ECE 300
Spring Semester, 2004
HW Set #5

February 26, 2004 **Name** **Dr.Green**
wlg

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations**. **<u>Be sure to show how you got your answers.</u>** Each problem counts 5 points.

5.18

$$v(t) = \begin{cases} 50t \text{ V} & 0 <= t <= 1 \text{ ms} \\ -50t + 0.1 \text{ V} & 1 \text{ ms} <= t <= 2 \text{ ms} \\ 50t - 0.1 \text{ V} & 2 \text{ ms} <= t <= 3 \text{ ms} \\ -50t + 0.2 \text{ V} & 3 \text{ ms} <= t <= 4 \text{ ms} \\ 50t - 0.2 \text{ V} & 4 \text{ ms} <= t <= 6 \text{ ms} \\ 50 \text{ mV} & t >= 5 \text{ ms} \end{cases}$$

5.32 $i(t) = 500t^2$ A $0 <= t <= 1$ ms

$i(t) = 2t - 500t^2 - 10^{-3}$ A 1ms <= t <= 2 ms

$i(t) = 1$ ma $t > 2$ ms

5.45 $V_1 = 4$ V, $V_2 = 8$ V

5.58 L = 4 mH

5FE-3 W = 3.84 mJ

6.4 $i_0(t) = 2.67e^{-t/0.6}$ mA $v_c(t) = 5.33e^{-t/0.6}$ V $t > 0$

6.7 $v_o(t) = (48/11)(1 - e^{-11t/6})$ V $t > 0$; $v_o(t) = 0$ $t < 0$

6.23 $i_o(t) = -1.2$ A $t < 0$ :::::: $i_o(t) = 2 - 3.2e^{-3t}$ A $t > 0$

6.29 $i_o(t) = 2 + 0.5e^{-3.75t}$ mA

6.35 $v_o(t) = -6e^{-4t}$ V $t > 0$

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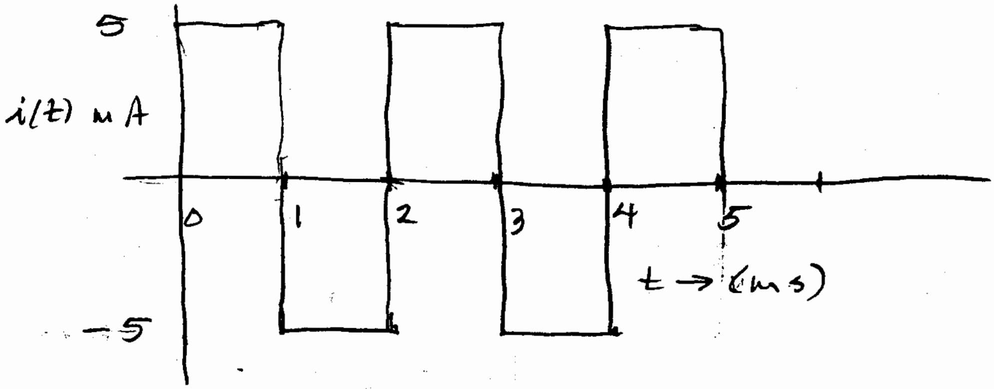
5.18 Waveform for current in a 100μF initially uncharged capacitor is shown below. Determine the waveform for the capacitor voltage.



$\boxed{0 \le t \le 1 \text{ ms}}$

$$i(t) = C\frac{dv}{dt}$$

$$\int_{\tau=0}^{\tau=t} dv(\tau) = \frac{1}{C}\int_{\tau=0}^{\tau=t} i(\tau)\,d\tau \qquad (1)$$

$$v(t) - v(0) = \frac{5\times10^{-3}}{0.1\times10^{-3}}\int_{\tau=0}^{\tau=t} d\tau = 50\tau\Big|_{\tau=0}^{\tau=t} = 50t$$

but $v(0) = 0$ so

$$\boxed{v(t) = 50t \qquad 0 \le t \le 1\text{ms}} \qquad (2).$$

Now $v(1\text{ ms}) = 50\times1\times10^{-3} = 0.05\text{V}$ (3)

$\boxed{1\text{ ms} \le t \le 2\text{ms}}$

$$\int_{\tau=1\text{ms}}^{\tau=t} dv(\tau) = 50\int_{\tau=1\text{msec}}^{\tau=t} d\tau \qquad (4)$$

3.18 cont.

