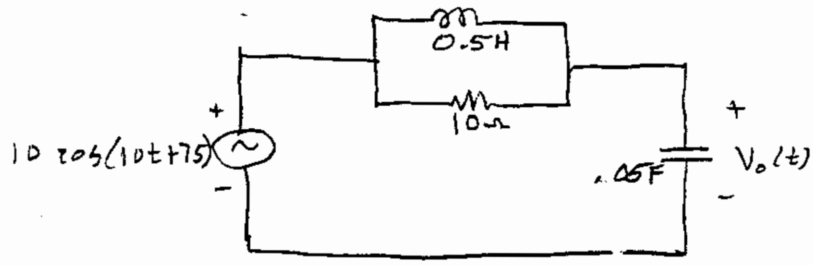
H.W. #4

(1) Determine $V_o(t)$ in the following ckt.

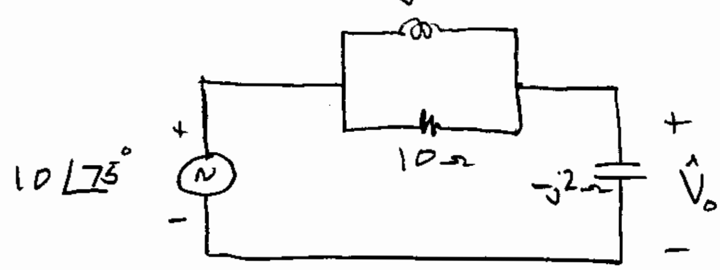


Change to a phasor circuit. $\omega = 10$ rad/sec

$.5H \longrightarrow j10 \times .5 = j5\,\Omega$

$.05F \longrightarrow \frac{-j}{10 \times .05} = -j2\,\Omega$

Redraw the circuit:



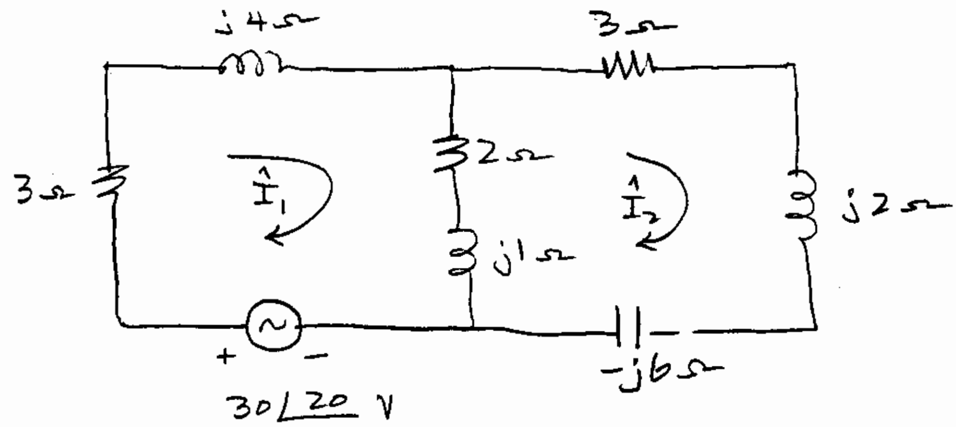
Apply voltage division:

$$\hat{V}_o = \frac{(10\angle 75)(2\angle -90)}{\frac{10 \times (5\angle 90)}{10+j5} - j2} = \frac{20\angle -15}{2+j2}$$

$$\hat{V}_o = 7.07\angle -60$$

$\therefore$ **$V_o(t) = 7.07\cos(10t - 60^\circ)$ V**

(2) Use mesh analysis to find $\hat{I}_1$ and $\hat{I}_2$ in the circuit below.



