Desk Copy

ECE 300
Spring Semester, 2004
HW Set #10

April 20, 2004 **Name** Green

wlg Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. <u>Be sure to show how you got your answers.</u>** Each problem counts 10 points except problem 15.12 counts 20 points.

15.2 $y_{11} = (1/6)$ S, $y_{12} = -(1/12)$ S, y_{21} - -(1/12) S, $y_{22} = (1/6)$ S

15.5 $y_{11} = (1/Z_1)$ S, $y_{22} = 0$, $y_{21} = (\Upsilon/Z_2)$ S, $y_{22} = (1/Z_2)$ S

15.7 $y_{11} = jw(C_1 + C_2)S$, $y_{12} = -jwC_2 S$, $y_{21} = g - jwC_2 S$, $y_{22} = \dfrac{1}{R} + jw(C_2 + C_3)\ S$

15EF-1 $V_1 = 36$ V

15.12 In addition to the Z parameters, also find the H parameters.

$z_{11} = 400$ ohms, $z_{12} = 15$ ohms, $z_{21} = -2\text{x}10^{-6}$ ohms, $z_{22} = 50$ ohms

$h_{11} = 1\ \text{k}\Omega$, $h_{12} = 3e^{-4}$, $h_{21} = 40$, $h_{22} = 20\ \mu$ S: $\Delta H = 8e^{-3}$

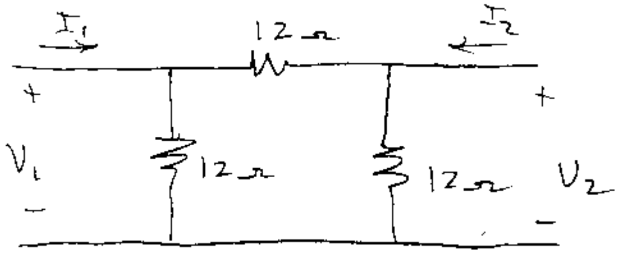
15.18 $$\frac{V_2}{V_s} = \frac{h_{21}R_L}{h_{12}h_{21}R_L - (1 + h_{22}R_L)(R_1 + h_{11})}$$

15.24 $A = \dfrac{-1}{\gamma}$, $B = \dfrac{-Z_2}{\gamma}$, $C = \dfrac{-1}{\gamma Z_1}$, $D = \dfrac{-Z_2}{\gamma Z_1}$

15.27 Answers: No answers given. On your on.

15.39 A = 3, B = j8 Ω, C = 3 – j1 S, D = 3 + j8 vd 7

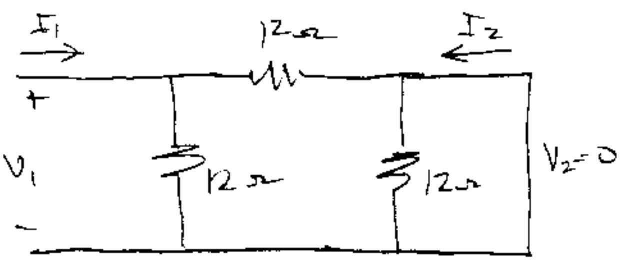
15.2 Find the Y-parameters for the following



$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \qquad y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$



$y_{11} = \dfrac{I_1}{V_1}$, where $V_1 = I_1 \times 6$

$$\boxed{y_{11} = \frac{1}{6}\ S}$$

$y_{21} = \dfrac{I_2}{V_1}$ where $V_1 = -12 I_2$

so

$$\boxed{y_{12} = -\frac{1}{12}\ S}$$

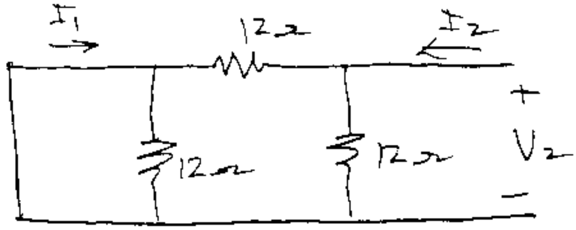
continued

wkg

continue

15.2

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{where } I_1 = \frac{-V_2}{12}$$

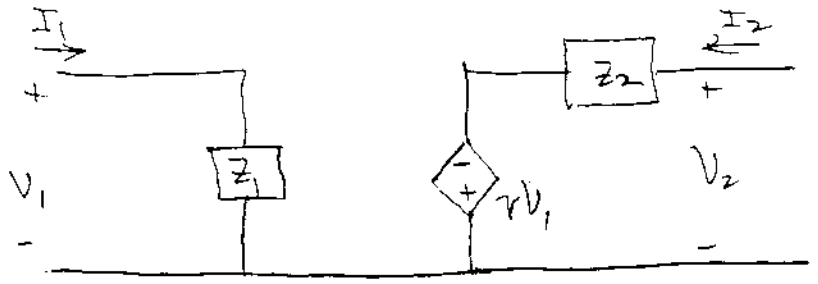
$$\boxed{y_{12} = -\frac{1}{12}\ S}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \quad \text{where } V_2 = I_2 \times 6$$