From (4)

$$v(t) - v(1\,ms) = -50\,\tau \Big|_{\tau=1\,ms}^{\tau=t} = -50(t - 1\,msec)$$

$$v(t) - 0.05 = -50t + 0.05$$

$$\boxed{v(t) = -50t + 0.1} \qquad 1\,ms \le t \le 2\,ms \qquad (5)$$

Note:

$$V(2\,ms) = -50 \times 2 \times 10^{-3} + 0.1 = -0.1 + 0.1 = 0$$

$\boxed{V(2\,ms) = 0\ V}$ use in next eq.

$2\,ms \le t \le 3\,ms$

$$\int_{\tau=2\,ms}^{\tau=t} dv(\tau) = +50 \int_{\tau=2\,ms}^{\tau=t} d\tau = 50\,\tau \Big|_{\tau=2\,ms}^{\tau=t} = 50(t - 2\,ms)$$

$$v(\tau) - v(2m) = 50(t - 2\,ms)$$

$$V(t) = 50t - 0.1 + 0$$

$$\boxed{v(t) = 50t - 0.1} \qquad 2\,ms \le t \le 3\,ms \qquad (6)$$

note:

$$v(3\,ms) = 50 \times 3 \times 10^{-3} - 0.1 = 0.15 - 0.1$$

$$V(3\,ms) = 0.05\,V \quad \text{use next}$$

$\boxed{3\,ms \le t \le 4\,ms}$

$$\int_{\tau=3\,ms}^{\tau=t} dv(\tau) = -50 \int_{\tau=3\,ms}^{\tau=t} d\tau$$

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5.18 continued

From previous equation:

