1. (a) The electric field of an electromagnetic (EM) plane wave traveling in a perfect insulator can be written as
\[ E(z,t) = E_0 \cos(2\pi ft - \beta_0 z + \phi). \]
For such a wave traveling in free space with a frequency \( f = 3 \) GHz, find the wavelength \( \lambda_0 \) and the propagation constant \( \beta_0 \).

(b) If the EM wave is traveling downward in seawater, which is conductive, it can be written as
\[ E(z,t) = E_0 e^{-\alpha z} \cos(2\pi ft - \beta z + \phi), \]
where \( z \) is the depth, and the decay constant \( \alpha \) quantifies how fast the attenuation is. In a certain region, the phase velocity of the wave is \( v_p = c/9 \), where \( c \) is the speed of light in free space, and \( \alpha = 0.01 \) m\(^{-1} \). For \( f = 3 \) GHz and \( E_0 = 100 \) V/m, what are the wavelength \( \lambda \) and the propagation constant \( \beta \), and what is the amplitude at \( z = 90 \) m?

**Solution:**

(a) In free space,
\[ \lambda_0 = \frac{c}{f} = \frac{(3 \times 10^8 \text{ m/s})}{(3 \times 10^9 \text{ Hz})} = 0.1 \text{ m} \]
\[ \beta_0 = \frac{2\pi}{\lambda} = 20\pi \text{ m}^{-1} \]

(b) In seawater,
\[ \lambda = \frac{c}{9f} = \frac{1}{90} \text{ m} = 0.011 \text{ m} \]
\[ \beta = \frac{2\pi}{\lambda} = 180\pi \text{ m}^{-1} \]

The amplitude at \( z \) is
\[ E_0 e^{-\alpha z} = (100 \text{ V/m}) e^{-0.01 \times 90} = 40.7 \text{ V/m} \]
2. With an unknown load connected to a slotted air line, a voltage standing wave ratio $S = 2$ is recorded by a standing wave indicator and minima are found at 19 cm, 27 cm, 35 cm, ... on the scale. When the load is replaced by a short circuit, the minima are at 16 cm, 24 cm, 32 cm, ... The characteristic impedance of the line is $Z_0 = 50 \Omega$.

(a) Calculate the wavelength $\lambda$ and the frequency $f$. (Hint: what information do you get from that the slotted line is an air line?)

(b) Find the reflection coefficient $\Gamma$ at the load $Z_L$, and then use the Smith chart to find $Z_L$.

(c) (Optional, meaning you get extra points on top of 100; do it if you have time) Verify your result of $Z_L$ by actually performing the calculation from $\Gamma$.

Solution:

(a)

\[
\text{\lambda}/2 = 8 \text{ cm} \\
\lambda = 16 \text{ cm}
\]

Air line $\Rightarrow v_p = c \\
f = (3 \times 10^8 \text{ m/s}) / (0.16 \text{ m}) = 1.875 \times 10^9 \text{ Hz} = 1.875 \text{ GHz}

(b)

\[|\Gamma| = (S - 1) / ((S + 1) = 1/3\]

(c) (Optional, meaning you get extra points on top of 100; do it if you have time) Verify your result of $Z_L$ by actually performing the calculation from $\Gamma$.

By Smith chart, $z_L = 1.4 - 0.75j$  
Partial credit for getting the right normalized $z$ is 2 points

Partial credit for getting the right $\theta$ is 3 points
3. A load with an impedance of \((100 - j50) \Omega\) is to be matched to a 50 \(\Omega\) lossless line with a shorted stub. Determine
   a) How far should the stub be away from the load in terms of wavelengths \((d/\lambda)\);
   b) The required stub admittance and impedance (give actual values in S and \(\Omega\), respectively, not the normalized), and whether it is inductive or capacitive;
   c) The length of the stub in wavelengths \((l_s/\lambda)\); and
   d) The standing wave ratios of the transmission line between the stub and the load, that of the stub, and that of the transmission line before the stub (i.e. between the generator and the stub, no matter how far away the generator is away from the stub). \textbf{Caution:} you need to find 3 SWRs for 3 segments.

\textbf{Solution:}

1. \(z_L = (100 - j50)/50 = 2 - j\)

2. Using Smith chart, we find \(y_L = 0.4 + 0.2j\)

3. a) Solution 1: \(d/\lambda = 0.161 - 0.036 = 0.125\), where \(y(d) = 1 + j\)
   Solution 2: \(d/\lambda = 0.338 - 0.036 = 0.302\), where \(y(d) = 1 - j\)

   b) Solution 1: To cancel \(y(d)\), we need \(y_{\text{stub}} = -j\).
      \(Y_{\text{stub}} = -j / (50 \Omega)\)
      \(Z_{\text{stub}} = j50 \Omega\)
      The required stub impedance is inductive.

   Solution 2: To cancel \(y(d)\), we need \(y_{\text{stub}} = j\).
      \(Y_{\text{stub}} = j / (50 \Omega)\)
      \(Z_{\text{stub}} = -j50 \Omega\)
      The required stub impedance is capacitive.

   \textbf{Caution:} you need to find 3 SWRs for 3 segments.

3. c) Solution 1: \(y_{\text{stub}} = -j\)
   \(l_s/\lambda = 0.125\)

   Solution 1: \(y_{\text{stub}} = j\)
   \(l_s/\lambda = 0.375\)

   \textbf{Caution:} you need to find 3 SWRs for 3 segments.

4. d) For both solutions, \(S = 3.4\) between the load and the stub by Smith chart, \(S = 1\) between the stub and the generator (impedance matched!), and \(S = \infty\) for the stub (short!).

   \textbf{Caution:} you need to find 3 SWRs for 3 segments.
4. Generate a bounce diagram for the voltage \( v(z,t) \) for a 2-m long, lossless transmission line characterized by \( Z_0 = 75 \, \Omega \), if the line is fed by a step voltage applied at \( t = 0 \) by a generator circuit with \( V_g = 100 \, \text{V} \) and \( R_g = 225 \, \Omega \). The line is terminated in a load \( Z_L = 30 \, \Omega \). The transmission line is filled with a dielectric such that the phase velocity \( v_p = \frac{2c}{3} \), where \( c \) is speed of light in vacuum.

Use the bounce diagram to plot the voltage waveforms at
\begin{itemize}
  \item[a)] Midway of the line, and
  \item[b)] The generator.
\end{itemize}

You need to give the voltage value for every plateau in the two waveforms. It is sufficient to plot the voltage waveforms for 3 round trips.

Then, find
\begin{itemize}
  \item[c)] The steady state voltage and current.
\end{itemize}

**Solution:**

\begin{align*}
\{ v_p &= 2c/3 = 2 \times 10^8 \, \text{m/s} \\
T &= l/v_p = 2 \, \text{m} / (2 \times 10^8 \, \text{m/s}) = 10^8 \, \text{s} = 10 \, \text{ns} \\
\Gamma_L &= (30 - 75) / (30 + 75) = -3/7 \\
\Gamma_g &= (225 - 75) / (225 + 75) = \frac{1}{2} \\
V_1^+ &= 100 \, \text{V} \times 75/(75+225) = 25 \, \text{V} \\
\}
\end{align*}

\begin{align*}
c) \quad v(\infty) &= 100 \, \text{V} \times 30/(30+225) = 11.8 \, \text{V} \\
i(\infty) &= 11.8 \, \text{V} / (30 \, \Omega) = 3.9 \, \text{mA}
\end{align*}