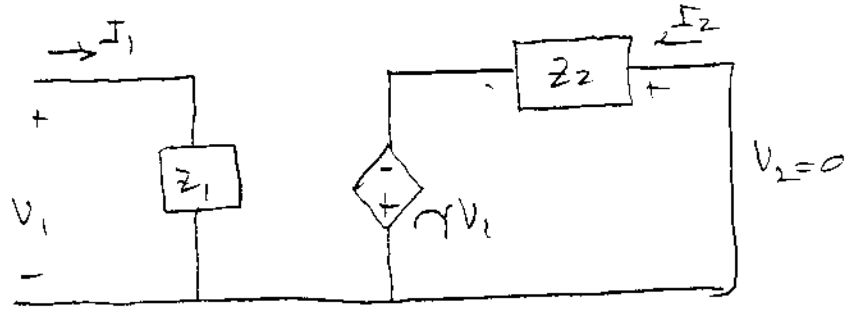
$$y_{22} = \frac{1}{6}\ S$$

$$Y = \begin{bmatrix} \frac{1}{6}S & -\frac{1}{12}S \\ -\frac{1}{12}S & \frac{1}{6}S \end{bmatrix}$$

15.5 Find the Y parameters for the following.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$



$$I_1 = \frac{V_1}{Z_1}; \qquad \frac{I_1}{V_1} = \frac{1}{Z_1}$$

$$\boxed{y_{11} = \frac{1}{Z_1}\ S}$$

$$I_2 Z_2 - \gamma V_1 = 0$$

$$I_2 Z_2 = \gamma V_1$$

$$\frac{I_2}{V_1} = \frac{\gamma}{Z_2}$$

$$\boxed{y_{21} = \frac{\gamma}{Z_2}\ S}$$

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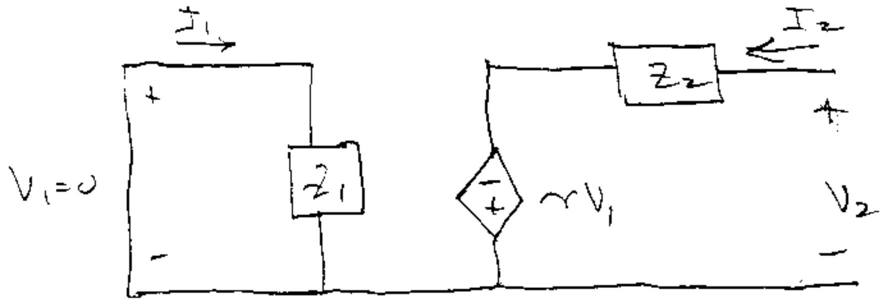
15.5

$$y_{21} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



with $V_1 = 0$, $I_1 = 0$

$$\therefore \quad \frac{I_1}{V_2} = 0$$

$$\boxed{y_{21} = 0}$$

$$-V_2 + I_2 Z_2 - rV_1 = 0$$

$$V_1 = 0$$

$$I_2 Z_2 = V_2$$

$$\frac{I_2}{V_2} = \frac{1}{Z_2}$$

$$y_{22} = \frac{1}{Z_2} \quad S$$

$$Y = \begin{bmatrix} \frac{1}{Z_1} S & 0 \\ \frac{r}{Z_2} S & \frac{1}{Z_2} S \end{bmatrix}$$

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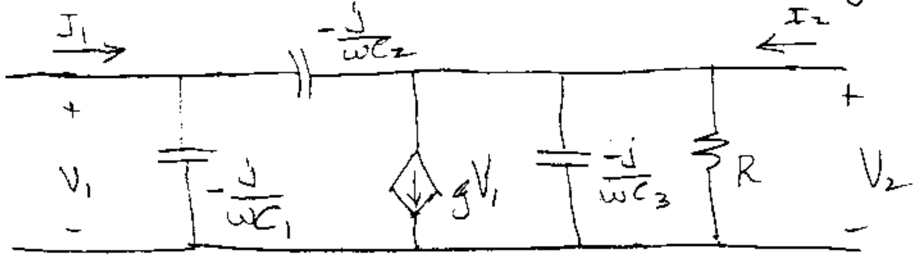
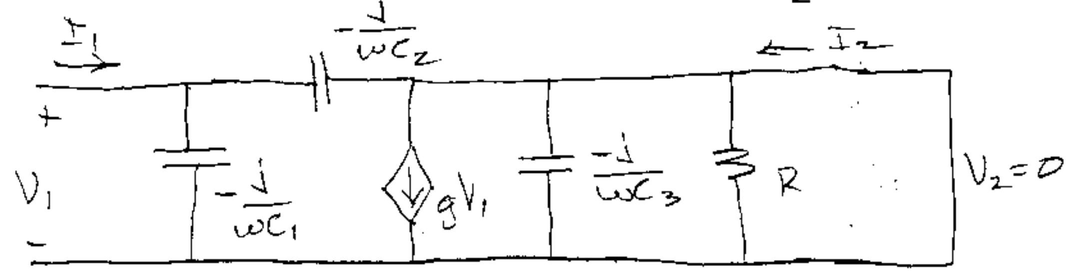
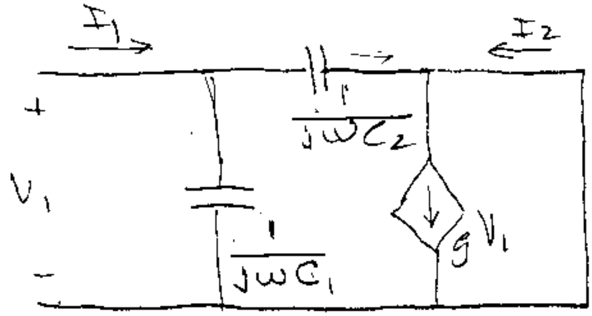
15.7 Find the Y parameters for the following:



