Chapter 10, Problem 12.

PS ML

By nodal analysis, find i_o in the circuit of Fig. 10.61.

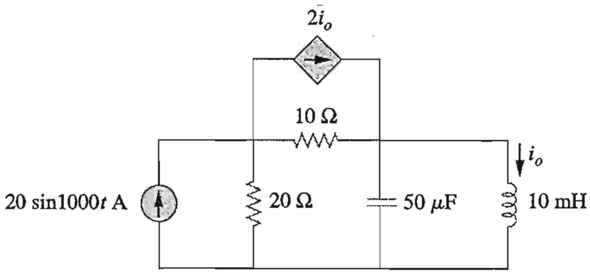


Figure 10.61
For Prob. 10.12.

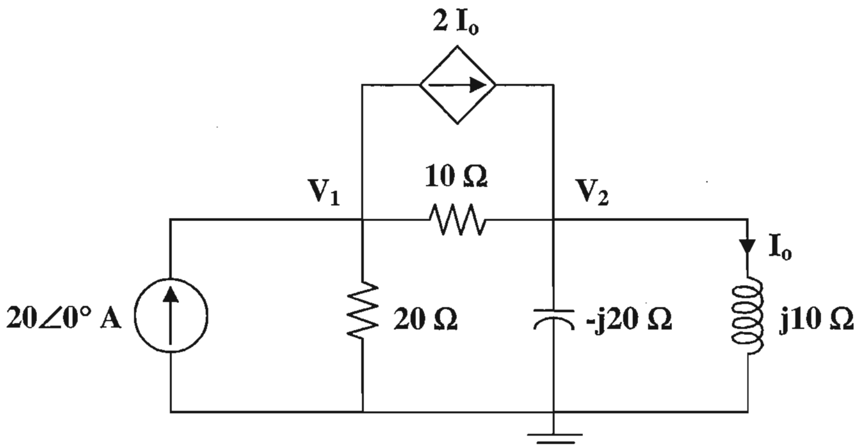
Chapter 10, Solution 12.

$$20\sin(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

The frequency-domain equivalent circuit is shown below.



At node 1,

$$20 = 2\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10},$$ where

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\,\mathbf{V}_1 = (-3 + j2)\,\mathbf{V}_2$$

or

$$\mathbf{V}_1 = (1 + j1.5)\,\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\,\mathbf{V}_2 - (2 + j4)\,\mathbf{V}_2 = (1 + j0.5)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle\text{-}116.6°$$

Therefore, $i_o(t) =$ **35.74 sin(1000t – 116.6°) A**

Chapter 10, Problem 16.

ps ML

Use nodal analysis to find $\mathbf{V}_x$ in the circuit shown in Fig. 10.65.

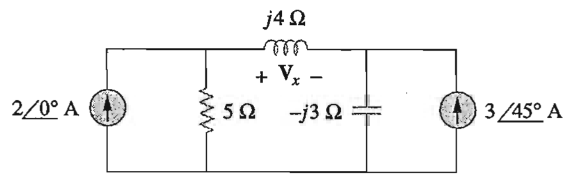
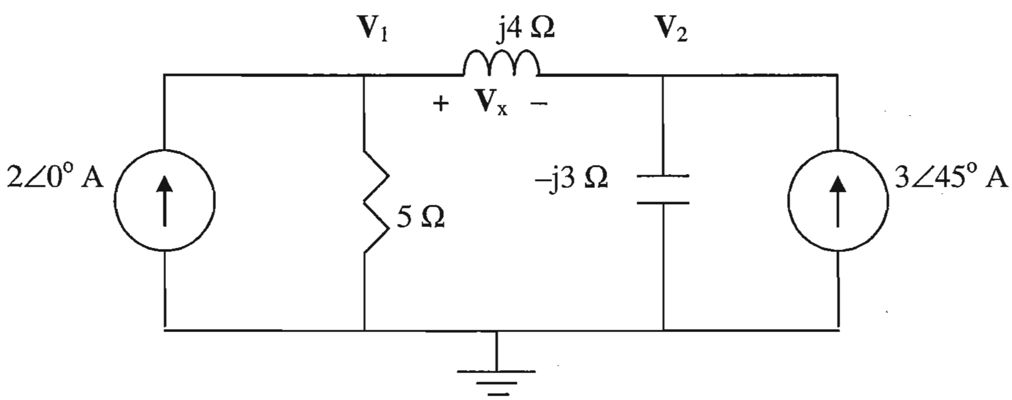


Figure 10.65
For Prob. 10.16.

