Photonic Devices

Light absorption and emission

Transitions between discrete states

Transition rate determined by the two states: Fermi’s golden rule

Absorption and emission of a semiconductor

“Vertical” transition (momentum conservation).
Why? (Why the photon’s momentum is negligible? – See next page)

Can we make decent light emitters (LEDs) based on band-to-band transitions from an indirect gap semiconductor (e.g. Si)?
Can we make decent light detectors (photodiodes) from an indirect gap semiconductor?
Why?
Energy of the photon

\[ E = h \nu = \frac{h}{\lambda} = \frac{hc}{\lambda} = \frac{2\pi hc}{\lambda} \]

\[ E = \frac{1.24 \mu m}{\lambda} \text{ eV} \]

For \( \lambda \sim 1 \mu m \), \( E \sim 1 \text{ eV} \), on the order of \( E_g \) of many semiconductors.

Momentum of the photon

\[ p = \hbar k \]

\[ k = \frac{2\pi}{\lambda} \quad \lambda \sim 1 \mu m \]
Compare the $k$ of a photon to the dimension of the Brillouin zone:

For an indirect gap semiconductor

$$\Delta k_{\text{valley}} \sim \frac{2\pi}{a}, \quad a \sim 5\text{Å}$$

$$k \ll \Delta k_{\text{valley}} \quad k \sim 0.$$

Momentum conservation $\Rightarrow$ vertical transition.

($3$-particle process involving a phonon, small probability)
Strong field in depletion region sweeps electrons to n side, holes to p side. When dark, no photo-generation. $p_n(x_n) < \text{equilibrium value}$ (i.e. excess concentration < 0) because holes swept by increased field.
Forward biased

Like an npn BJT with BC junction forward biased

Forward is not necessarily externally applied. For example, open circuit voltage $V_{oc} > 0$. 

$q(\phi_i - V_a)$

$F_{p}$

$qV_a$

$E_c$

$E_{F,n}$

$E_{F,p}$

$E_n$

$I$

$V$

$V_a$

$\text{dark}$

$\text{dark}$

$\eta_p(x)$

$\rho_n(x)$

$\rho_{no}$
\[ I = I_s \left( e^{\frac{qV}{kT}} - 1 \right) - I_p \]

With large enough negative bias, \( I_p \) proportional to light intensity.
One problem w/ photodiodes.

Only the region \((L_p + x_d + L_n)\) useful.

The PIN photo diode

Is there band bending in the i region? Why?
Here, subscript 0 means equilibrium.

Intraband relaxation time \( \sim \text{ps} \)
Interband recombination time \( \sim \text{ns} \)

The concept of quasi-Fermi levels

Within \((L_n + x_d + L_p)\)

\[np > n_i^2\]

(Not necessarily population inversion)

Where does the recombination primarily happen?
Semiconductor optical amplifiers and lasers

What do you already know about lasers?
(How are lasers different from other light sources?)
For discrete levels

Spontaneous emission: Into all possible photon modes. How LEDs work. Stimulated emission: Duplicate the incoming photon. **Amplification.** Amplification + positive feedback → oscillation

The positive feedback and mode (not just frequency) selection mechanism:

Optical cavity

$$2L = n\lambda = n \frac{c}{\nu}$$

$$\nu = n \frac{c}{2L}$$

Refractive index

Like a state of electrons, a mode of photons is defined by frequency (energy $h\nu = \hbar\omega$), wave vector (momentum $\hbar\mathbf{k} = (h/\lambda)\mathbf{\hat{k}}$), and polarization (spin).
Necessary condition for amplification: population inversion

Absorption

Stimulated emission (photon “cloning”)

\[ E_1, \text{population } N_1 \]

\[ E_2, \text{population } N_2 \]

# of photons of a mode absorbed per unit time per unit volume \( \propto N_1 \).

# of photons of a mode cloned per unit per unit volume \( \propto N_2 \).

Same proportional constant

\[ \Rightarrow \text{ # of photons of a mode gained per unit time per unit volume } \propto (N_2 - N_1). \]

\[ (N_2 - N_1) > 0 \iff \text{net gain: population inversion} \]

Many modes within the transition spectral linewidth are amplified upon population inversion.