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$





$$I_1 = V_1(j\omega C_1 + j\omega C_2)\ ;\ \frac{I_1}{V_1} = j\omega C_1 + j\omega C_2$$

$$\boxed{y_{11} = (j\omega C_1 + j\omega C_2)\ S}$$

$$I_2 + j\omega C_2 V_1 = gV_1$$

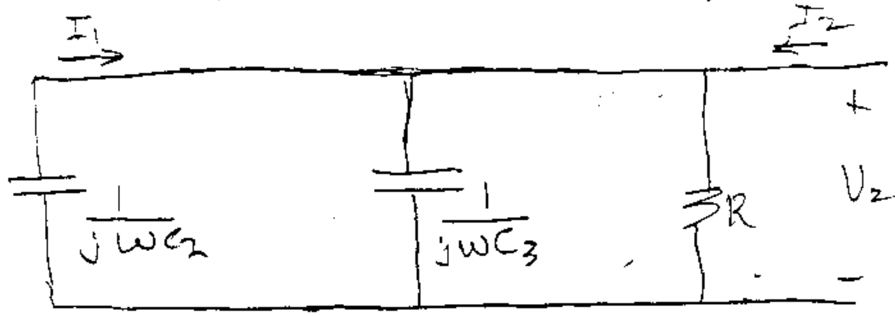
$$\frac{I_2}{V_2} = g - j\omega C_2$$

$$\boxed{y_{21} = (g - j\omega C_2\ S}$$

wkg continued

15.7

$$y_{21} = \frac{I_1}{V_2}\Bigg|_{V_1=0} \qquad y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1=0}$$



$$V_2 = -\frac{I_1}{j\omega C_2} \;; \quad \frac{I_1}{V_2} = -j\omega C_2$$

$$\boxed{y_{21} = -j\omega C_2 \ S}$$

$$I_2 = \frac{V_2}{R} + j\omega C_3 V_2 + j\omega C_2 V_2$$

$$y_{22} = \frac{I_2}{V_2} = R + j\omega C_2 + j\omega C_3$$

$$Y = \begin{bmatrix} (j\omega C_1 + j\omega C_2)S & (-j\omega C_2)S \\ (9 - j\omega C_2)S & (R + j\omega C_2 + j\omega C_3)S \end{bmatrix}$$

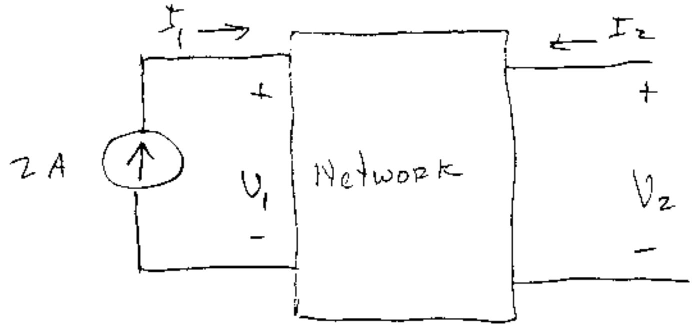
wly

15 EF-1

<u>Given</u>

$$Y = \begin{bmatrix} \frac{1}{14}\,S & -\frac{1}{21}\,S \\ -\frac{1}{21}\,S & \frac{1}{7}\,S \end{bmatrix}$$

A 2A current source is connected to the input terminals as shown below.



Find V_1:

$$2 = \left(\frac{1}{14}\right)V_1 - \frac{1}{21}V_2 \qquad (1)$$

$$0 = -\frac{1}{21}V_1 + \frac{1}{7}V_2 \qquad (2)$$

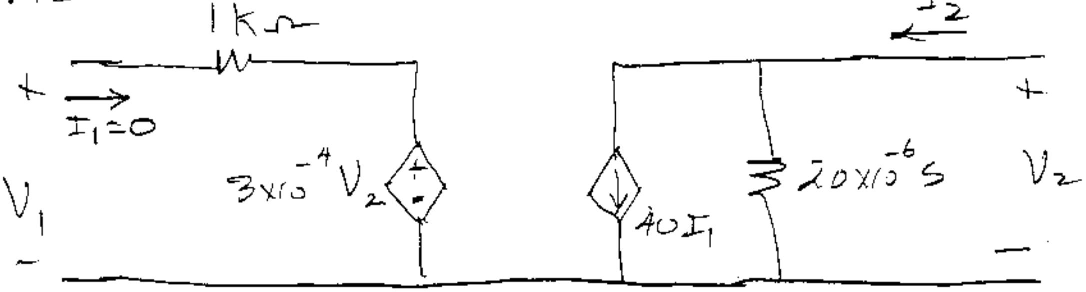
multiply (1) by 3 and add

$$6 = \frac{3}{14}V_1 - \frac{1}{21}V_1$$

$$42 = \frac{3}{2}V_1 - \frac{V_1}{3} = \frac{9V_1 - 2V_1}{6}$$

$$\boxed{V_1 = 36\ V}$$

15.12



$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$V_1 = 3 \times 10^{-4} V_2 \quad \text{but} \quad V_2 = \frac{I_2}{20 \times 10^{-6}}$$

$$V_1 = \frac{3 \times 10^{-4}}{20 \times 10^{-6}} I_2 = 3 \times 10^{-4} \times 50 \times 10^{3} I_2$$

$$\boxed{Z_{12} = 15\ \Omega}$$

with $I_1 = 0$;

