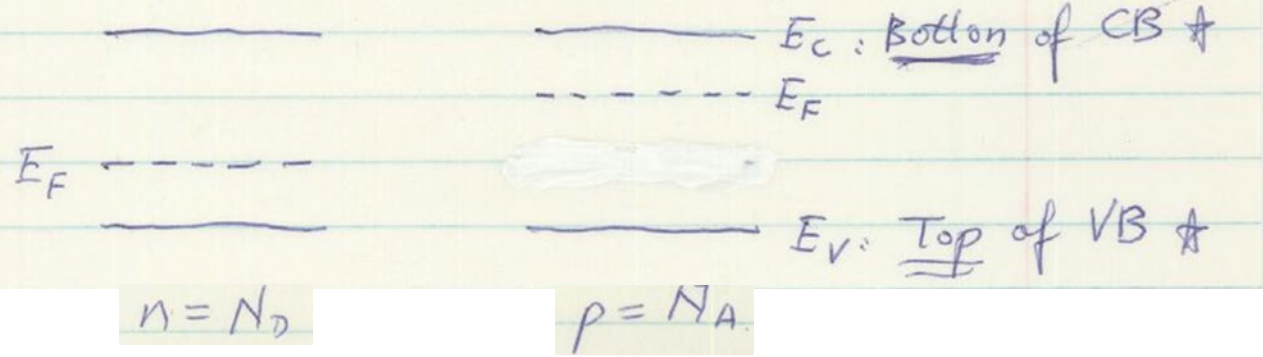
The pn junction

homojunction

Two pieces of the same semiconductor, one n, one p.



$n = N_D$ $\qquad$ $p = N_A$

Fermi distribution

Fermi level

Density of states

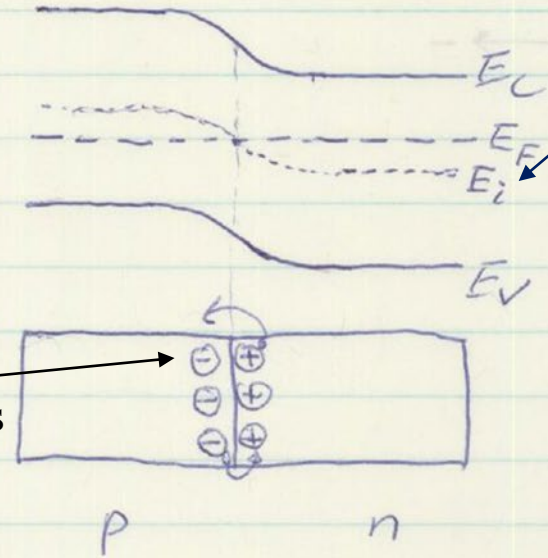
Carrier density

Units of these things

$$n = n_i \, e^{\frac{E_F - E_i}{k_B T}}$$

$$p = n_i \, e^{\frac{E_i - E_F}{k_B T}}$$

What happens when we put them in contact to each other?



The intrinsic level: the Fermi energy of the intrinsic semiconductor. For Si, it's about at the mid-gap. We discussed why.

Depletion region more or less intrinsic (depleted), but cannot really be

Diffusion: from high concentration to low concentration

Drift: from high potential to low $\underbrace{\text{potential}}_{\text{electrostatic}}$

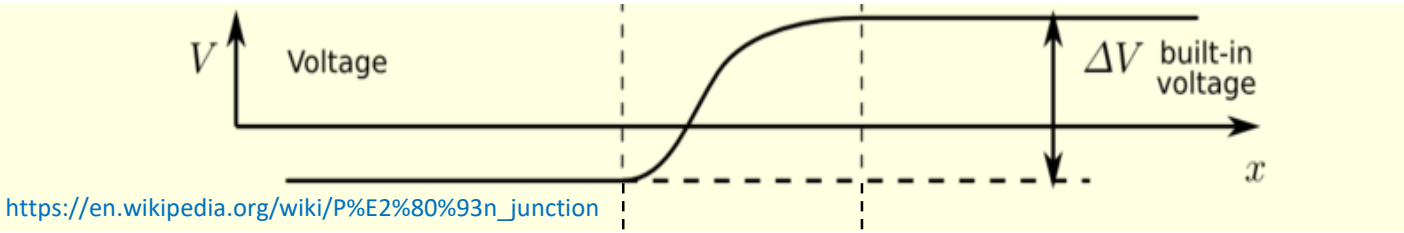
(Holes drift uphill in this kind of diagrams, since energies are plotted for electrons.)