A limited # of modes are then selected by the optical cavity.
Population inversion in a semiconductor

\[ E_2 = \frac{\hbar^2 k^2}{2m_n^*} + E_g \]

\[ E_1 = -\frac{\hbar^2 k^2}{2m_p^*} \]

\[ E_2 - E_1 = \hbar \omega = \hbar \nu \]

\[
\begin{align*}
f_n(E_2) &= \frac{1}{e^{\frac{E_2 - E_{F_n}}{k_B T}} + 1} \\ &\approx e^{-\frac{E_2 - E_{F_n}}{k_B T}}
\end{align*}
\]

\[
\begin{align*}
f_p(E_1) &= \frac{1}{e^{\frac{E_1 - E_{F_p}}{k_B T}} + 1} \\ &\approx 1 - e^{-\frac{E_1 - E_{F_p}}{k_B T}}
\end{align*}
\]
\[ E_2 - E_1 = \frac{\hbar^2 k^2}{2m_n^*} + \frac{\hbar^2 k^2}{2m_p^*} + E_g = \hbar \omega = h \nu \]

\[ \frac{\hbar^2}{2} \left( \frac{1}{m_n^*} + \frac{1}{m_p^*} \right) k^2 = h \nu - E_g \]

Let \( \frac{1}{m_r} = \frac{1}{m_n^*} + \frac{1}{m_p^*} \)

\( m_r \): the reduced effective mass of the e-h pair

\[ \frac{\hbar^2 k^2}{2m_r} = h \nu - E_g \quad \iff \quad k^2 = \frac{2m_r}{\hbar^2} (h \nu - E_g) \]

Density of states of conduction band e’s

Joint density of states (a factor of the “proportional constant”)

\[ \rho(\nu) = \frac{(2m_r)^{\frac{3}{2}}}{\pi \hbar^2} (h \nu - E_g)^{\frac{1}{2}} \propto \sqrt{h \nu - E_g} \]
Weak injection

Probability of emission (e at $E_2$, and there’s a hole at $E_2$ for it to fall into):

$$ f_e (\nu) = f_n (E_2) [1 - f_p (E_1)] $$

Probability of absorption:

$$ f_a (\nu) = [1 - f_n (E_2)] f_p (E_1) $$

Probability of net gain in number of photons:

$$ f_e (\nu) - f_a (\nu) = f_n (E_2) - f_p (E_1) $$
Digression: The LED works by spontaneous emission.

Spontaneous emission rate:
(Emissions per time per volume per frequency interval)

\[ r_{sp}(\nu) = \frac{l}{\tau_r} \rho(\nu) f_e(\nu) \]

At equilibrium,

\[ f_e(\nu) = e^{-\frac{E_z-E_F}{k_B T}} e^{-\frac{E_e-E_F}{k_B T}} = e^{-\frac{E_z-E_e}{k_B T}} = e^{-\frac{\hbar \nu}{k_B T}} \]

\[ r_{sp}(\nu) = \frac{l}{\tau_r} \rho(\nu) f_e(\nu) \]

\[ = \frac{l}{\tau_r} \left( \frac{2m_r}{\pi \hbar^2} \right)^{\frac{3}{2}} \sqrt{\hbar \nu - E_g} \cdot e^{-\frac{\hbar \nu}{k_B T}} \]

You can calculate the optical power emitted by a slab of semiconductor.
At room temperature, a 2 micron layer of GaAs emits \(1.5 \times 10^{-20}\) W/cm\(^2\).
You have to either heat it up (incandescence) or “pump” e-h pairs into it (electroluminescence).

