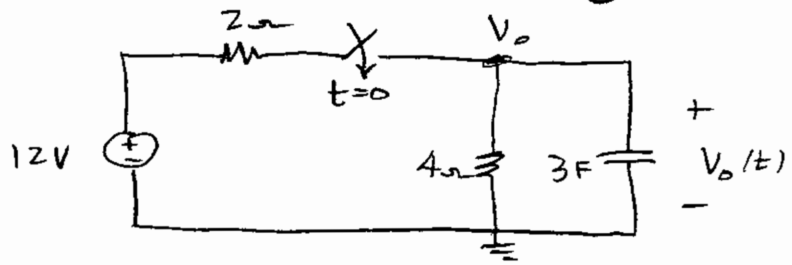
# Home Work #5

1

Find $V_o(t)$ for the following circuit.




$V_o(0^+) = 0$ voltage across the capacitor cannot change instantaneously.

$$V_o(\infty) = \frac{12 \times 4}{6} = 8\ V$$

Write a node equation;

$$3\frac{dV_o}{dt} + \frac{V_o}{4} + \frac{V_o - 12}{2} = 0$$

$$3\frac{dV_o}{dt} + 0.75 V_o = 6$$

$$\frac{dV_o}{dt} + 0.25 V_o = 2$$

$V_{op} = K$

$.25K = 2$

$K = 8$

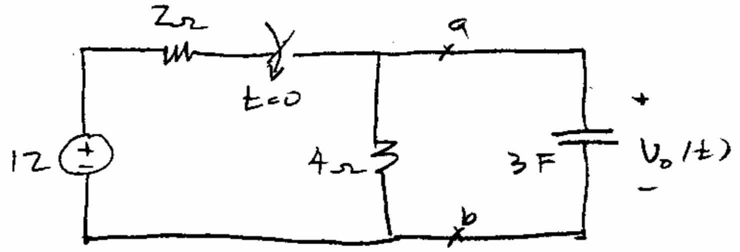
$$V_o = 8 + K_c e^{-0.25t}$$

$V_o(0) = 0 = 8 + K_c$

$K_c = -8$

$$\boxed{V_o(t) = 8(1 - e^{-0.25t})}$$

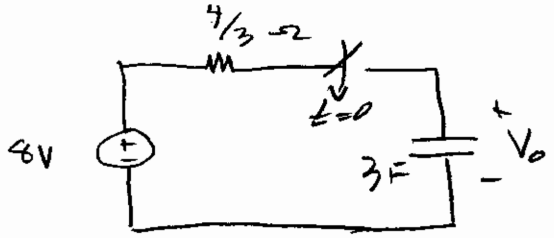
① 2nd solution:




Form the Thevenin equivalent to the left of a-b.

$$R_{TH} = \frac{4\times 2}{4+2} = \frac{8}{6} = \frac{4}{3}\ \Omega$$

$$V_{TH} = \frac{12\times 4}{4+2} = 8V$$




using

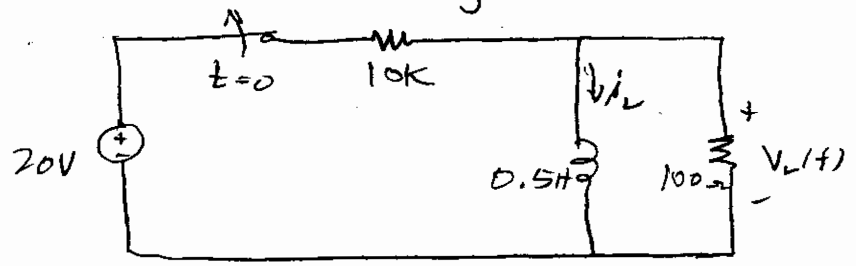
$$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]\, e^{-\frac{t}{\tau}}$$

$$\tau = R_{eq}C = \frac{4}{3}\times 3 = 4$$

$$V_o(\infty) = 8V, \quad V_o(0^+) = 0$$

$$V_o(t) = 8(1 - e^{-0.25t})\ V$$

(2) For the following circuit;

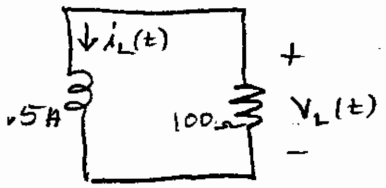



(a) Develop the DE that allows you to solve for $V_L(t)$.