Diffusion and drift are against each other.
Equilibrium is reached eventually.

In the transition region (the junction!), you don't have

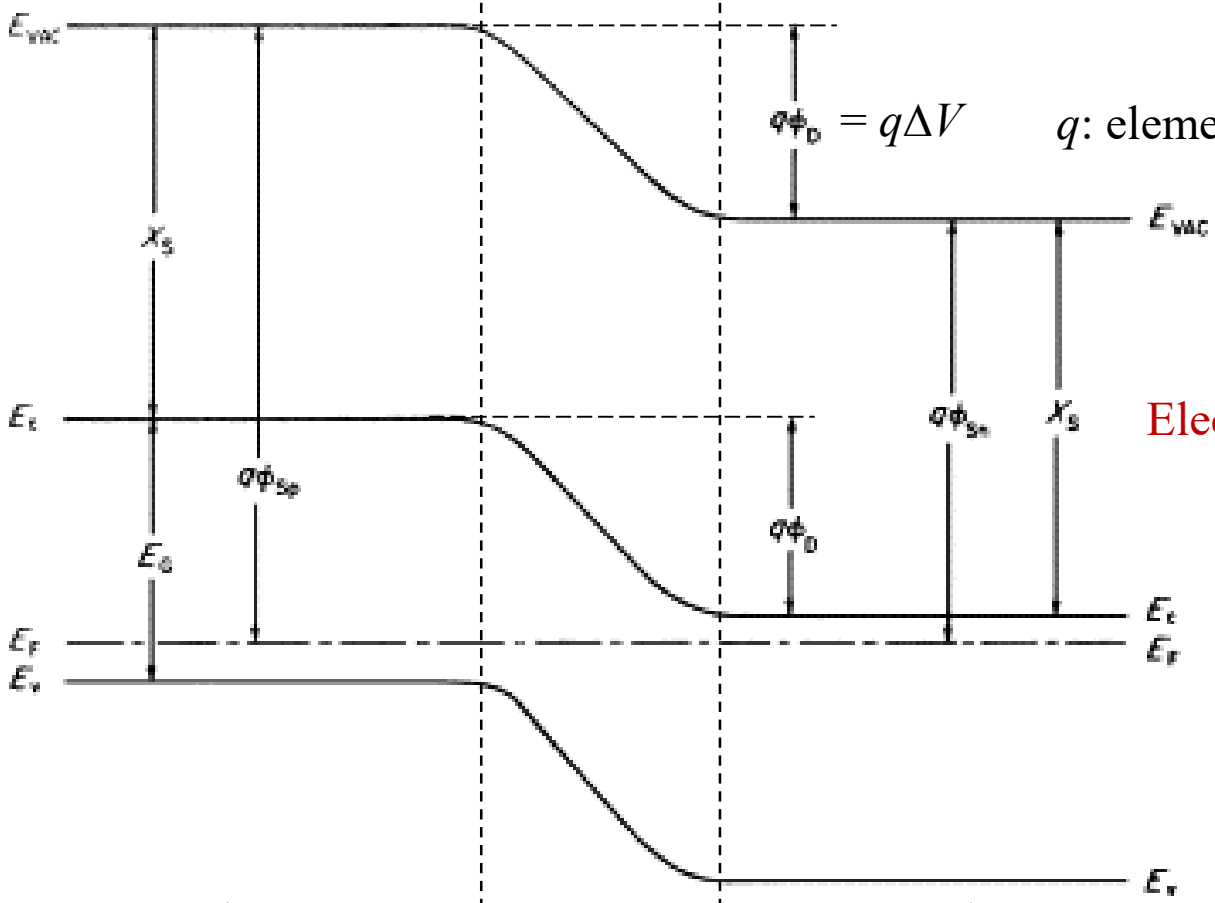
$$n = N_D \quad \text{or} \quad p = N_A$$