This and many other figures are adapted from Saleh & Teich, *Fundamentals of Photonics*. 
We are not interested in making incandescent light bulbs from semiconductors. Let’s look at LEDs.

\[ f_e (\nu) = f_n (E_2) [1 - f_p (E_1)] \]

LED: (weak injection)
\[ f_e (\nu) = e^{-\frac{E_2 - E_{Fn}}{k_B T}} e^{\frac{E_1 - E_{Fp}}{k_B T}} \]
\[ = e^{-\frac{E_{Fn} - E_{Fp}}{k_B T}} e^{-\frac{h\nu}{k_B T}} \]

\[ r_{sp} (\nu) = \frac{1}{\tau_r} \rho (\nu) f_e (\nu) \]
\[ = e^{-\frac{E_{Fn} - E_{Fp}}{k_B T}} \]

Compare to the equilibrium case:
\[ r_{sp0} (\nu) = \frac{1}{\tau_r} \rho (\nu) f_e (\nu) \]
\[ = \frac{1}{\tau_r} \frac{(2m_r)^{\frac{3}{2}}}{\pi \hbar^2} \sqrt{\hbar \nu - E_g} \cdot e^{-\frac{h\nu}{k_B T}} \]
LED weak injection:

\[ r_{sp}(\nu) = \frac{1}{\tau_r} \rho(\nu) f_e(\nu) = e^{\frac{E_{Fn} - E_F}{k_B T}} r_{spo}(\nu) \]

Equilibrium:

\[ r_{spo}(\nu) = \frac{1}{\tau_r} \rho(\nu) f_e(\nu) = \frac{1}{\tau_r} \frac{(2m_r)^{\frac{3}{2}}}{\pi \hbar^2} \sqrt{\hbar \nu - E_g} \cdot e^{-\frac{\hbar \nu}{k_B T}} \]

Peak at \( E_g + \frac{1}{2} k_B T \)

Same peak, same width, enhanced by an exponential factor

Compare with “incandescence”

Photons can escape before getting absorbed. Therefore we consider \( f_e \), instead of net \( f_e - f_a \).

Photons are emitted into all modes.
Now let’s talk about semiconductor optical amplifiers and lasers

Stimulated emission rate: (Emissions per time per volume per frequency interval)

\[ r_{st} (\nu) = \phi_{\nu} \frac{\lambda^2}{8\pi \tau_r} \rho (\nu) f_e (\nu) \]

Absorption rate: (Photons absorbed per time per volume per frequency interval)

\[ r_{ab} (\nu) = \phi_{\nu} \frac{\lambda^2}{8\pi \tau_r} \rho (\nu) f_a (\nu) \]

Incoming photon flux (photons per time per area per frequency interval)

Compare with the spontaneous emission rate:

\[ r_{sp} (\nu) = \frac{1}{\tau_r} \rho (\nu) f_e (\nu) \]

The origin of the \( \lambda^2 \) dependence: density of modes

\[ \mathcal{M} (\nu) = \frac{8\pi \nu^2}{c^3} \]

The photon density of modes is similar to the electron density of states \( D(E) \). The difference: \( E = \hbar^2 k^2/(2m^*) \) for e’s, \( E = h \nu = c\hbar k \) for photons. Simply put, the larger the \( E \), the more \( k \) states in an interval \( dE \).
For spontaneous emission, photons are emitted into all modes.

\[ \gamma_{sp} (\nu) = \frac{1}{\tau_r} \rho (\nu) f_e (\nu) \]

For stimulated emission, photons are emitted into the one mode of the incoming photon.

\[ \gamma_{st} (\nu) = \phi \nu \frac{\lambda^2}{8 \pi \tau_r} \rho (\nu) f_e (\nu) \]

\[ \gamma_{ab} (\nu) = \phi \nu \frac{\lambda^2}{8 \pi \tau_r} \rho (\nu) f_a (\nu) \]

For absorption, the absorbed photon is of a specific mode. Thus the modal density should be in the denominator.

\[ M (\nu) = \frac{8 \pi \nu^2}{c^3} \]

Thus the \( \lambda^2 \) dependence.
A medium absorbs at equilibrium.  \[ I(x) = I(0) e^{-\alpha(x)x} \]

