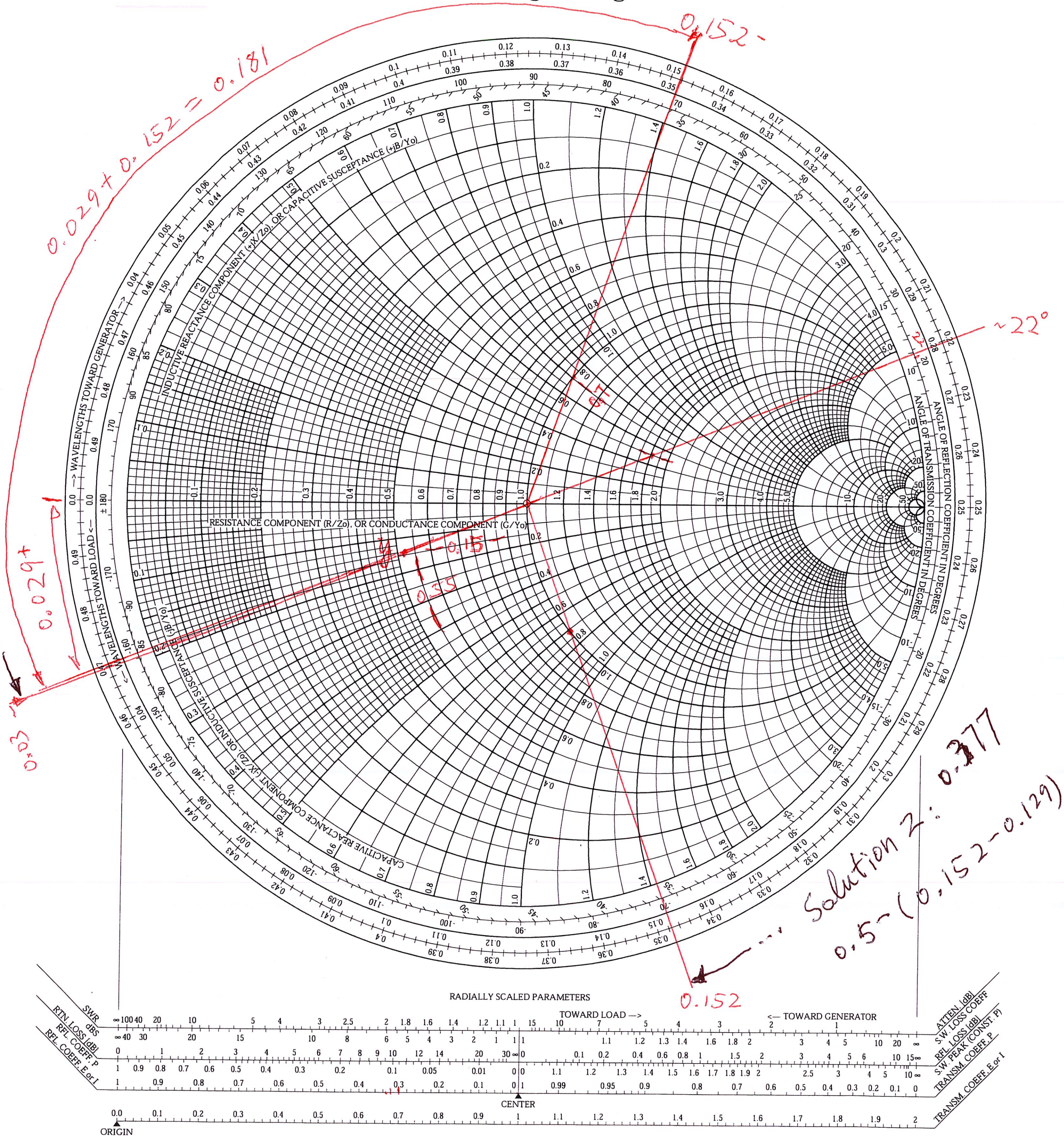
$S = \infty$ for the stub. Short! 4

(5) The voltage reflection coefficient at the load $\Gamma_L = (1/3)\angle 21.9°$. 4

The voltage reflection coefficient at the short-circuit termination of the stub $\Gamma_S = -1$. 2