$$3\hat{I}_1 + j4\hat{I}_1 + 2(\hat{I}_1 - \hat{I}_2) + j1(\hat{I}_1 - I_2) = 30\angle 20$$

$$\boxed{(5 + j5)\hat{I}_1 - (2 + j1)\hat{I}_2 = 30\angle 20}$$

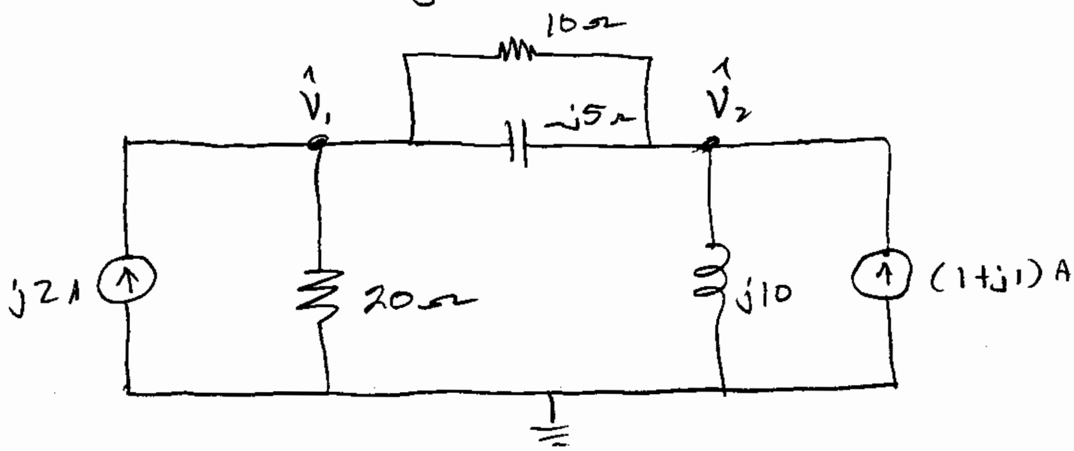
$$(2 + j)(\hat{I}_2 - \hat{I}_1) + (3 - j4)\hat{V}_2 = 0$$

$$\boxed{-(2 + j)\hat{I}_1 + (5 - j3)\hat{I}_2 = 0}$$

$$\begin{bmatrix} 5+j5 , & -2-j1 \\ -2-j1 , & 5-j3 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 30\angle 20 \\ 0 \end{bmatrix}$$

$\hat{I}_1 = 4.67\angle -20.2^\circ$ A $\quad \hat{I}_2 = 1.79\angle 37.4^\circ$ A

(3) Use Nodal analysis to find $\hat{V}_1$ and $\hat{V}_2$.



$$10 \| (-j5) = \frac{(50\angle -90)}{(10 - j5)} = 2 - j4 = 4.47\angle -63.4^\circ$$

At $\hat{V}_1$

$$\frac{\hat{V}_1}{20} + \frac{(\hat{V}_1 - \hat{V}_2)}{4.47\angle -63.4} = 2\angle 90^\circ$$

$$.05V_1 + (.1 + j.2)V_1 + (-.1 - j.2)V_2 = 2\angle 90$$

$$\boxed{(.15 + j.2)\hat{V}_1 + (-0.1 - j0.2)\hat{V}_2 = 2\angle 90}$$

At $\hat{V}_2$

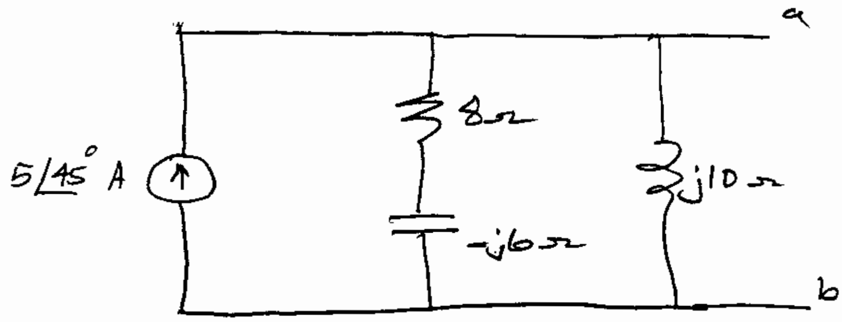
$$\frac{\hat{V}_2 - \hat{V}_1}{4.47\angle -63.4} + \frac{\hat{V}_2}{j10} = (1 + j1)$$

$$(.1 + j.2)V_2 + (-0.1 - j0.2)V_1 - j0.1V_2 = 1 + j1$$

$$\boxed{(-0.1 - j0.2)\hat{V}_1 + (0.1 + j0.1)\hat{V}_2 = (1 + j1)}$$

$\hat{V}_1 = 22.9\angle 132.3^\circ$ V $\qquad \hat{V}_2 = 27.9\angle 140.6^\circ$ V

(4) Find the Thevenin circuit looking into terminals a-b



First, Z_{TH}: Open the current source, look into a-b and find the impedance.