$$V_2 = 50 \times 10^{3} \times I_2$$

$$\boxed{Z_{22} = 50 \times 10^{3}\ \Omega}$$

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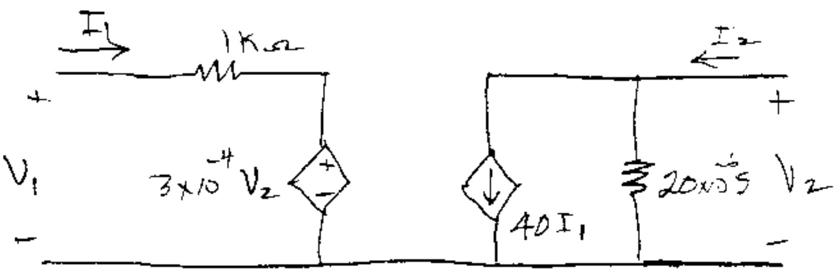
15.12

Given the following circuit. Find the z-parameters.



$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$V_1 = 1\times10^{3}I_1 + 3\times10^{-4}V_2 = 1\times10^{3}I_1 + \frac{3\times10^{-4}(-40)I_1}{20\times10^{-6}}$$

$$V_1 = 400\,I_1 \;;\; \frac{V_1}{I_1} = z_{11}$$

$$\boxed{z_{11} = 400\,\Omega}$$

Now, $I_2 = 0$, so

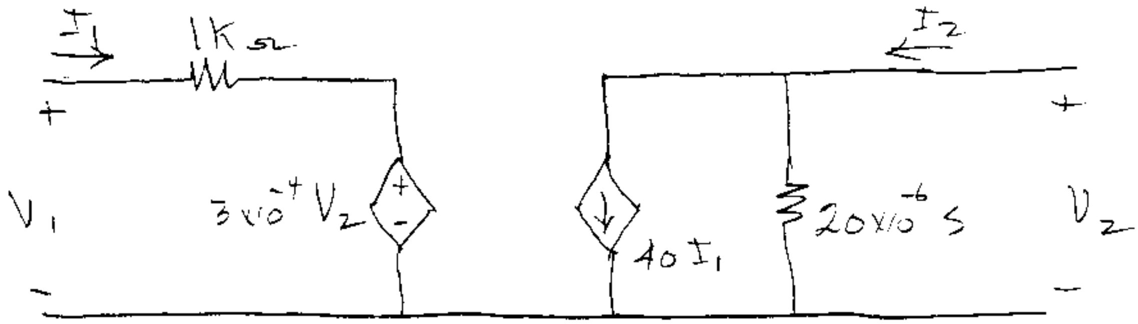
$$V_2 = -40I_1 \times \frac{1}{20\times10^{-6}} = -2\times10^{6}I_1$$

$$z_{21} = -2\times10^{6}\,\Omega \qquad ?$$

continued

15.12

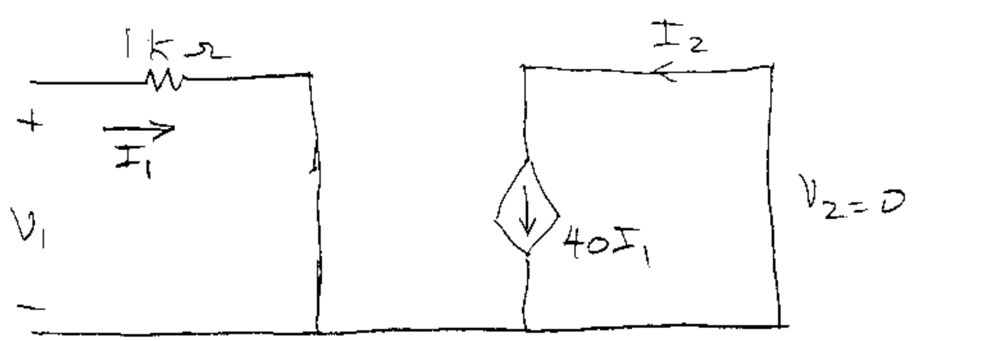
Find the hybrid parameters for the following