$$v(t) - v(3\,ms) = -50\tau\Big|_{\tau=3ms}^{\tau=t} = -50(t-3\,ms)$$

$$v(t) - 0.05 = -50t + 0.15$$

$$\boxed{v(t) = -50t + 0.2} \qquad 3ms \le t \le 4ms \qquad (7)$$

$$v(4ms) = -50\times 4ms + 0.2 = -0.2 + 0.2 = 0$$

$$\boxed{v(4ms) = 0}$$ use next

$4ms \le t \le 5ms$

$$\int_{\tau=4ms}^{\tau=t} dv(\tau) = 50\int_{\tau=4ms}^{\tau=t} d\tau$$

$$v(t) - v(4ms) = 50\tau\Big|_{\tau=4ms}^{\tau=t} = 50(t-4ms)$$

$$v(t) = 50t - 0.2 + 0$$

$$\boxed{v(t) = 50t - 0.2} \qquad 4ms \le t \le 5ms \qquad (8)$$

$$v(5m) = 50\times 5ms - 0.2 = 0.25 - 0.2 = 0.05V$$

$$\boxed{v(5ms) = 50\,mV \qquad 5ms \le t} \qquad (9)$$

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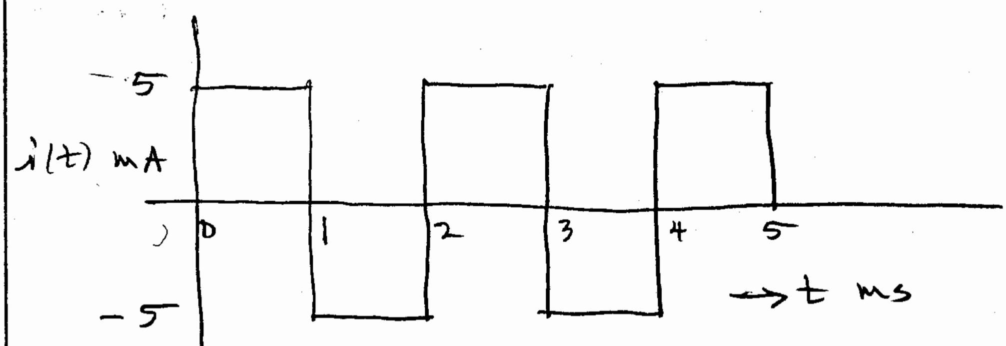
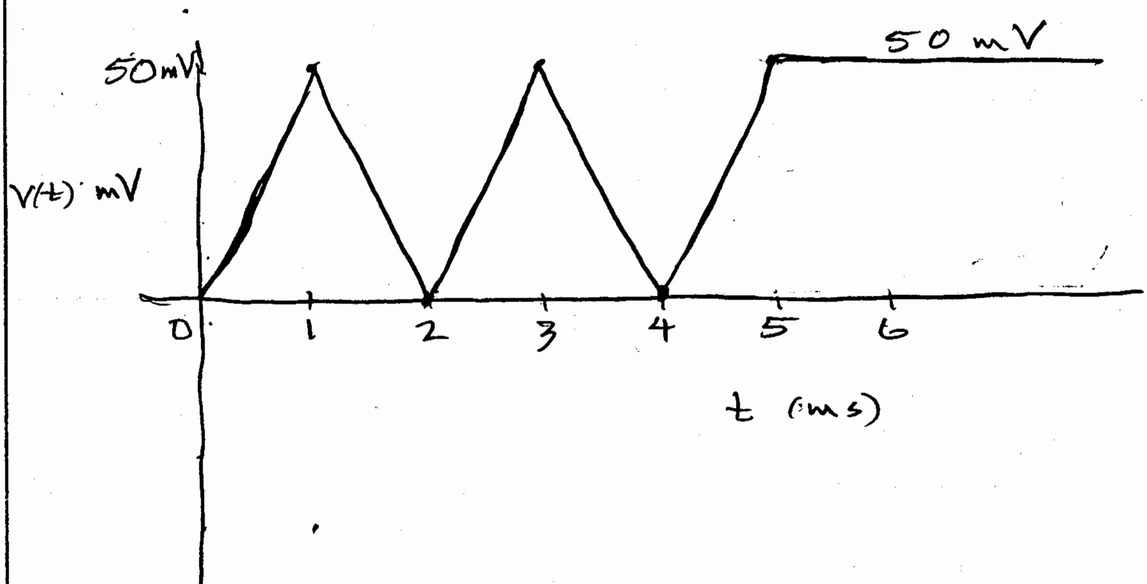
5.18 continued

Summary from (2), (5), (6), (7), (8) and (9)

$$v(t) = \begin{cases} 50t \quad V & 0\,ms \le t \le 1\,ms \\ -50t + 0.1 \quad V & 1\,ms \le t \le 2\,ms \\ 50t - 0.1 \quad V & 2\,ms \le t \le 3\,ms \\ -50t + 0.2 \quad V & 3\,ms \le t \le 4\,ms \\ 50t - 0.2 \quad V & 4\,ms \le t \le 5\,ms \\ 50\,mV & t \ge 5\,ms\,\infty \end{cases}$$





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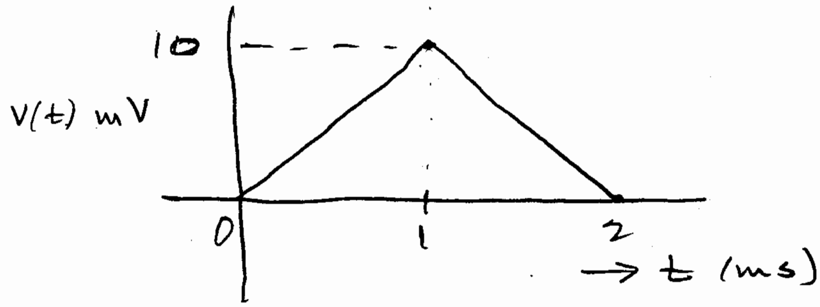
5.32 The voltage across a 10 mH inductor is shown below. Determine the waveform for the inductor current. $V(t)=0,\ t<0$



For $0 \le t \le 1$ msec

$$V_L(t) = L\frac{di(t)}{dt}$$

$$\int_{\tau=0}^{\tau=t} di(\tau) = \frac{1}{L}\int_{\tau=0}^{\tau=t} V(\tau)\,d\tau \qquad L = 10\,mH$$

over this range $V(\tau) = 10t$

so, integrating we have

$$i(t) - i(0) = \frac{10}{.01}\int_{\tau=0}^{\tau=t} \tau\,d\tau = \frac{1000}{2}\tau^2\Big|_{\tau=0}^{\tau=t}$$

$i(0)=0$

$$\boxed{i(t) = 500t^2 \qquad 0 \le t \le 1\ ms} \quad (1)$$

Note from above

$$i(1\,ms) = 500\,(1\times10^{-3})^2 = 500\times1\times10^{-6}$$

$$i(1\,msec) = 0.5\ mA \qquad (2)$$

use in next part.