The Complete Smith Chart

Black Magic Design

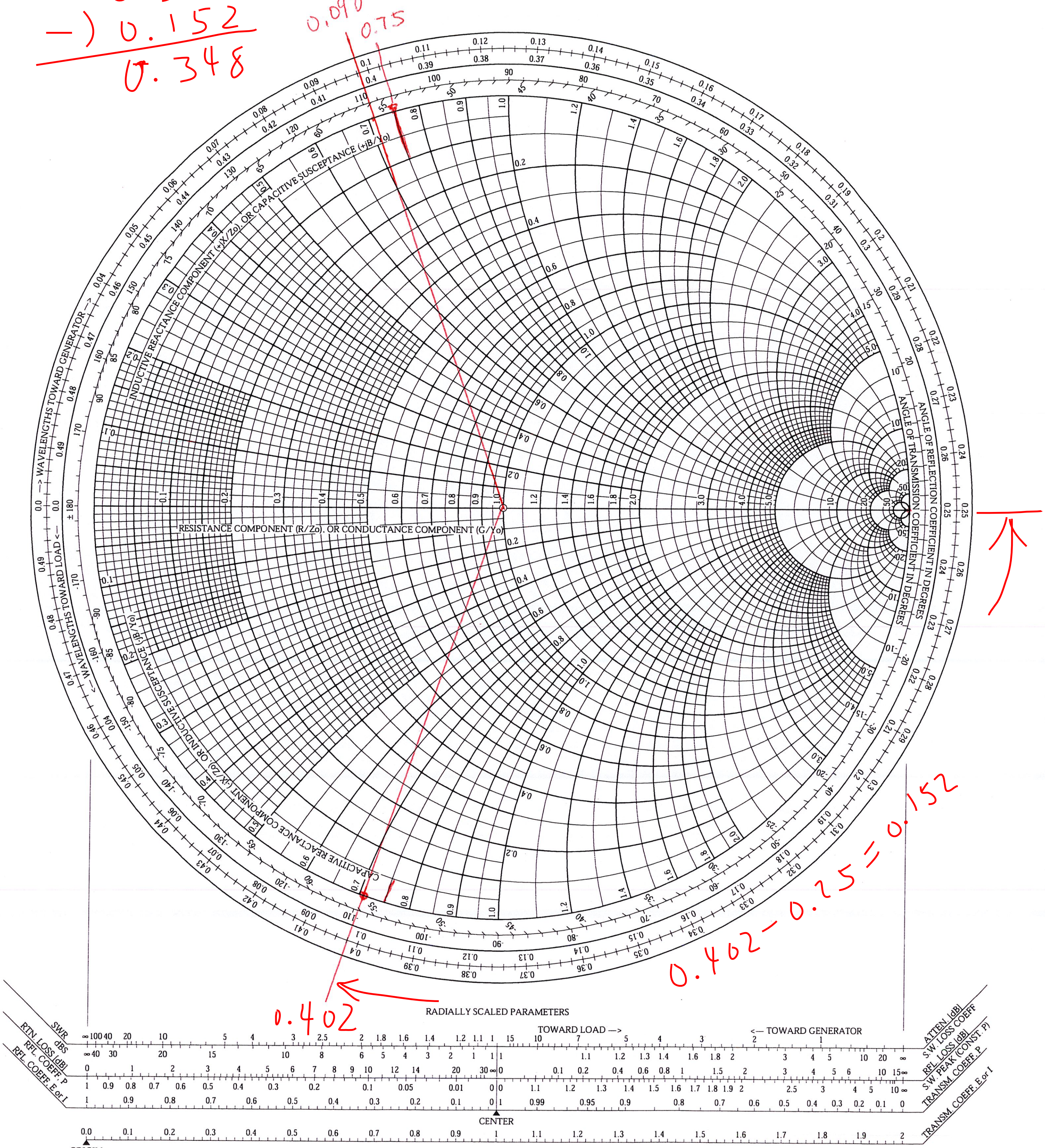


Solution 2

The Complete Smith Chart

Black Magic Design

$$\begin{array}{r} 0.5 \\ -)\ 0.152 \\ \hline 0.348 \end{array}$$



Problem 4 Total 14 points

Generate the voltage bounce diagram for a 2-m long lossless transmission line of characteristic impedance $Z_0 = 75\ \Omega$ and phase velocity $v_p = 2c/3$ (where c is the speed of light), assuming that the line is fed by a voltage pulse, with a width $\tau = 5$ ns and a height $V_g = 1$ V, applied at time $t = 0$ by a generator circuit with $R_g = 25\ \Omega$. The line is terminated in a load $R_L = 225\ \Omega$. Use the bounce diagram to plot $v(t)$ at the load.