Chapter 10, Solution 16.

Consider the circuit as shown in the figure below.



At node 1,

$$-2+\frac{V_1-0}{5}+\frac{V_1-V_2}{j4}=0 \qquad (1)$$

$$(0.2-j0.25)V_1+j0.25V_2=2$$

At node 2,

$$\frac{V_2-V_1}{j4}+\frac{V_2-0}{-j3}-3\angle 45^\circ=0 \qquad (2)$$

$$j0.25V_1+j0.08333V_2=2.121+j2.121$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 0.2-j0.25 & j0.25 \\ j0.25 & j0.08333 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 2 \\ 2.121+j2.121 \end{bmatrix}$$

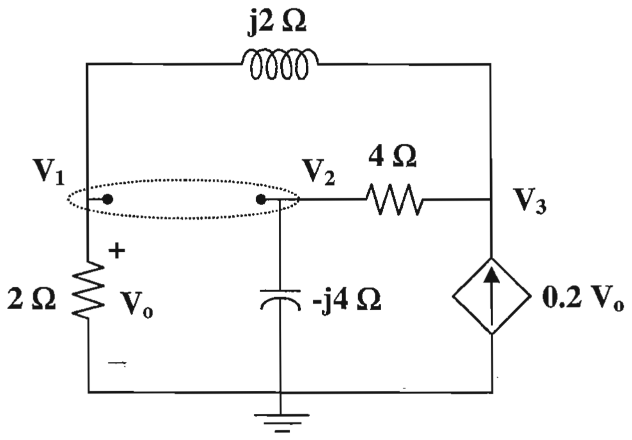
Solving this using MATLAB, we get,

```
>> Y=[(0.2-0.25i),0.25i;0.25i,0.08333i]

Y =

  0.2000 - 0.2500i      0 + 0.2500i
       0 + 0.2500i      0 + 0.0833i

>> I=[2;(2.121+2.121i)]

I =

  2.0000
  2.1210 + 2.1210i

>> V=inv(Y)*I

V =

  5.2793 - 5.4190i
  9.6145 - 9.1955i
```

$V_s = V_1 - V_2 = -4.335 + j3.776 = \underline{\mathbf{5.749\angle 138.94^\circ\ V}}$.

Chapter 10, Solution 19.

We have a supernode as shown in the circuit below.



Notice that $\mathbf{V}_o = \mathbf{V}_1$.

At the supernode,

$$\frac{\mathbf{V}_3 - \mathbf{V}_2}{4} = \frac{\mathbf{V}_2}{-j4} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{j2}$$

$$0 = (2 - j2)\,\mathbf{V}_1 + (1 + j)\,\mathbf{V}_2 + (-1 + j2)\,\mathbf{V}_3 \qquad (1)$$

At node 3,

$$0.2\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_3}{j2} = \frac{\mathbf{V}_3 - \mathbf{V}_2}{4}$$

$$(0.8 - j2)\,\mathbf{V}_1 + \mathbf{V}_2 + (-1 + j2)\,\mathbf{V}_3 = 0 \qquad (2)$$

Subtracting (2) from (1),

$$0 = 1.2\mathbf{V}_1 + j\mathbf{V}_2 \qquad (3)$$

But at the supernode,

$$\mathbf{V}_1 = 12\angle 0° + \mathbf{V}_2$$

or

$$\mathbf{V}_2 = \mathbf{V}_1 - 12 \qquad (4)$$

Substituting (4) into (3),

$$0 = 1.2\mathbf{V}_1 + j(\mathbf{V}_1 - 12)$$

$$\mathbf{V}_1 = \frac{j12}{1.2 + j} = \mathbf{V}_o$$

$$\mathbf{V}_o = \frac{12\angle 90°}{1.562\angle 39.81°}$$

$$\mathbf{V}_o = \underline{\mathbf{7.682\angle 50.19°\ V}}$$

Chapter 10, Solution 25.