Band diagram of pn junction



https://en.wikipedia.org/wiki/P%E2%80%93n_junction

Built-in voltage



$= q\Delta V$

q: elementary charge

Vacuum level

Electron affinity

One Fermi level in equilibrium

p type

n type

https://www.sciencedirect.com/topics/computer-science/energy-band-diagram

Richard H. Bube, in Encyclopedia of Physical Science and Technology (Third Edition), 2003

Field

Think about this eq. graphically (esp. with depletion approximation)

Remember Gauss's law

$$\varepsilon_0 \varepsilon_r \frac{d\mathcal{E}}{dx} = \text{charge density}$$

$$= q\,(N_D - N_A + p - n)$$

Remember E_c is the $\overbrace{\text{potential}}^{\text{electrostatic}}$ energy?

So, $-\dfrac{E_c}{q}$ is the electric (electrostatic) potential.

Work from definition

$$\Rightarrow \quad \mathcal{E} = -\frac{d}{dx}\left(-\frac{E_c}{q}\right) = \frac{1}{q}\frac{dE_c}{dx} \quad \checkmark$$

$$\frac{d\mathcal{E}}{dx} = \frac{1}{q}\frac{d^2E_c}{dx^2}$$

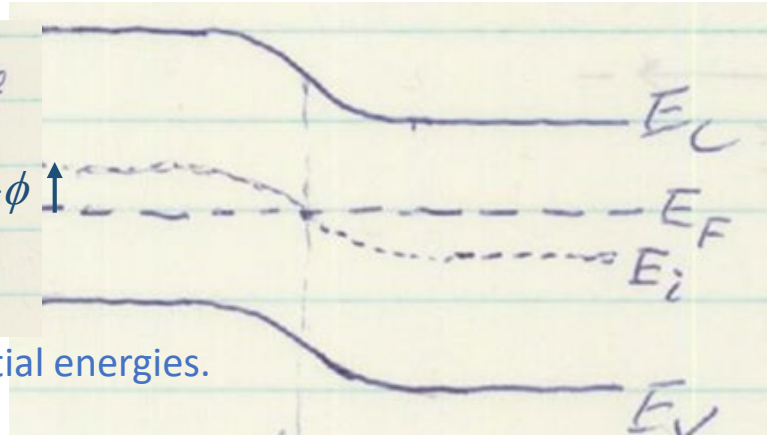
Put into Gauss's law:

Poisson Eq.

$$\frac{1}{q}\frac{d^2E_c}{dx^2} = \frac{q}{\varepsilon_0\varepsilon_r}\,(N_D - N_A + p - n)$$

Let's define some reference for the electric potential.

$$\phi = -\frac{1}{q}(E_i - E_F)$$



Pay attention to signs of electrostatic potentials and potential energies.

Sidenote: the unit of ϕ is V.

The electron (or hole) goes across 1 V. the energy gained or lost is 1 eV.

— a convenient unit.

Power = I V

1 W = 1 A · V

Energy = Q V

1 J = 1 C · V

How to convert eV to J?

$$1\ \text{eV} = 1.6\times10^{-19}\ \text{C}\cdot\text{V}$$
$$= 1.6\times10^{-19}\ \text{J}$$

Now. we have the Poisson Eq as:

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\varepsilon_0 \varepsilon_r}(N_D - N_A + p - n) \quad (1)$$

N_D & N_A are known.

$$n = n_i e^{\frac{E_F - E_i}{k_B T}} = n_i e^{+\frac{q\phi}{k_B T}} \quad (2)$$

$$p = n_i e^{\frac{E_i - E_F}{k_B T}} = n_i e^{-\frac{q\phi}{k_B T}} \quad (3)$$

3 Eq's. 3 unknowns. so we have solutions.

ϕ, n, p.