$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$



$$\boxed{h_{11} = \frac{V_1}{I_1} = 1000\,\Omega}$$

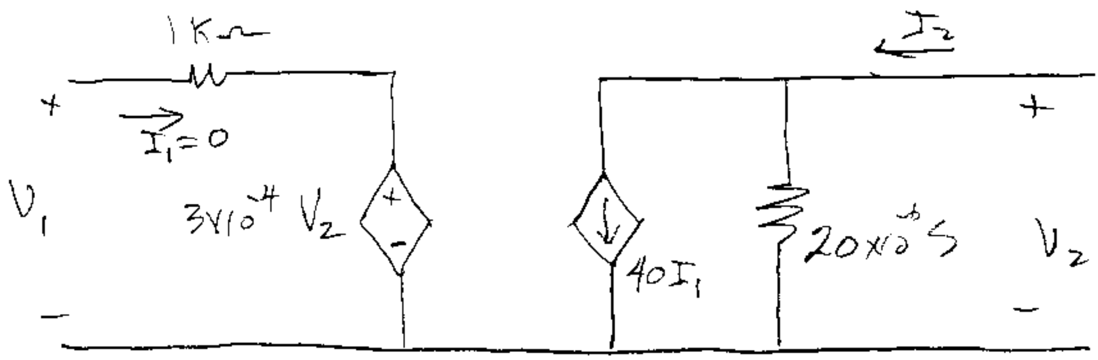
$$I_2 = 40 I_1$$

$$\boxed{h_{21} = 40}$$

CONTINUED

15.12

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



with $I_1 = 0$

$$V_1 = 3\times10^{-4} V_2$$

$$\boxed{h_{12} = \frac{V_1}{V_2} = 3\times10^{-4}}$$

with $I_1 = 0$

$$V_2 = \frac{I_2}{20\times10^{-6}}$$

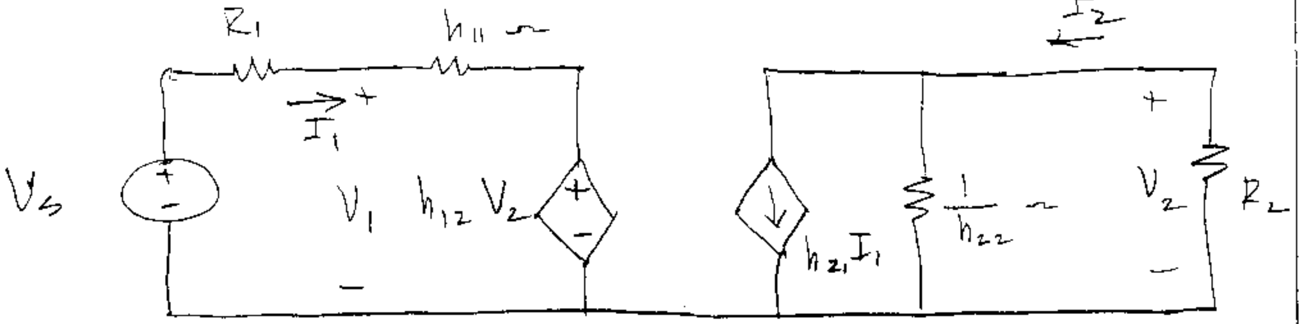
$$\frac{I_2}{V_2} = 20\times10^{-6}\text{ S}$$

$$\boxed{h_{22} = 20\times10^{-6}\text{ S}}$$

$$H = \begin{bmatrix} 1\text{ K}\Omega & 3\times10^{-4} \\ 40 & 20\times10^{-6}\text{ S} \end{bmatrix}$$

5.18

Given



We know

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

We know:

$$V_1 = V_s - I_1 R_1$$

$$V_2 = -R_L I_2 \longrightarrow I_2 = -\frac{V_2}{R_L}$$

Determine V_2/V_s

$$V_s - I_1 R_1 = h_{11} I_1 + h_{12} V_2$$

$$\boxed{V_s = (R_1 + h_{11}) I_1 + h_{12} V_2}$$

and

$$-\frac{V_2}{R_L} = h_{21} I_1 + h_{22} V_2$$

$$\boxed{0 = h_{21} I_1 + \left(h_{22} + \frac{1}{R_L}\right) V_2}$$

5.18 continued

$$\begin{array}{cc} I_1 & V_2 \\ \left[\begin{array}{cc} R_1 + h_{11} & h_{12} \\ h_{21} & \dfrac{1 + h_{22}R_L}{R_L} \end{array}\right] & \end{array} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}$$

By Cramer's rule:

$$V_2 = \frac{\begin{vmatrix} R_1 + h_{11} & V_s \\ h_{21} & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + h_{11} & h_{12} \\ h_{21} & \dfrac{1 + h_{22}R_L}{R_L} \end{vmatrix}}$$

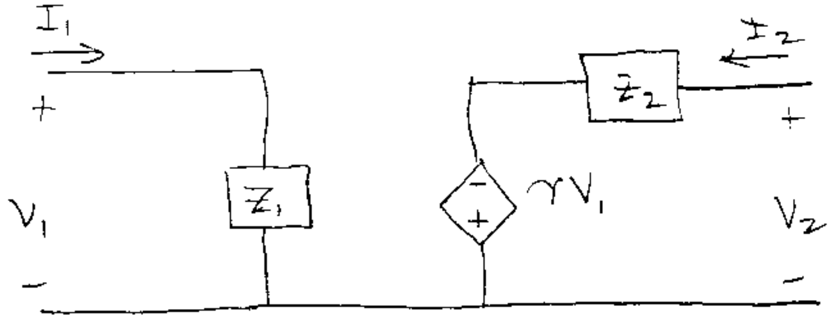
$$V_2 = \frac{-h_{21}V_s}{\frac{1}{R_L}(R_1 + h_{11})(1 + h_{22}R_L) - h_{12}h_{21}}$$

$$V_2 = \frac{h_{21}R_L V_s}{h_{12}h_{21} - (1 + h_{22}R_L)(R_1 + h_{11})}$$

$$\boxed{\frac{V_2}{V_s} = \frac{h_{21}R_L}{h_{12}h_{21} - (1 + h_{22}R_L)(R_1 + h_{11})}}$$