5.32 continued

$\boxed{1ms \le t \le 2ms}$

$$\int_{\tau=1ms}^{\tau=t} di(\tau) = \frac{1}{L}\int_{\tau=1ms}^{\tau=t} v(\tau)\,d\tau \qquad (3)$$

$L = 0.01$

$$v(\tau) = -10(\tau - 2ms) \qquad (4)$$

substitute (4) into 3 and integrate

$$i(t) - i(1ms) = -1000\int_{\tau=1ms}^{\tau=t}(\tau - 2ms)\,d\tau$$

remember $i(1ms) = 0.5\times10^{-3}A$

$$= -1000\left[\frac{\tau^2}{2} - 2ms\,\tau\right]\Big|_{\tau=1ms}^{\tau=t}$$

$$= -500(t^2 + 1\times10^{-6}) + 2\tau\Big|_{1ms}^{\tau=t}$$

$$i(t) - i(1ms) = -500t^2 + .5\times10^{-3} + 2t - 2\times10^{-3}$$

$$i(t) = -500t^2 + 2t + 0.5\times10^{-3} - 2\times10^{-3} + .5\times10^{-3}$$

$$\boxed{i(t) = (-500t^2 + 2t - 1\times10^{-3})A \qquad 1ms \le t < 2ms}$$

$$i(2ms) = -500\times(2\times10^{-3})^2 + 2\times2\times10^{-3} - 1\times10^{-3}$$

$$i(2ms) = -2\times10^{-3} + 4\times10^{-3} - 1\times10^{-3}$$

$$\boxed{i(2ms) = 1\times10^{-3}A \qquad 2ms \le t \le \infty}$$

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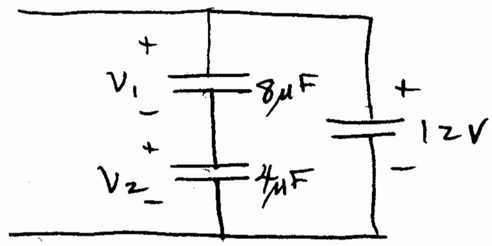
5.45 The capacitors shown below have been connected for some time and have reached their present values. Find V_1 and V_2.



C_{eq} for the 8μF & 4μF in series is

$$\frac{4\times 8}{4+8} = \frac{32}{12} = \frac{8}{3}\ \mu F$$

$$q_{8+4} = C_{eq}\,12 = \frac{8}{3}\mu F \times 12 = 32\,\mu C$$

$$q_{8\mu} = 32\mu C = q_{4\mu F}$$

$$q_{8\mu F} = (32\mu C) = C_8 V_8 = 8\mu F\, V_8$$

$$V_{8\mu F} = \frac{32}{8} = 4\,V$$

$$\boxed{V_{8\mu F} = 4V = V_1}$$

$$V_{4\mu F} = 12 - V_{8\mu F} = 12-4 = 8V$$

$$\boxed{V_{4\mu F} = 8\,V = V_2}$$

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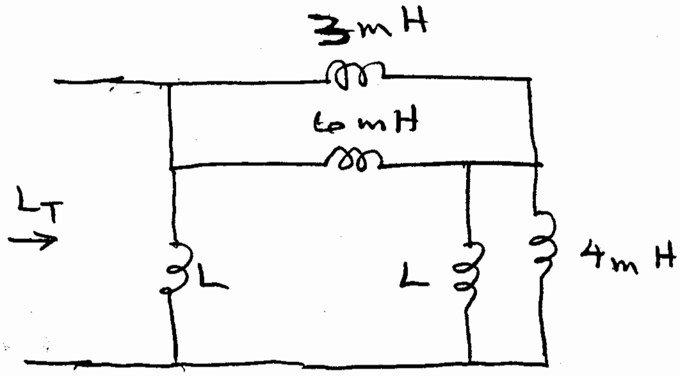
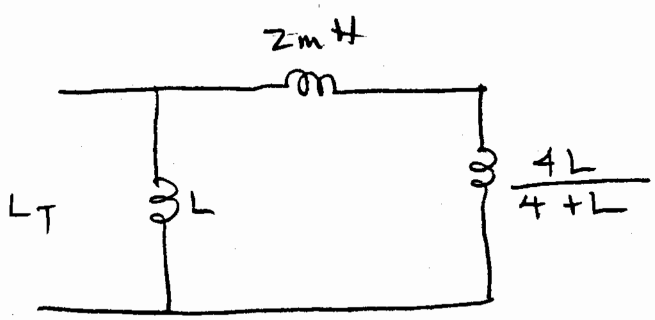
5.58