But no analytical solution!

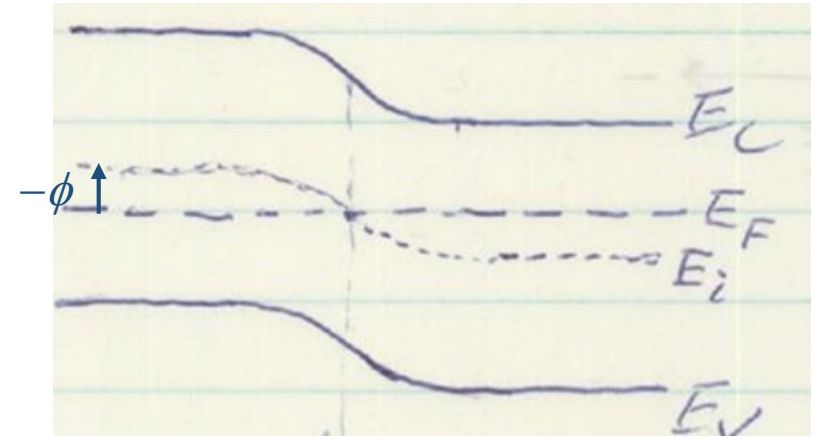
We could solve them numerically, but we like analytical expressions since they provide physical insights.

The depletion approximation

n and p depend on ϕ exponentially.