$$\omega = 2$$

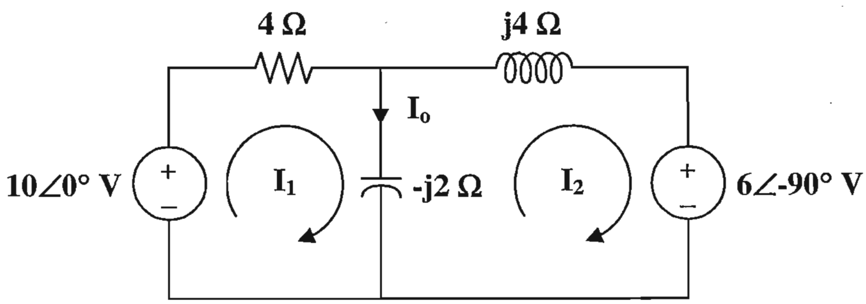
$$10\cos(2t) \longrightarrow 10\angle 0°$$

$$6\sin(2t) \longrightarrow 6\angle -90° = -j6$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$0.25\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

The circuit is shown below.



For loop 1,

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$
$$5 = (2 - j)\mathbf{I}_1 + j\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$
$$\mathbf{I}_1 + \mathbf{I}_2 = 3 \qquad (2)$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\Delta = 2(1-j), \qquad \Delta_1 = 5 - j3, \qquad \Delta_2 = 1 - j3$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{4}{2(1-j)} = 1 + j = 1.414\angle 45°$$

Therefore, $i_o(t) =$ **1.4142 cos(2t + 45°) A**

Chapter 10, Problem 30.

PS ML

Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 120\cos(100t + 90°)$ V, $v_{s2} = 80\cos 100t$ V.

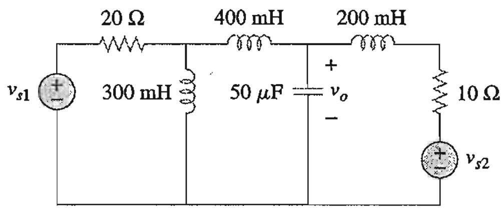
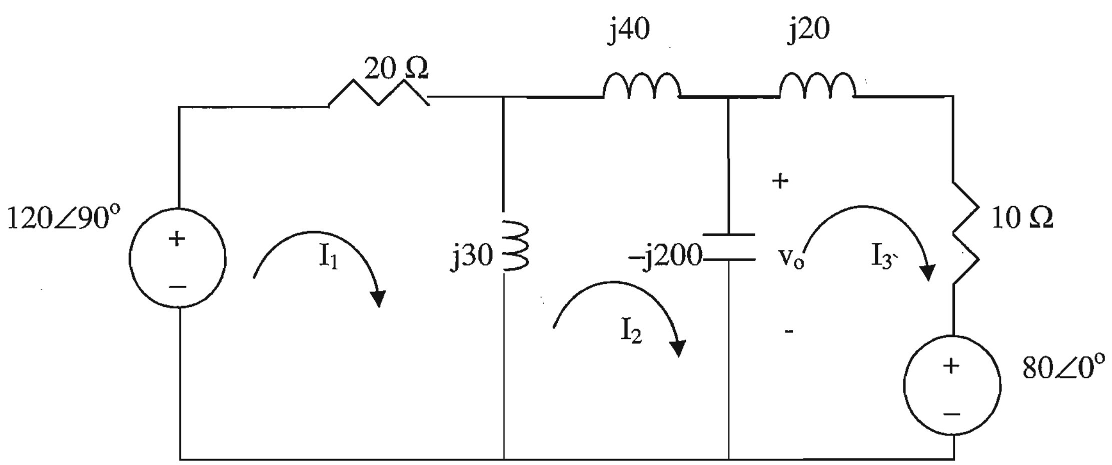


Figure 10.78
For Prob. 10.30.

Chapter 10, Solution 30.

$$300mH \longrightarrow j\omega L = j100x300x10^{-3} = j30$$
$$200mH \longrightarrow j\omega L = j100x200x10^{-3} = j20$$
$$400mH \longrightarrow j\omega L = j100x400x10^{-3} = j40$$
$$50\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j100x50x10^{-6}} = -j200$$

The circuit becomes that shown below.



