The pn junction

Units of these things

What happens when we put them in contact to each other?
Depletion region more or less intrinsic (depleted), but cannot really be

Diffusion and drift are against each other. Equilibrium is reached eventually.

In the transition region (the junction!), you don’t have

$\nu = N_0$ or $\rho = N_A$. 

The intrinsic level: the Fermi energy of the intrinsic semiconductor. For Si, it’s about at the mid-gap. We discussed why.

(Holes drift uphill in this kind of diagrams, since energies are plotted for electrons.)
Built-in voltage

$$q \Delta V$$  
$q$: elementary charge

Vacuum level

Electron affinity

One Fermi level in equilibrium
Think about this eq. graphically (esp. with depletion approximation)

Work from definition

\[ \varepsilon_0 \varepsilon_r \frac{d \varepsilon}{dx} = \text{charge density} \]
\[ = q \left( N_d - N_A + p - n \right) \]

Remember \( E \varepsilon \) is the potential energy.

\[ - \frac{E \varepsilon}{q} \]

\[ \Rightarrow \varepsilon = - \frac{d}{dx} \left( - \frac{E \varepsilon}{q} \right) = \frac{1}{q} \frac{dE \varepsilon}{dx} \]

\[ \frac{d \varepsilon}{dx} = \frac{1}{q} \frac{d^2 \varepsilon}{dx^2} \]

Put into Gauss’s law:

\[ \frac{1}{q} \frac{d^2 \varepsilon}{dx^2} = \frac{q}{\varepsilon_0 \varepsilon_r} \left( N_d - N_A + p - n \right) \]
Let's define some reference for the electric potential:
\[
\phi = -\frac{1}{q} (E_i - E_f)
\]

Pay attention to signs of electrostatic potentials and potential energies.

Side note: the unit of \( \phi \) is V.

The electron (or hole) goes across 1V, the energy gained or lost is 1 eV.

A convenient unit.

Power = I V

\( 1 W = 1 A \cdot V \)

Energy = Q V

\( 1 J = 1 C \cdot V \)

How to convert eV to J?

\[ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \cdot \text{V} = 1.6 \times 10^{-19} \text{ J} \]
We could solve them numerically, but we like analytical expressions since they provide physical insights.
The depletion approximation

\[ n = n_i \cdot e^{\frac{E_F - E_i}{k_B T}} = n_i \cdot e^\frac{-\varphi}{k_B T} \]

\[ p = n_i \cdot e^{\frac{E_i - E_F}{k_B T}} = n_i \cdot e^{-\frac{\varphi}{k_B T}} \]

\( n, p \) depend on \( \phi \) exponentially.

When \( \phi \) is not too much larger than \( k_B T \), \( n \approx 0 \) and \( p \approx 0 \).

\[ n_i = 1.45 \times 10^{10} / \text{cm}^3 \] for Si

\( e \approx 2.718, e^2 \approx 7.4, e^4 \approx 55, e^5 \approx 148 \)

\[ n = 10^{12} / \text{cm}^3 \approx 0 \]

???
On the $n$ side of the depletion region,
$$\frac{d\varepsilon}{dx} = \frac{q}{\varepsilon_0 \varepsilon_r} N_D$$

The ionized donors are the only charge.
$$\frac{d\varepsilon}{dx} = \text{Const.} \quad \text{what should} \\
\varepsilon(x) \text{ look like?}$$

Boundary condition known.
$$\varepsilon(x_n) = 0$$

$$\therefore \varepsilon(x) = -\frac{q N_D}{\varepsilon_0 \varepsilon_r} (x - x_d)$$

Similarly, on the $p$ side.
$$\varepsilon(x) = -\frac{q N_A}{\varepsilon_0 \varepsilon_r} (x + x_p)$$

On the $n$ side,
$$\phi(x) = \phi(x_n)$$

$$-\int_{x_n}^{x} \varepsilon(x) \, dx$$
$$= \phi_n - \frac{q N_D}{2 \varepsilon_0 \varepsilon_r} (x - x_n)^2$$
Similarly, on the $p$ side,

$$\phi(x) = \phi_p + \frac{qN_A}{2\varepsilon_0 \varepsilon} (x + x_p)^2$$

Define

$$\phi_n = \phi(x_n) = -\frac{l}{q} \left[ E_{F_n} - E_F \right]$$

In the $n$ side neutral region

$$N_n = n = n_i e^{\frac{E_F - E_i}{kT}}$$

$$\therefore \phi_n = \frac{kT}{q} \ln \frac{N_n}{n_i}$$

Similarly, on the $p$ side,

$$\phi_p = -\frac{kT}{q} \ln \frac{N_A}{n_i}$$

The built-in potential

$$\phi_i = \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_0N_A}{n_i}$$
Why?

At \( x = 0 \), where the \( n \) joins the \( p \),

\[
\phi (0) = \phi_n - \frac{q N_0}{2 \varepsilon_0 \varepsilon_r} x_n^2
\]

and

\[
\phi (0) = \phi_p + \frac{q N_0}{2 \varepsilon_0 \varepsilon_r} x_p^2
\]

\[\therefore \phi_n - \phi_p \equiv \phi_i = \left\{ \frac{q}{2 \varepsilon_0 \varepsilon_r} \left( N_0 x_n^2 + N_p x_p^2 \right) \right\}
\]

Recall that \( N_0 x_n = N_A x_p \)

\[x_n + x_p = \sqrt{2 \frac{\varepsilon_0 \varepsilon_r}{q} \phi_i \left( \frac{1}{N_0} + \frac{1}{N_A} \right)}\]

It's easy for you to write the expressions for \( x_n \) & \( x_p \).
Schottky junctions

$E_{\text{vac}}$: vacuum level

work function of metal

ionization potential

electron affinity

$\phi_M > \phi_S$ (work function of semiconductor)

Fermi levels line up; depletion approximation.
Fermi levels line up; depletion approximation.

Rectifying Schottky junction:

Barrier height
\[ \phi_b = \phi_m - \chi \]

Built-in potential
\[ \phi_i = \phi_m - \phi_s = E_{FS} - E_F \]

The depletion region width
\[ x_d = \sqrt{\frac{2 \phi_i \varepsilon \varepsilon_0}{q N_D}} \]
Schottky ohmic contact:

More e’s here than in bulk

We stopped here on Tue 8/31. Class canceled for Thu 9/2. Review this slide set off-line.