If “pumped”, net gain is possible:  \[ I(x) = I(0) e^{\gamma x} \]

\[ \gamma(\nu) = \frac{\lambda^2}{8\pi \tau_r} \rho(\nu) \left[ f_{e}(\nu) - f_{a}(\nu) \right] \]

\[ = \frac{\lambda^2}{8\pi \tau_r} \rho(\nu) \left[ f_{n}(E_{2}) - f_{p}(E_{1}) \right] \]

\[ = \frac{1}{8\pi} \frac{(2m_{e})^{2}}{\pi h^{2}} \frac{\lambda^2}{\tau_r} \sqrt{(h\nu - E_{g})} \left[ f_{n}(E_{2}) - f_{p}(E_{1}) \right] \]
\[ Y(\nu) = \frac{\lambda^2}{8\pi^2\tau_r} \rho(\nu) [f_e(\nu) - f_a(\nu)] \]
\[ = \frac{\lambda^2}{8\pi\tau_r} \rho(\nu) [f_n(E_x) - f_p(E_i)] \]
\[ = \frac{1}{8\pi} \left( \frac{\hbar}{\pi\hbar^2} \right)^\frac{3}{2} \frac{\lambda^2}{\tau_r} \left[ (\hbar \nu - E_g) \right] [f_n(E_x) - f_p(E_i)] \]

Notations used in figures:
- \( f_g = f_e - f_a \)
- \( f_c(E_x) \equiv f_n(E_x) \)
- \( f_v(E_i) \equiv f_p(E_i) \)
A digression: net absorption at equilibrium

\[ \alpha(\nu) = -\gamma(\nu) \]

\[ \alpha \approx \frac{\lambda^2}{\tau_r} \sqrt{h\nu - E_g} \left[ f(E_2) - f(E_1) \right] \]

\[ \approx \frac{h^2}{\tau_r} \frac{1}{(h\nu)^2} \sqrt{h\nu - E_g} \]

Calculated absorption of GaAs
\[ Y(\nu) = \frac{\lambda^2}{8\pi\tau_r} \, \rho(\nu) \left[ f_e(\nu) - f_a(\nu) \right] \]
\[ = \frac{\lambda^2}{8\pi\tau_r} \, \rho(\nu) \left[ f_n(E_z) - f_p(E_i) \right] \]
\[ = \frac{1}{8\pi} \left( \frac{2m_r}{\pi\hbar^2} \right) \frac{\lambda^2}{\tau_r} \frac{1}{(h\nu - E_g)} \left[ f_n(E_z) - f_p(E_i) \right] \]

Notations used in figures:

- \( f_g = f_e - f_a \)  \( E_{fc} \equiv E_{Fn} \)
- \( f_c(E_z) \equiv f_n(E_z) \)  \( E_{fv} \equiv E_{Fp} \)
- \( f_v(E_i) \equiv f_p(E_i) \)

Net gain: amplification.

To have net gain (for some frequencies), you must have \( E_{Fn} - E_{Fp} > E_g \).

(the "population inversion" in semiconductors)

Amplification with positive feedback: oscillation
Laser diode

(a)

(b)

http://www.star.le.ac.uk/~zrw/courses/lect4313.html
$E_g$

$p$ type

$E_F$

Holes

$n$ type

Fermi level

(a)

$eV_f$

$hv$

(b)

$T = 0$

$E_{fc} \equiv E_{Fn}$

$E_{fv} \equiv E_{FP}$
The positive feedback and frequency selection mechanism:

Optical cavity

\[ 2L = n\lambda = n\frac{c}{\nu} \]

\[ \nu = n\frac{c}{2L} \]

To have oscillation, you must have net gain (of the cavity, not just the medium).

There are losses in the medium other than band-to-band absorption, which is already accounted for in net medium gain \( \gamma(\nu) \), or \( \gamma_0(\nu) \) in the right figure.

Other losses include scattering and free carrier absorption in the medium, as well as transmission at cavity ends (reflection \( R < 1 \)).

You can prorate the transmission loss to the medium:

\[ \gamma(\nu) \geq \alpha_{\text{loss}} + \frac{\ln(R_1 R_2)}{L} \]
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\[ \gamma(\nu) \geq \alpha_{\text{loss}} + \frac{\ln(R_1R_2)}{L} \]

Loss other than band-to-band absorption

\[ \nu = n \frac{c}{2L} \]

The concept of “hole burning”. No spectral hole burning in semiconductors due to fast intra-band processes. But there’s spatial hole burning.
Example: spectrum of an InGaAsP laser
Threshold behavior

Intensity

Current density