Wlg

15.24 Find the A, B, C, D parameters for the following



$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

with $I_2 = 0$

$$V_2 = -rV_1 \; ; \quad \frac{V_1}{V_2} = -\frac{1}{r}$$

$$\therefore \boxed{A = -\frac{1}{r}}$$

\#

with $I_2 = 0$

$$V_2 = -rV_1 = -r(I_1 Z_1)$$

so

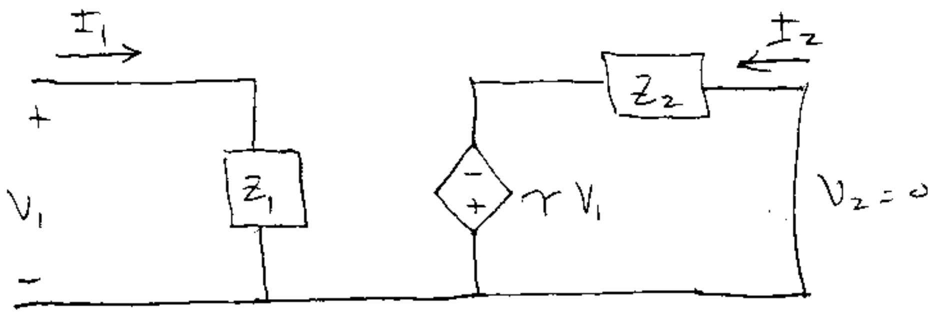
$$\frac{I_1}{V_2} = -\frac{1}{r Z_1}$$

$$\boxed{C = -\frac{1}{r Z_1}}$$

continued

15.24

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$



with $V_2 = 0$

$$Z_2 I_2 - r V_1 = 0$$

$$\frac{V_1}{-I_2} = \frac{-Z_2}{r}$$

$$B = -\frac{Z_2}{r}$$

Now

$$V_1 = \frac{Z_2}{r} I_2$$

but $V_1 = I_1 Z_1$ so

$$I_1 Z_1 = \frac{Z_2}{r} I_2$$

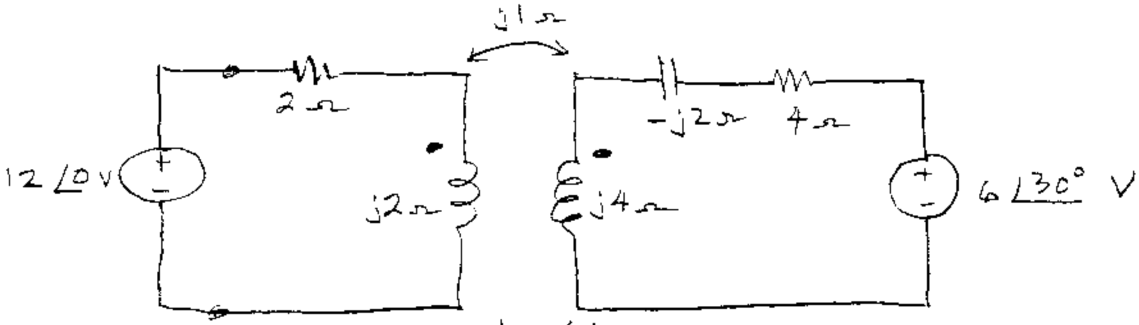
$$D = \frac{I_1}{-I_2} = -\frac{Z_2}{Z_1 r}$$

$$\begin{bmatrix} -\frac{1}{r} & -\frac{Z_2}{r} \\ -\frac{1}{\ } & -\frac{Z_2}{\ } \end{bmatrix}$$

wlg

15.27

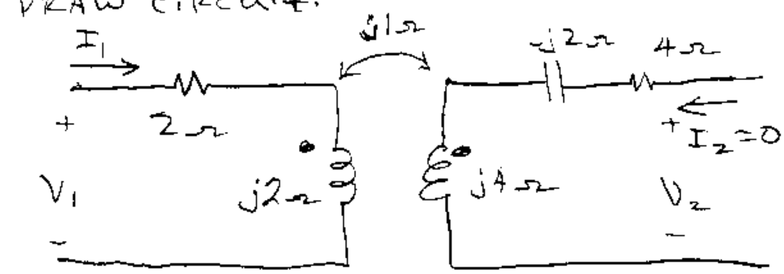
Given the network below. Find the A, B, C, D parameters and then find I_o using terminal conditions.



ckt #1

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

DRAW circuit:



ckt #2

$$I_1 = \frac{V_1}{2+j2} \qquad (1)$$

and

$$V_2 = (j1)\,I_1 \qquad (2)$$

substitute (1) into (2)

$$V_2 = \frac{(j1)\,V_1}{2+j2} \qquad (3)$$

$$\frac{V_1}{V_2} = \frac{2+j2}{j} = 2-j2$$

$$\boxed{A = 2 - j2}$$

15.27 continuing from circuit 2

$$C = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

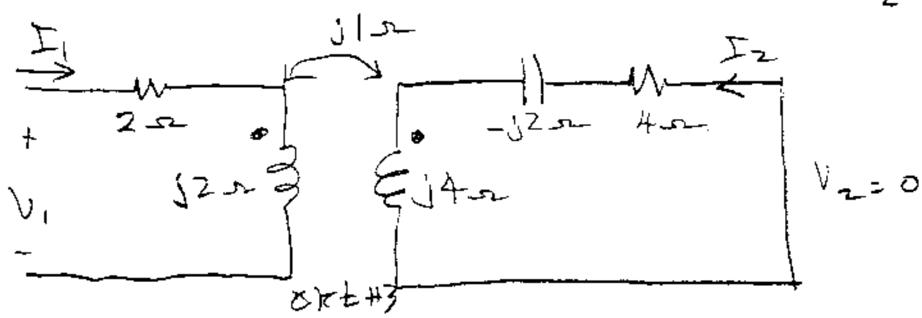
Now

$$V_2 = (j1)\, I_1 \qquad \text{(induced)} \quad I_2 = 0$$

$$\frac{I_1}{V_2} = \frac{1}{j} = -j\ S$$

$$\boxed{C = -j1\ S}$$

$$B = \frac{V_1}{-I_2}\bigg|_{V_2=0} \qquad D = \frac{I_1}{-I_2}\bigg|_{V_2=0}$$



loop 1 $\quad V_1 = (2+j2)I_1 + (j1)\, I_2 \qquad (4)$

loop 2 $\quad (4+j2)\, I_2 + (j1)\, I_1 = 0 \qquad (5)$

or

$$I_1 = -\frac{(4+j2)}{j} I_2 = (-2+j4)\, I_2 \qquad (6)$$

substitute (6) into (4)