$$n = n_i e^{\frac{E_F - E_i}{k_B T}} = n_i e^{+\frac{q\phi}{k_B T}}$$

$$p = n_i e^{\frac{E_i - E_F}{k_B T}} = n_i e^{-\frac{q\phi}{k_B T}}$$



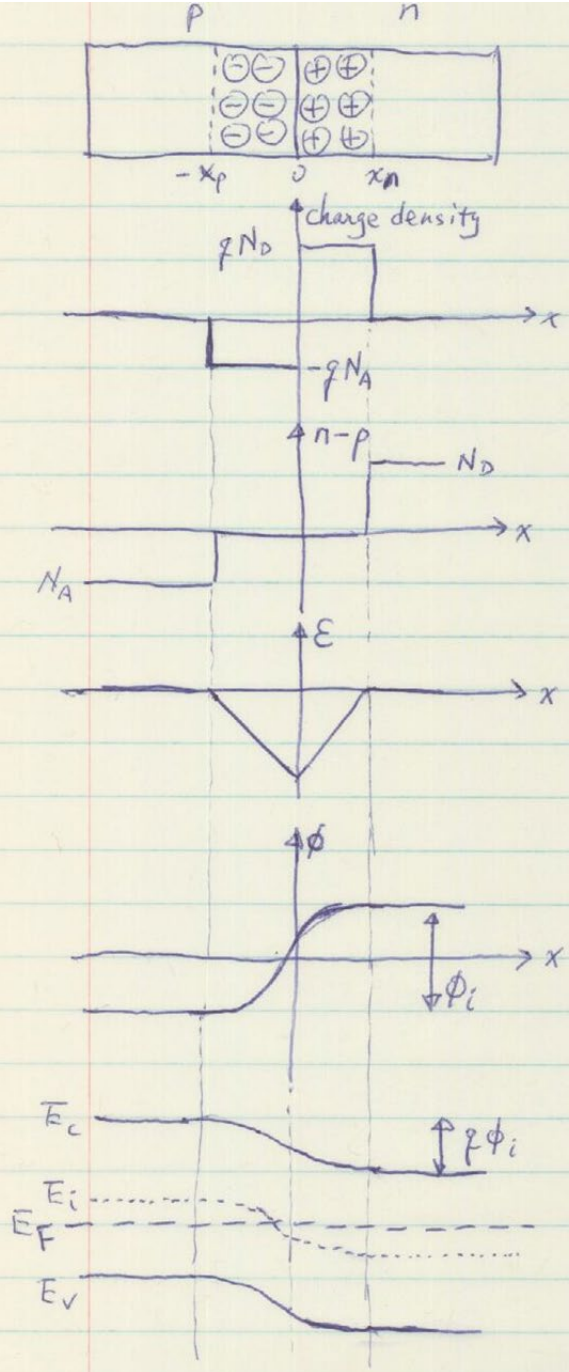
$$\phi = -\frac{1}{q}(E_i - E_F)$$

When ϕ is not too much larger than $k_B T$,
$n \approx 0$ and $p \approx 0$.

$n_i = 1.45 \times 10^{10}/\text{cm}^3$ for Si

$e \approx 2.718$, $e^2 \approx 7.4$, $e^4 \approx 55$, $e^5 \approx 148$

$n = 10^{12}\ /\text{cm}^3 \approx 0$
???



depletion or neutral

On the n side of the depletion region.

$$\frac{d\mathcal{E}}{dx} = \frac{q}{\varepsilon_0 \varepsilon_r} N_D$$

The ionized donors are the only charge.

$\frac{d\mathcal{E}}{dx} = \text{Const}$ what should $\mathcal{E}(x)$ look like?

Boundary condition known.

$\mathcal{E}(x_n) = 0$

$$\therefore \mathcal{E}(x) = -\frac{qN_D}{\varepsilon_r \varepsilon_0}(x - x_d)$$

Similarly, on the p side

$$\mathcal{E}(x) = -\frac{qN_A}{\varepsilon_0 \varepsilon_r}(x + x_p)$$

On the n side,

$$\phi(x) = \phi(x_n) - \int_{x_n}^{x} \mathcal{E}(x)\,dx$$

$$= \phi_n - \frac{qN_D}{2\varepsilon_0 \varepsilon_r}(x - x_n)^2$$

Similarly, on the p side,

$$\phi(x) = \phi_p + \frac{qN_A}{2\varepsilon_0\varepsilon_s}(x + x_p)^2$$

Define $\phi_n = \phi(x_n) = -\frac{1}{q}\left[E_i(x_n) - E_F\right]$

In the n side neutral region

$$N_D = n = n_i e^{\frac{E_F - E_i}{kT}}$$

$$\therefore \phi_n = \frac{kT}{q} \ln \frac{N_D}{n_i}$$

Similarly, on the p side,

$$\phi_p = -\frac{kT}{q} \ln \frac{N_A}{n_i}$$

The built-in potential

$$\phi_i = \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_D N_A}{n_i}$$

at $x=0$, where the n joins the p.

$$\phi(0) = \phi_n - \frac{qN_D}{2\varepsilon_0\varepsilon_r} x_n^2$$

and

$$\phi(0) = \phi_p + \frac{qN_A}{2\varepsilon_0\varepsilon_r} x_p^2$$

$$\therefore \phi_n - \phi_p \equiv \phi_i = \frac{q}{2\varepsilon_0\varepsilon_r}\left(N_D x_n^2 + N_A x_p^2\right)$$

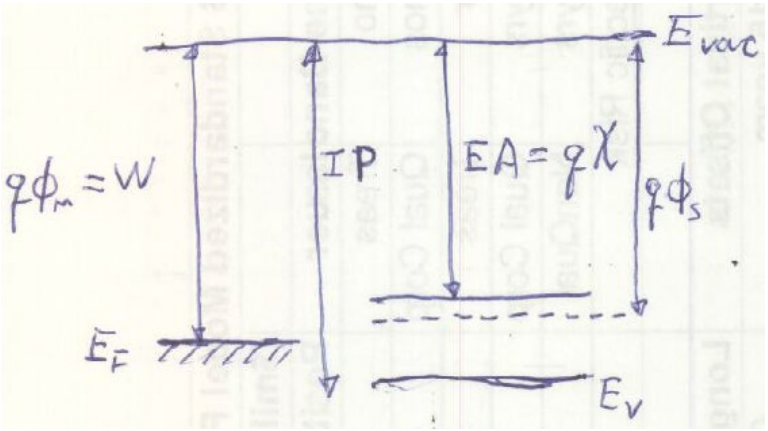
Why?