With the switch closed, $i_L$ reach steady state. The inductor looks like a short circuit.

$$i_L(0^-) = \frac{20}{10K} = 2\times10^{-3}\ A$$

After the switch is open we have




$$L\frac{di_L(t)}{dt} + Ri_L(t) = 0 \qquad L=0.5,\ R=100\Omega$$

(b)

$$\frac{di_L(t)}{dt} + \frac{R}{L}i_L(t) = 0$$

$$\frac{di_L(t)}{dt} + 200i_L(t) = 0$$

$$i_L(t) = Ke^{-200t}$$

$$i_L(0^-) = i_L(0^+) = 2\times10^{-3}$$

$$\boxed{i_L(t) = 0.002e^{-200t}\ A}$$

(2) continued

(c) What is the time constant?

$$\tau = \frac{L}{R} = \frac{0.5}{100} = 0.005 \text{ sec}$$

$\tau = 5$ msec

(d) When will the current reach 0.001 A

Take

$$i_L(t) = 0.002e^{-200t}$$

Set it equal to 0.001

$$0.001 = 0.002\,e^{-200t}$$

so, $e^{-200t} = 0.5$

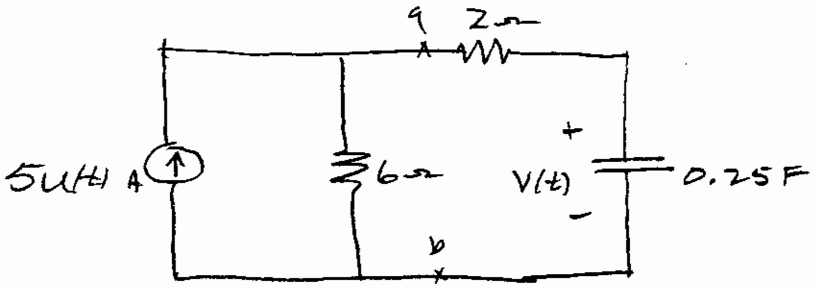
$$-200t \ln e = \ln 0.5 = -0.693$$

$$\ln e = 1$$

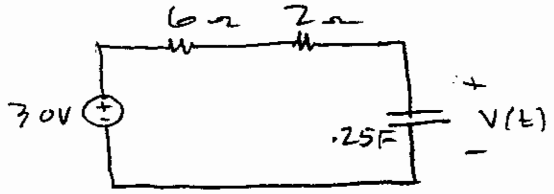
$$-200t = -0.693$$

$$\boxed{t = 0.003 \text{ seconds}}$$

(3) For the following circuit, find $v(t)$, $t \geq 0$.




Make a Thevenin to the left of a-b




$$0.25 \times 8 \frac{dV_c}{dt} + V_c = 30$$

$$\frac{dV_c}{dt} + 0.5V_c = 15$$

Solution:

$$V_c(t) = 30(1 - e^{-0.5t})\ V$$

#

Step-by-step

$$V(t) = V(\infty) + [V(0^+) - V(\infty)]\, e^{-\frac{t}{\tau}}$$

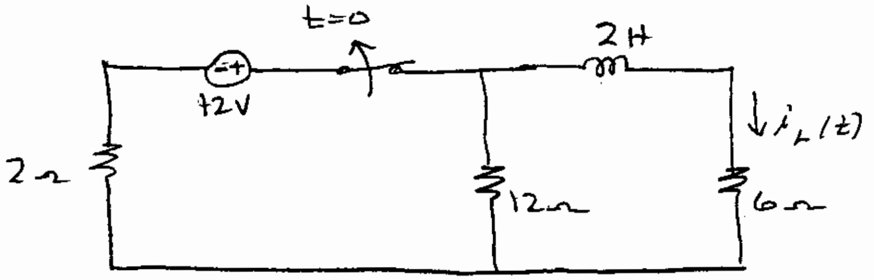
$$\tau = R_{eq}C = 8 \times 0.25 = 2$$

$$V(\infty) = 30\text{V}, \quad V(0) = 0$$

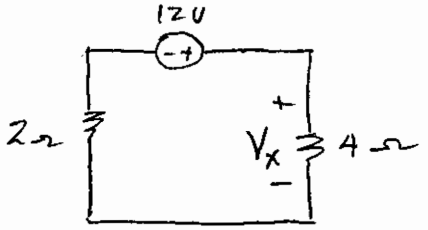
$\therefore$

$$v(t) = 30(1 - e^{-0.5t})\ \text{V}$$

(4) Find $i_L(t)$ in the circuit below.