15.27

$$V_1 = (2+j2)(-2+j4)\,I_2 + (j1)\,I_2$$

$$V_1 = (-12+j4)\,I_2 + j\,I_2$$

$$V_1 = -12+j5)\,I_2$$

$$\frac{V_1}{I_2} = -12+j5$$

$$\frac{V_1}{-I_2} = (+12-j5)\ S$$

$$\boxed{B = (+12-j5)\ S}$$

\#

$$D = \frac{I_1}{-I_2}$$

From (6)

$$\frac{I_1}{I_2} = -2+j4$$

so

$$\frac{I_1}{-I_2} = \boxed{(+2-j4) = D}$$

$$\begin{bmatrix} 2-j2 & +12-j5\ S \\ -j1\ S & +2-j4 \end{bmatrix} \quad (7)$$

continued

15.27 continued

Attach $V_1 = 12\angle 0\ V$

$V_2 = 6\angle 30^\circ\ V$

$I_o = -I_2$

solve for I_o

$$V_1 = (2-j2)V_2 - (12-j5)I_2$$

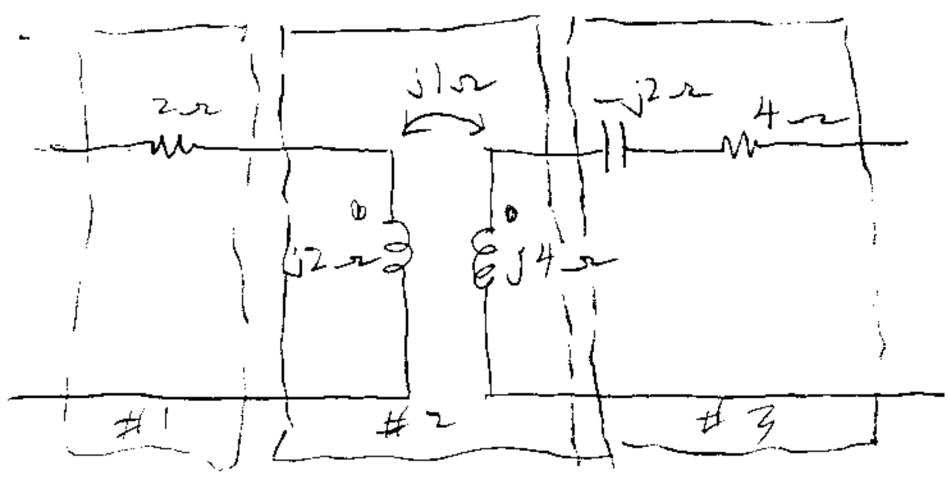
$$12\angle 0 = (2-j2)6\angle 30 + (12-j5)I_o$$

$$I_o = \frac{\left(12 - (2-j2)(6\angle 30)\right)}{(12-j5)}$$

$$\boxed{\begin{aligned} I_o &= 0.48\angle 157.6\ A \\ I_o &= -0.48\angle -22.4\ A \end{aligned}}$$

As an alternate solution,

break the network into 3 parts

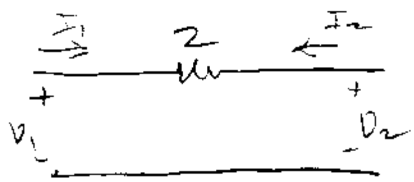


15.27 continued

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

#1

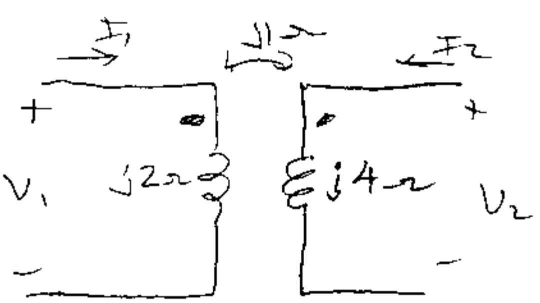


$$A_1 = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \; ; \quad C_1 = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0; \; B_1 = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{-2I_2}{-I_2} = 2$$