Find L in the ckt below so that $L_T = 2\,mH$

Redrawing the ckt gives





Assuming L → mH's

$$L_T = L \parallel \left(2mH + \frac{4L(mH)^2}{(4+L)\,mH}\right)$$

$$= L \parallel \left(\frac{8 + 2L + 4L}{4+L}\right) = L \parallel \frac{(8+6L)}{4+L}$$

$$= \frac{L \times \left(\frac{8+6L}{4+L}\right)}{L + \left(\frac{8+6L}{4+L}\right)} = \frac{L(8+6L)}{4L+L^2+8+6L}$$

$$2\,mH = \frac{6L^2+8L}{L^2+10L+8}$$

$$2L^2 + 20L + 16 = 6L^2 + 8L$$

$$4L^2 - 12L - 16 = 0 \Rightarrow L^2 - 3L - 4$$

$$L_T = (L-4)(L+1) = 0$$

$$\boxed{L_T = 4\,mH}$$

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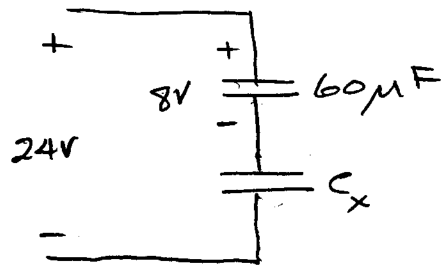
5FE-3 Two capacitor below have been connected for a long time and have reached their present values. Determine C_x and the energy stored in C_x.



<u>Upper capacitor:</u>

$$q = CV = 60 \times 10^{-6} \times 8 = 0.48 \times 10^{-3}\,C$$

<u>Lower capacitor:</u>

Voltage on the capacitor

$$V_{cx} = 24 - 8 = 16V$$

$$q_{cx} = q_{60\mu F} = 0.48 \times 10^{-3}\,C$$

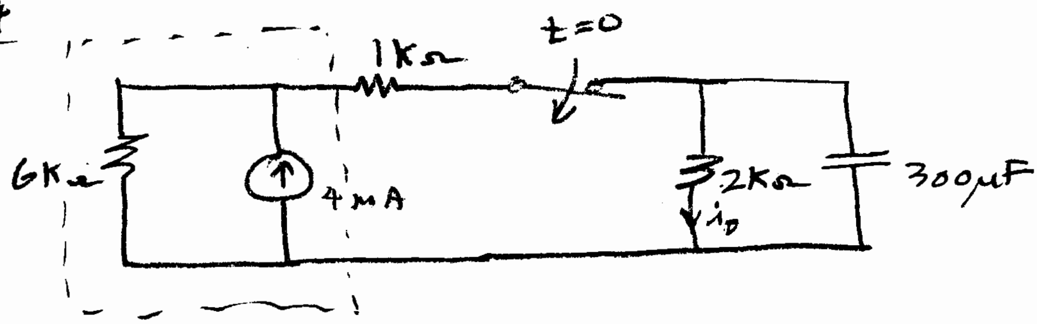
$$C_x = \frac{q_{cx}}{V_{cx}} = \frac{0.48 \times 10^{-3}}{16} = 0.03 \times 10^{-3}\,F$$

$$\boxed{C_x = 30\,\mu F}$$

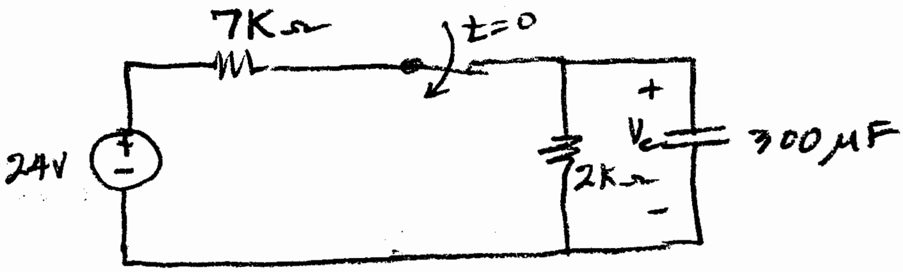
$$W = \frac{1}{2} C V^2 = \frac{1}{2} \times 30 \times 10^{-6} \times (16)^2$$