With the switch closed for a long time, the inductor is a short circuit. We can find $i_L(0^-)$ from the following:

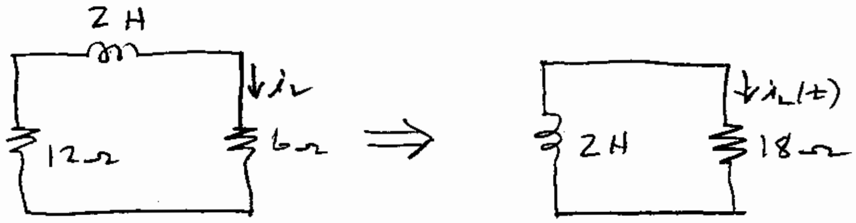



$$6 \| 12 = \frac{72}{18} = 4\,\Omega$$

$$V_x = \frac{12 \times 4}{6} = 8\text{ V}$$

$$\therefore\ i_L(0^-) = \frac{8\text{ V}}{6\,\Omega} = \frac{4}{3}\text{ A}^{*} = i_L(0^+)$$

After the switch is opened we have




$$L\frac{di_L}{dt} + Ri_L = 0$$

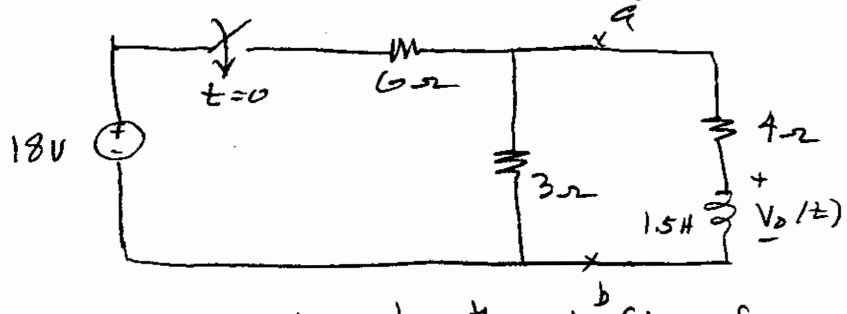
$$2\frac{di_L}{dt} + 18i_L = 0$$

$$\frac{di_L}{dt} + 9i_L = 0 \Rightarrow i_L = Ke^{-9t}\text{ A}$$

$$\boxed{i_L = \frac{4}{3}e^{-9t}\text{ A}}$$

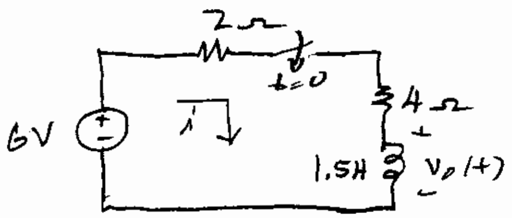
⑤ Determine $V_o(t)$ for the circuit below.

Assume $V_o(0^-) = 0$




circuit 1

Use Thevenin to the left of a-b.




circuit 2

$$V_o(t) = L\frac{di}{dt}$$

$$di = \frac{1}{L} V_o(t)\,dt$$

$$\int_0^t di = \frac{1}{L}\int_0^t V_o(t)\,dt$$

$$i(t) = \frac{1}{L}\int_0^t V_o(t)\,dt + i(0)$$ Assume $i(0) = 0$

For the circuit;

$$6\,i(t) + V_o(t) = 6$$

$$6 \times \frac{1}{1.5}\int_0^t V_o(t)\,dt + V_o(t) = 0 \qquad (1)$$

Take the Derivative of (1)

$$4V_o(t) + \frac{dV_o(t)}{dt} = 0$$

$$\frac{dV_o(t)}{dt} + 4V_o(t) = 0 \qquad (2)$$

(5) continued

The solution to Eq (2) is

$$V_o(t) = K e^{-4t}$$

We need to know $V_o(0^+)$. We know $V_o(0^-) = 0$. However, the voltage across an inductor can change instantaneously.

From circuit 2 we know that when the 6 volts is applied, the current cannot change instantaneously. That means the voltage drops across the $2\Omega$ and $4\Omega$ resistors are zero, Therefore all the 6V applied is across the inductor. Thus, $V_o(0^+) = 6V$

$$V_o(0^+) = 6 = K e^0 = K$$

so

$$\underline{V_o(t) = 6 e^{-4t} \text{ V}}$$