$$D_1 = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{-I_2}{-I_2} = 1$$

#1
$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

#



$$A_2 = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{j2I_1}{j1I_1} = 2$$

$$C_2 = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{jI_1} = -j\ S$$

15.27 continued

$$B_2 = \frac{V_1}{-I_2}\Big|_{V_2=0}$$

$$j4I_2 + j1I_1 = 0 \qquad (\text{loop } 2)$$

$$I_1 = \frac{-j4I_2}{j} = -4I_2$$

$$V_1 = j2I_1 + jI_2 \qquad (\text{loop } 1)$$

$$V_1 = -j8I_2 + jI_2 = -j7I_2$$

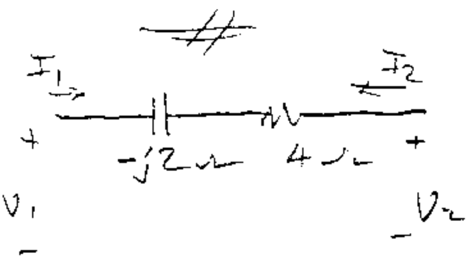
$$\frac{V_1}{I_2} = -j7$$

$$\frac{V_1}{-I_2} = \boxed{j7 = B_2}$$

$$D = \frac{I_1}{-I_2}\Big|_{V_2=0} = \frac{-4I_2}{-I_2} = 4$$

#2
$$\begin{bmatrix} 2 & j7\Omega \\ -j15 & 4 \end{bmatrix}$$

#3



$$A_3 = \frac{V_1}{V_2}\Big|_{I_2=0} = 1, \qquad C_3 = \frac{I_1}{V_2}\Big|_{I_2=0} = 0$$

$$B_3 = \frac{V_1}{-I_2}\Big|_{V_2=0} = \frac{-(4-j2)I_2}{I_2} = 4-j2\ \Omega$$

$$D_3 = \frac{I_1}{-I_2}\Big|_{V_2=0} = \frac{-I_2}{-I_2} = 1$$

#3

$$\begin{bmatrix} 1 & 4-j2\ \Omega \\ 0 & 1 \end{bmatrix}$$

(#1) (#2) (#3)

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & j7 \\ -j & 4 \end{bmatrix}\begin{bmatrix} 1 & 4-j2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2-j2, & 8+j7 \\ -j & 4 \end{bmatrix}\begin{bmatrix} 1 & 4-j2 \\ 0 & 1 \end{bmatrix}$$

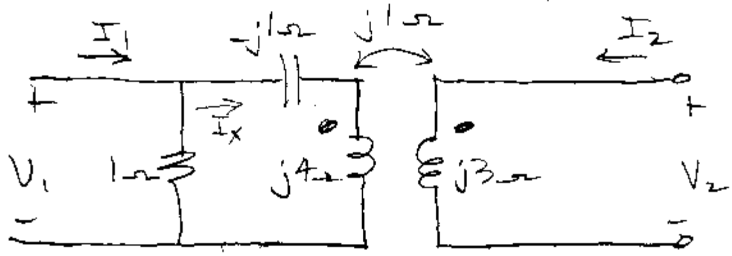
$$\begin{bmatrix} 2-j2 & 12-j5 \\ -j & 2-j4 \end{bmatrix}$$

same as (7)

Proceed as before to solve for I_0

15.39

Find the transmission parameters for the following.



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

with $V_2 = 0$

$$V_2 = (j1)I_x = j1\left(\frac{V_1}{j3}\right) = \frac{V_1}{3}$$

so

$$\boxed{\frac{V_1}{V_2} = 3 = A}$$

#

From above diagram

$$I_x = \frac{I_1 \times 1}{1+j3}$$

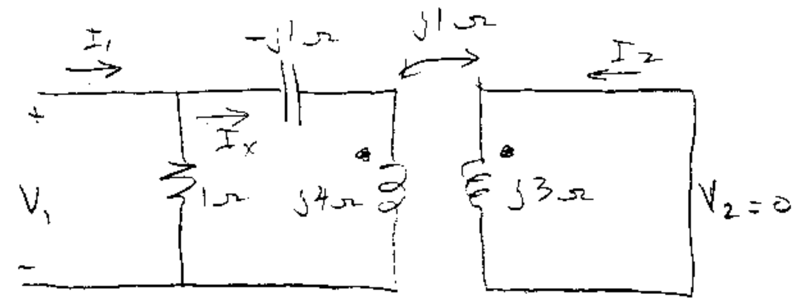
$$V_2 = j1(I_x) = \frac{(j1)I_1}{1+j3}$$

$$\boxed{\frac{I_1}{V_2} = \frac{1+j3}{j} = 3 - j1 = C}$$

continued

15.39

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$



$$(j3)I_2 + (j1)I_x = 0 \qquad (1)$$

but

$$V_1 = (-j1)I_x + (j4)I_x + jI_2 \qquad (1A)$$

$$I_x = \frac{V_1 - jI_2}{j3} \qquad (2)$$

(2) into (1)

$$j3I_2 + (j1)\left[\frac{V_1 - jI_2}{j3}\right] = 0$$

or

$$j9I_2 + V_1 - jI_2 = 0$$

$$V_1 = -j8I_2$$

$$\frac{V_1}{-I_2} = \boxed{j8\Omega = B}$$

continued

15.39

From (1A)

$$V_1 = jI_2 + j3I_x \qquad (3)$$

and

$$V_1 = (I_1 - I_x)\,1 \qquad (4)$$

Equate (3) to (4)

$$I_1 - I_x = jI_2 + j3I_x$$

$$I_x = \frac{I_1 - jI_2}{1+j3} \qquad (5)$$

Substitute (5) into (1)

$$j3I_2 + (j1)\left(\frac{I_1 - jI_2}{1+j3}\right) = 0$$

$$j3(1+j3)I_2 + jI_1 + I_2 = 0$$

$$(-8+j3)\,I_2 = -jI_1$$

$$\frac{I_1}{I_2} = \frac{(-8+j3)}{-j} = -3 - j8$$

$$\frac{I_1}{-I_2} = 3+j8$$

$$D = \frac{I_1}{-I_2} = 3+j8$$

$$\begin{bmatrix} 3 & j8\,\Omega \\ 3-j1 & 3+j8 \end{bmatrix}$$