$$\boxed{W = 3.84\,mJ}$$

6.4



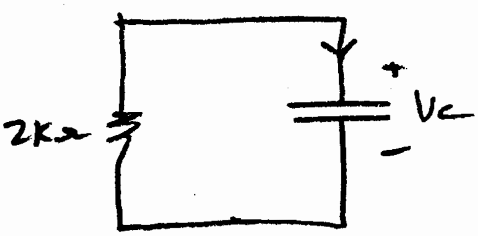
Change current source to voltage source



$$V_c(0^-) = \frac{24 \times 2K}{(2+7)K} = 5.33V$$

so $V_c(0^+) = V_c(0^-) = 5.33V$

After the switch opens



$$V_c(t) = V_c(0)\, e^{-\frac{t}{\tau}}$$

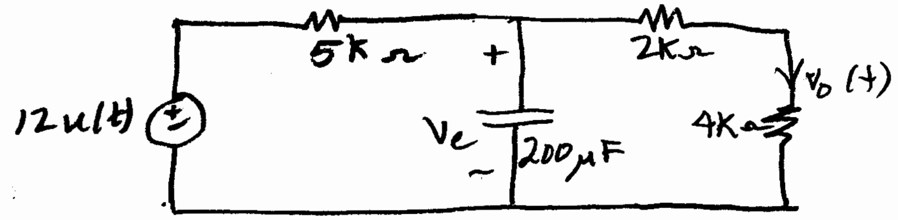
$\tau = RC = (2K)(.3 \times 10^{-3})$

$\tau = 0.6 = 6/10 = 3/5$

$$\boxed{V_c(t) = 5.33\, e^{-\frac{5t}{3}}\, V}$$

$$\boxed{i_o(t) = \frac{V_c(t)}{2K} = 2.67 e^{-\frac{5}{3}t}\ mA}$$

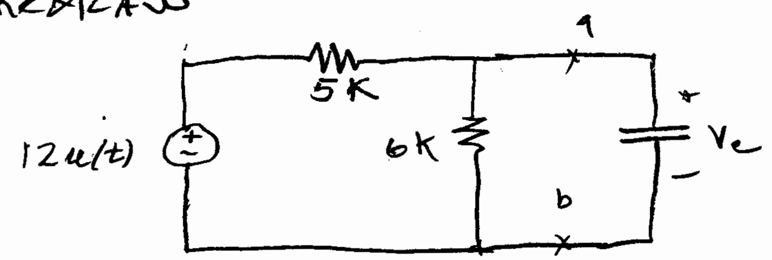
6.7 Use the d.e. approach to find $V_o(t)$ for the following. Prior to $t=0$, the circuit is set so that all Ic's are zero. After the switch is closed, the circuit is as follows



If you find $V_c(t)$ then

$$V_o(t) = \frac{V_c(t) \times 4K}{4K + 2K} \qquad (1)$$

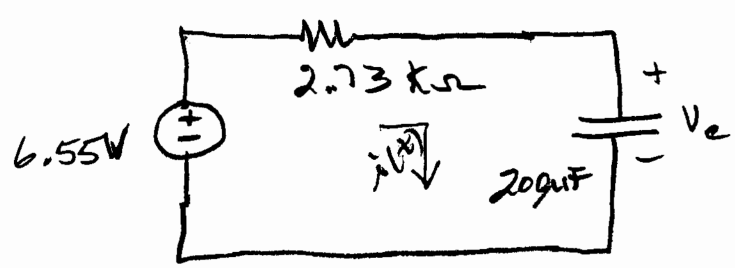
Redraw



The Thevenin to the left of a-b is

$$V_{TH} = \frac{12 \times 6}{11} = \frac{72}{11} = 6.55$$

$$R_{TH} = 6K \| 5K = 2.73 K\Omega$$



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6.7 cont.