Recall that $N_D x_n = N_A x_p$ $\Rightarrow$

$$x_n + x_p = \sqrt{2\frac{\varepsilon_0\varepsilon_r}{q}\phi_i\left(\frac{1}{N_D}+\frac{1}{N_A}\right)}$$

It's easy for you to write the expressions for x_n & x_p.

Schottky junctions



metal semiconductor
separated

E_{vac}: vacuum level

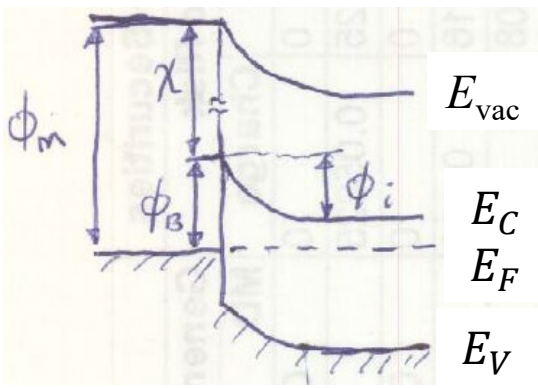
$W = q\phi_m$: work function of metal

W in eV, ϕ_m in V.

IP : ionization potential

$EA = q\chi$: electron affinity

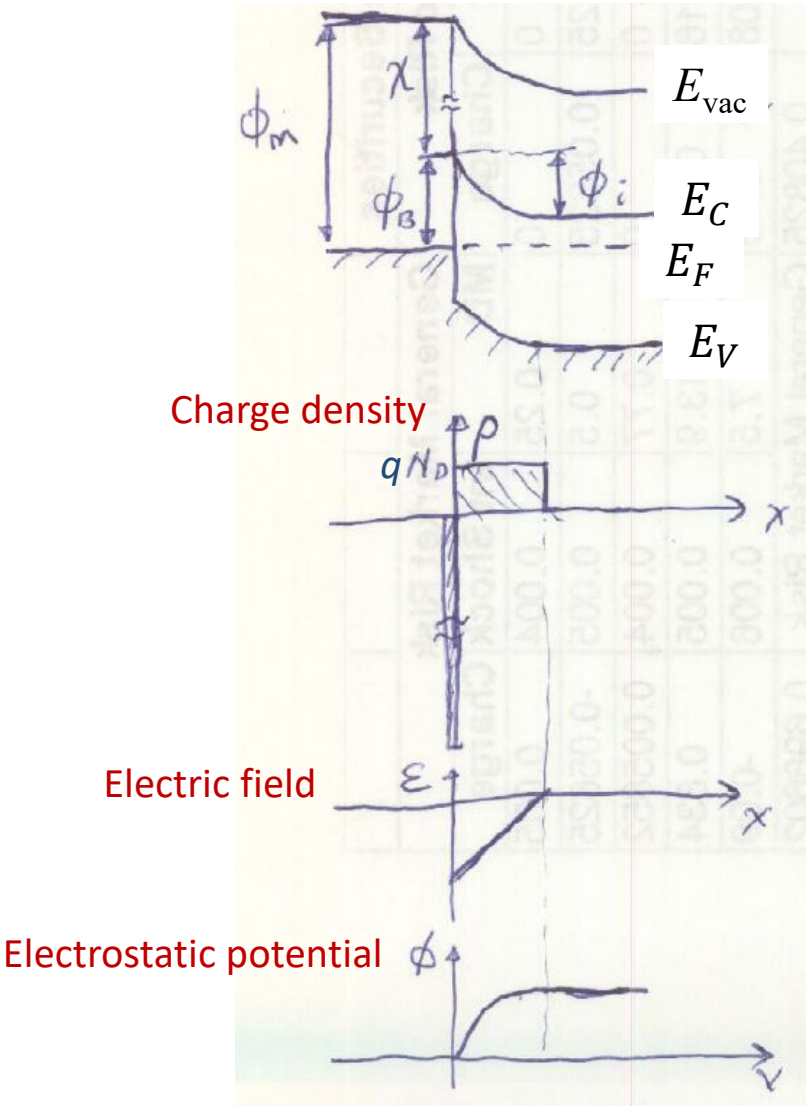
$\phi_m > \phi_s$ (work function of semiconductor)



in contact

Fermi levels line up; depletion approximation.