For mesh 1,

$-120<90^{o}+(20+j30)I_1-j30I_2=0 \quad \longrightarrow \quad j120=(20+j30)I_1-j30I_2 \qquad (1)$

For mesh 2,

$-j30I_1+(j30+j40-j200)I_2+j200I_3=0 \quad \longrightarrow \quad 0=-3I_1-13I_2+20I_3 \qquad (2)$

For mesh 3,

$80+j200I_2+(10-j180)I_3=0 \rightarrow -8=j20I_2+(1-j18)I_3 \qquad (3)$

We put (1) to (3) in matrix form.

$$\begin{bmatrix} 2+j3 & -j3 & 0 \\ -3 & -13 & 20 \\ 0 & j20 & 1-j18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} j12 \\ 0 \\ -8 \end{bmatrix}$$

This is an excellent candidate for MATLAB.

```
>> Z=[(2+3i),-3i,0;-3,-13,20;0,20i,(1-18i)]

Z =

   2.0000 + 3.0000i        0 - 3.0000i        0
  -3.0000            -13.0000            20.0000
        0                  0 +20.0000i   1.0000 -18.0000i

>> V=[12i;0;-8]

V =

        0 +12.0000i
        0
  -8.0000

>> I=inv(Z)*V

I =

   2.0557 + 3.5651i
   0.4324 + 2.1946i
   0.5894 + 1.9612i
```

$V_o = -j200(I_2 - I_3) = -j200(-0.157+j0.2334) = 46.68 + j31.4 = 56.26\angle 33.93^\circ$

v_o = **56.26cos(100t + 33.93° V**.

Chapter 10, Problem 58.

For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a-b*.

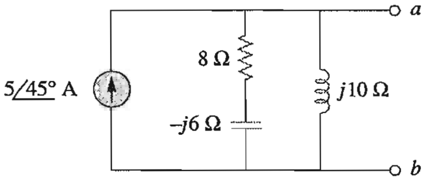
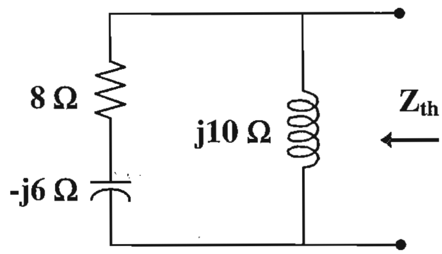


Figure 10.101
For Prob. 10.58.

Chapter 10, Solution 58.

Consider the circuit in Fig. (a) to find $\mathbf{Z}_{th}$.

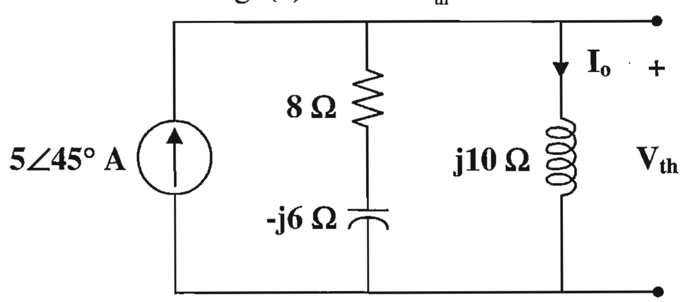


(a)

$$\mathbf{Z}_{th} = j10 \parallel (8 - j6) = \frac{(j10)(8 - j6)}{8 + j4} = 5(2 + j)$$

$$= \underline{\mathbf{11.18\angle 26.56^\circ\ \Omega}}$$

Consider the circuit in Fig. (b) to find $\mathbf{V}_{th}$.



(b)

$$\mathbf{I}_o = \frac{8 - j6}{8 - j6 + j10}(5\angle 45^\circ) = \frac{4 - j3}{4 + j2}(5\angle 45^\circ)$$

$$\mathbf{V}_{th} = j10\mathbf{I}_o = \frac{(j10)(4 - j3)(5\angle 45^\circ)}{(2)(2 + j)} = \underline{\mathbf{55.9\angle 71.56^\circ\ V}}$$