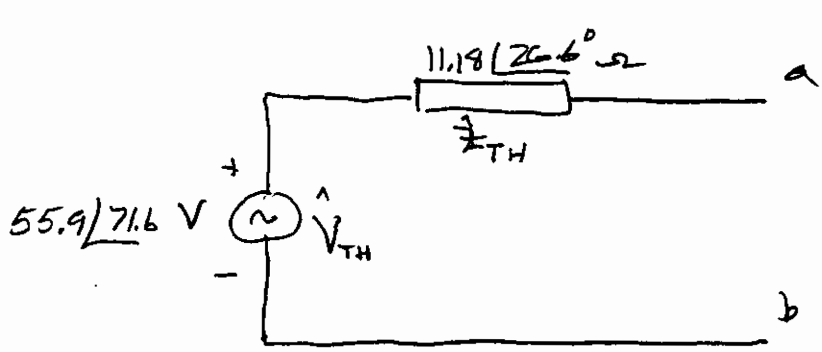
$$\hat{Z}_{TH} = (j10) \parallel (8-j6) = \frac{(10\angle 90)(8-j6)}{j10+8-j6}$$

$$\hat{Z}_{TH} = 10+j5 = 11.18\angle 26.6° \ \Omega$$

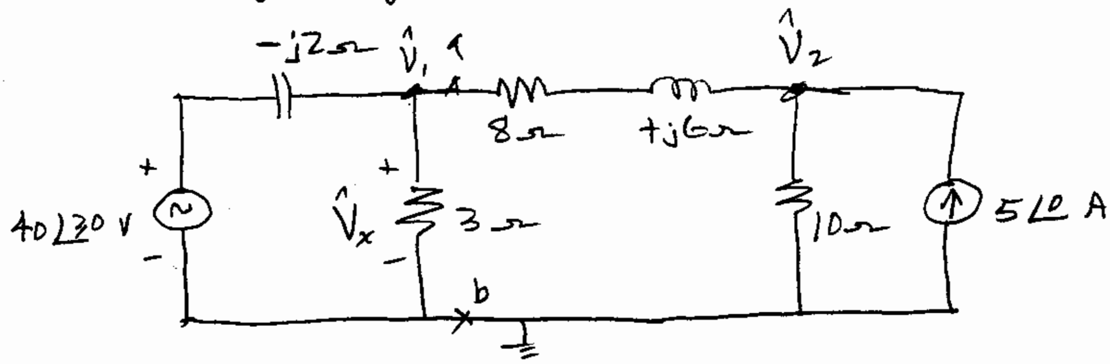
Find V_{TH}: Find the current in j10 using current splitting. Then multiply this current by j10 Ω.

$$\hat{V}_{TH} = \left(\frac{(5\angle 45)(8-j6)}{8-j6+j10}\right) \times j10$$

$$\hat{V}_{TH} = 55.9\angle 71.6° \ V$$



(5) Determine $\hat{V}_x$ in the following circuit using any method.



Use nodal analysis (my choice)

At $\hat{V}_1$

$$\frac{V_1 - 40\angle 30}{-j2} + \frac{\hat{V}_1}{3} + \frac{\hat{V}_1 - \hat{V}_2}{8 + j6} = 0$$

$$j0.5V_1 + 20\angle{-60} + 0.333V_1 + (0.08 - j0.06)V_1 - (0.08 - j0.06)V_2 = 0$$

$$(0.413 + j0.44)\hat{V}_1 + (-0.08 + j0.06)\hat{V}_2 = -20\angle{-60} = 20\angle 120^\circ$$

At $\hat{V}_2$

$$\frac{\hat{V}_2 - \hat{V}_1}{8 + j6} + \frac{\hat{V}_2}{10} = 5\angle 0$$

$$(0.08 - j0.06)V_2 - (0.08 - j0.06)\hat{V}_1 + 0.1\hat{V}_2 = 5\angle 0$$

$$(-0.08 + j0.06)\hat{V}_1 + (.18 - j0.06)\hat{V}_2 = 5\angle 0$$

$$\hat{V}_1 = \hat{V}_x = 29.4\angle 62.9^\circ \text{ V}$$