Fermi levels line up; depletion approximation.



Rectifying Schottky junction: $\phi_M > \phi_S$

Barrier height

$$\phi_B = \phi_M - \chi$$

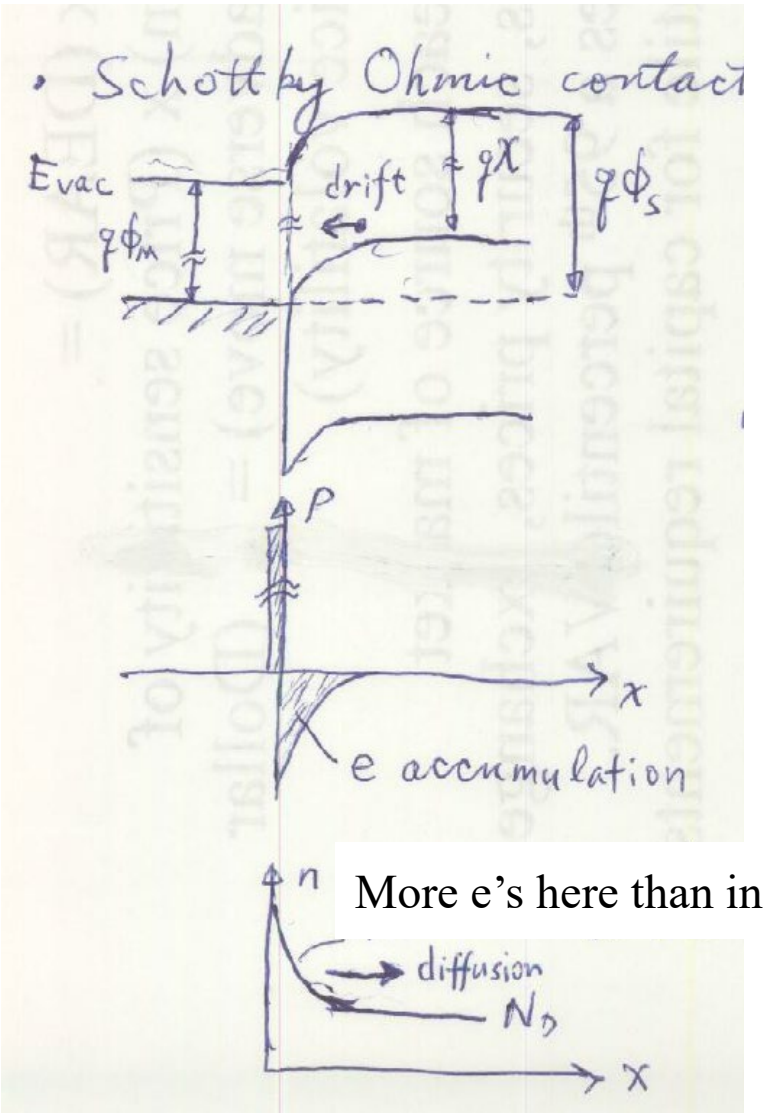
Built-in potential

$$\phi_i = \phi_M - \phi_S = E_{FS} - E_{FM}$$

The depletion region width

$$x_d = \sqrt{\frac{2\phi_i \varepsilon_0 \varepsilon_r}{q N_D}}$$

Schottky ohmic contact: $\phi_M < \phi_s$



More e's here than in bulk

We stopped here on Tue 8/31.
Class canceled for Thu 9/2.
Review this slide set off-line.