$$\tau = R_{eq} C = 2.73\times10^{3} \times 0.2\times10^{-3}$$

$$\boxed{\tau = 0.546}$$

The d.e. for the ckt is

$$R i(t) + V_c(t) = 6.55$$

$$RC \frac{dV_c}{dt} + V_c(t) = 6.55$$

$$\frac{dV_c}{dt} + \frac{V_c}{0.546} = \frac{6.55}{0.546}$$

$$\frac{dV_c}{dt} + 1.83 V_c(t) = 12 \qquad (2)$$

$$V_c(t) = V_f(t) + V_n(t)$$

$V_f(t) = K$ Assume this; substitute back into (2) gives

$$(K)1.83 = 12$$

$$K = 6.56$$

$$V_c(t) = 6.56 + K_2 e^{-1.83t}$$

At $t = 0^+$ $V_c(0^+) = V_c(0^-) = 0$

$\therefore K_2 = -6.56$

$$V_c(t) = 6.56\left(1 - e^{-1.83t}\right) V\ u(t)$$

using (1)

$$V_0(t) = 4.37\left(1 - e^{-1.83t}\right) V\ u(t)$$

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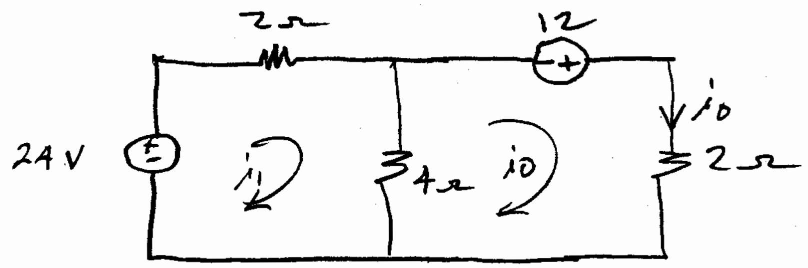
6.23 Work using d.e. approach. Find $i_o(t)$ for $t>0$ and just prior to $t=0$.

$t<0$ circuit is as below (coil shorted)

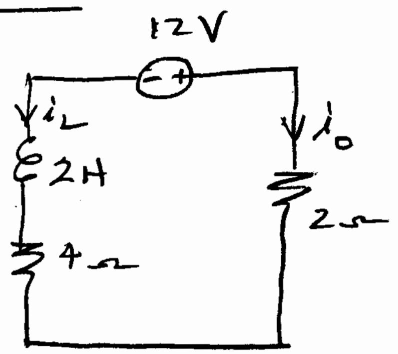


$$\begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} i_1 \\ i_o \end{bmatrix} = \begin{bmatrix} 24 \\ 12 \end{bmatrix}$$

$i_1(0^-) = 9.6$ A
$i_o(0^-) = 8.4$ A

$\therefore i_L(0^-) = i_1(0^-) - i_o(0^-) = 1.2$ (1)

For $t>0$



$i_L(t) = -i_o(t)$

$$2\frac{di_o}{dt} + 6i_o(t) = 12$$

$$\frac{di_o}{dt} + 3i_o(t) = 6 \qquad (2)$$

$i_{of} = K$ substitute into (2)

get $K = 2$.

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6.23 cont.

$$i_0(t) = 2 + K_2 e^{-\frac{t}{\tau}}$$

$R_{eq} = 6\Omega$, $L = 2H$

$$\tau = \frac{L}{R} = \frac{2}{6} = \frac{1}{3} \text{ sec}$$

$$i_0(t) = 2 + K_2 e^{-3t}$$

$i_0(0^+) = -i_L(0^-)$; current through L cannot change inst. so current through 2Ω is the negative of $i_L(0^-)$. From (1)