Find the steady state voltage and current.

Automatically received if bounce chart & waveform right

Solution:

The single trip time

$$T = \frac{\ell}{v_p} = \frac{2m}{2 \times 10^8 m/s} = 10^{-8} s = 10\ ns$$ 2

The incident voltage of the first single trip

$$V_1^+ = \frac{Z_0}{Z_0 + R_g} V_g = \frac{75}{75+25} \times 1V = 0.75V$$ 2

The reflection coefficient at the load

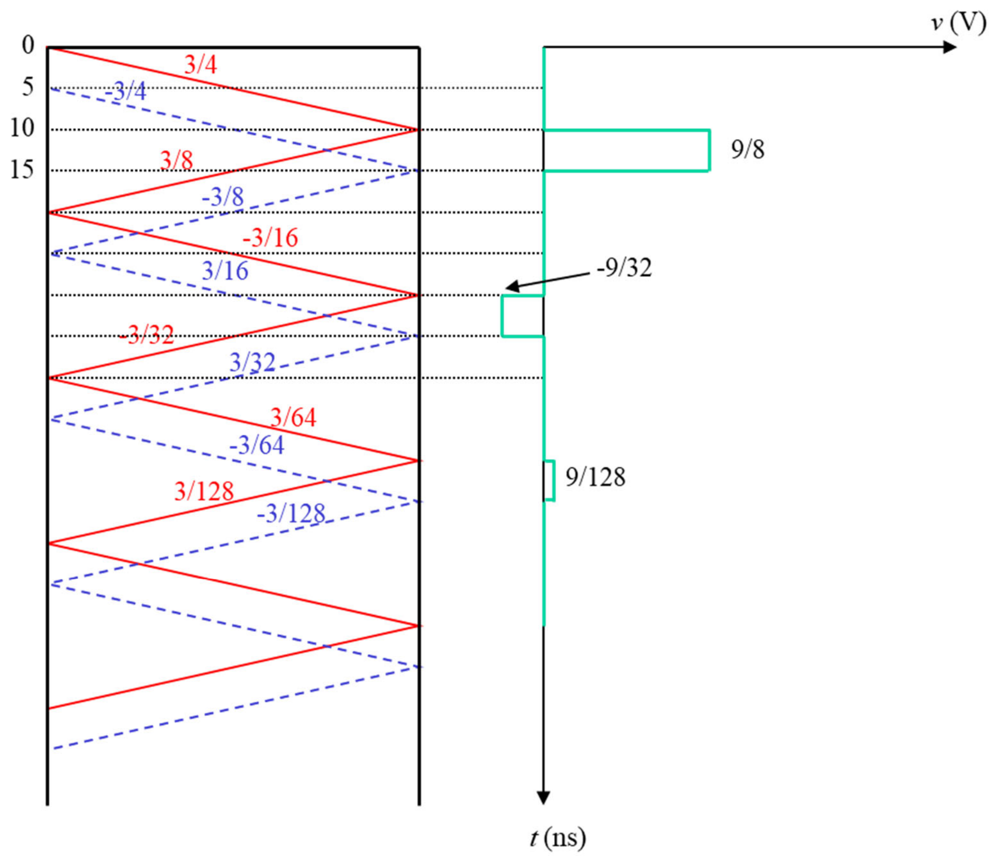
$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{225-75}{225+75} = \frac{150}{300} = \frac{1}{2}$$ 2

The reflection coefficient at the generator

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{25-75}{25+75} = -\frac{50}{100} = -\frac{1}{2}$$ 2

See next page for the bounce diagram and the waveform.

At the steady state, $v(\infty) = 0$, $i(\infty) = 0$, since the input is a pulse. 2



4 points for chart & waveform. Three pulses sufficient. Must indicate time and level for each pulse; any understandable means is acceptable.