$$-1.2 = 2 + K_2 e^{-3t}\Big|_{t=0}$$

$$-1.2 = 2 + K_2$$

$$K_2 = -3.2$$

$$\boxed{i_0(t) = 2 - 3.2e^{-3t} \text{ A} \quad t \geq 0}$$

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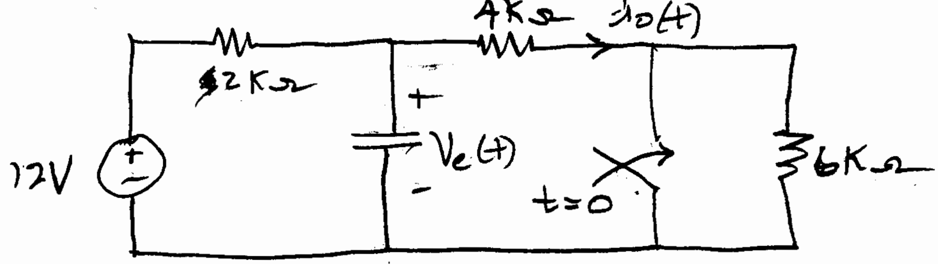
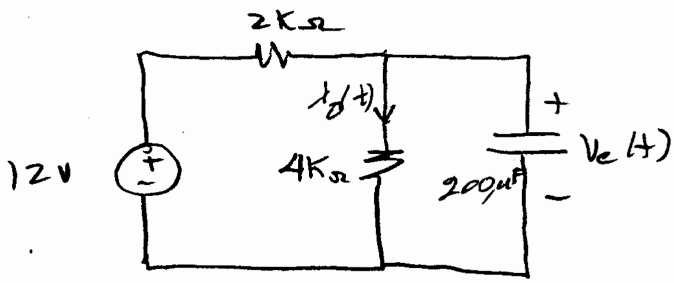
<u>6.29</u> Use step-by-step method to find $i_o(t)$ for $t>0$ in the following network.



<u>Before</u> the switch is closed, $V_c(t)$ can be found by using voltage division;

$$V_c(0^-) = \frac{12 \times 10K}{10K+2K} = 10$$

<u>After</u> the switch is closed the ckt becomes



Find $V_c(t)$, divide by 4KΩ to get $i_o(t)$.

$$R_{eq} = 4K \| 2K = \frac{4 \times 2\ K}{16} = 1.33K$$

$$\tau = R_{eq} \times C = (1.33 \times 10^3)(0.2 \times 10^{-3}) = 0.267$$

$$V_c(\infty) = \frac{12 \times 4K}{16K} = 3V$$

$$V_c(0^+) = V_c(0^-) = 10\ V$$

$$i_o(t) = K_1 + K_2 e^{-3.75t}$$

$$i_o(\infty) = \underline{K_1} = \frac{12}{6K} = \underline{2\ mA}$$

$$i_o(0^+) = \frac{V_c(0^+)}{4K} = \frac{10}{4}\ mA = 2.5\ mA$$

$$2.5_{mA} = K_1 + K_2$$

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6.29 cont

$$K_2 = 2.5\,mA - K_1 = 2.5\,mA - 2\,mA$$

$$K_2 = 0.5\ mA$$

$$i_o(t) = [2 + 0.5e^{-3.75t}]\,mA\ u(t)$$

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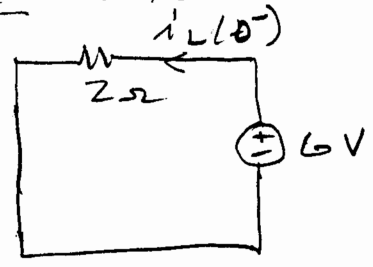
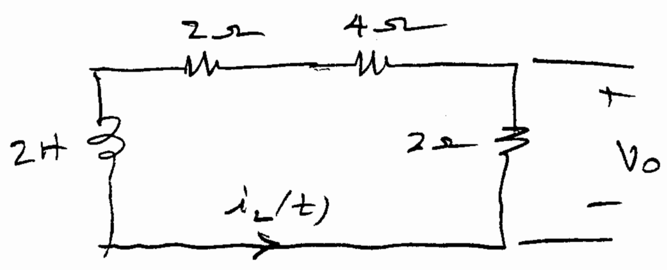
6.35 Use step-by-step to find $V_o(t)$ for $t>0$

For $t<0$ circuit is



$$i_L(0^-) = \frac{6}{2} = 3A$$

For $t>0$



$$R_{eq} = 2+4+2 = 8\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = 0.25 \text{ sec}$$

$$V_o = K_1 + K_2 e^{-4t}$$

$$V_o(\infty) = 0 \quad \therefore K_1 = 0$$

$$V_o(0^+) = -i_L(0^+)2 = -3\times 2 = -6V$$

$$\therefore V_o(0) = K_2 = -6$$

$$V_o(t) = -6e^{-4t}u(t)$$