Desk Copy

ECE 300
HW #12

wlg
Due: November 29, '07

Name wlg

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 10 points.

(11.5) Also find the power supplied by the source: $P_1 = 1.42$ W; $P_2 = 5.1$ W; $P_{source} = 6.52$ W

~~(11.8) $P_{40} = 43.8$ W~~

(11.15) $Z_L = (0.5 - j0.5)$ ohms; $P_{max} = 90$ W

(11.29) $I_{RMS} = 5.77$ A; $P_{12} = 400$ W

(11.31) $V_{RMS} = 2.94$ V

(11.42) (a) $S = (84.8 + j84.8)$ VA; (b) $I_{RMS} = 1.09$ A rms ; (c) $Z = (71.3 + j71.3)$ ohms;
(d) $R = 71.3$ ohms, $L = 0.189$ H

(11.47) (a) $S = 224\angle 60°$ VA; $P = 112$ W, $Q = 194$ VAR

(b) $S = 320\angle -45°$ VA; $P = 226.3$ W, $Q = -226.3$ VAR

(c) $S = 128\angle 30°$ VA; $P = 110.85$ W, $Q = 64$ VAR

(d) On your own

(11.55)
40 V source: $S = (-140 + j20)$ VA
capacitor: $S = -j250$ VA
resistor : $S = 290$ VA
inductor : $S = j130$ VA
50 V source: $S = (-150 + j100)$ VA

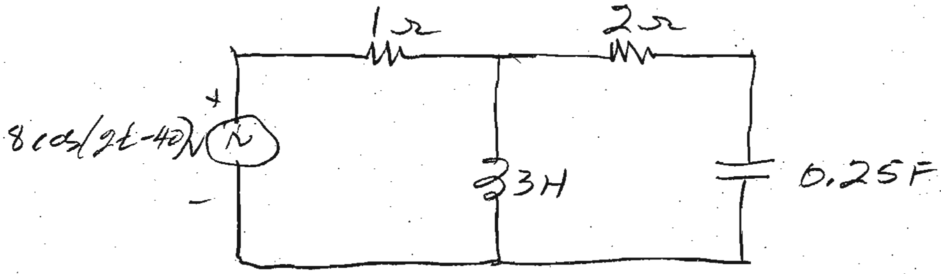
(11.60) $V_0 = 7.098\angle 32.3°$ V; pf – 0.8454 lagging

(11.70) $C = 69.45$ μF

wlg

ECE 300
HW #12
Fall, 2007

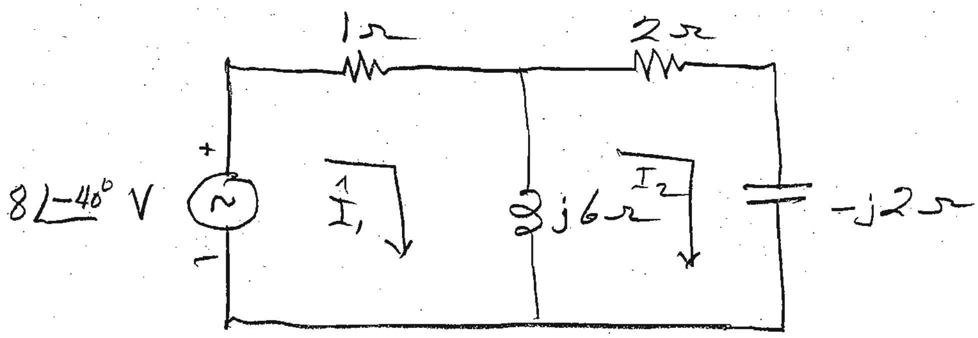
11.5 In the circuit below, find the average power delivered to each passive element.



Change to phasor circuit

$3H \rightarrow j2 \times 3 = j6\,\Omega$

$0.25F \rightarrow \frac{-j}{2 \times .25} = -j2\,\Omega$



$$\begin{bmatrix} (1+j6), & (0-j6) \\ (0-j6) & (2+j4) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 8\angle -40 \\ 0 \end{bmatrix}$$

$\vec{I}_1 = 1.68\angle -25.4^\circ\,A$ $\quad\vec{I}_2 = 2.26\angle 11.19^\circ\,A$

(11.5) continued

$$P_1 = \frac{|\vec{I}_1|^2 \times 1}{2} = \frac{(1.68)^2}{2} \times 1 = \boxed{1.41\ W}$$

$$P_2 = \frac{|\vec{I}_2|^2 \times 2}{2} = (2.26)^2 = \boxed{5.11\ W}$$

$$\Sigma(P_1 + P_2) = 1.41 + 5.11 = \underline{6.52\ W}$$

$$P_{source} = \frac{|\vec{V}_3| \times |\vec{I}_1|}{2} \cos(\theta_V - \theta_I)$$

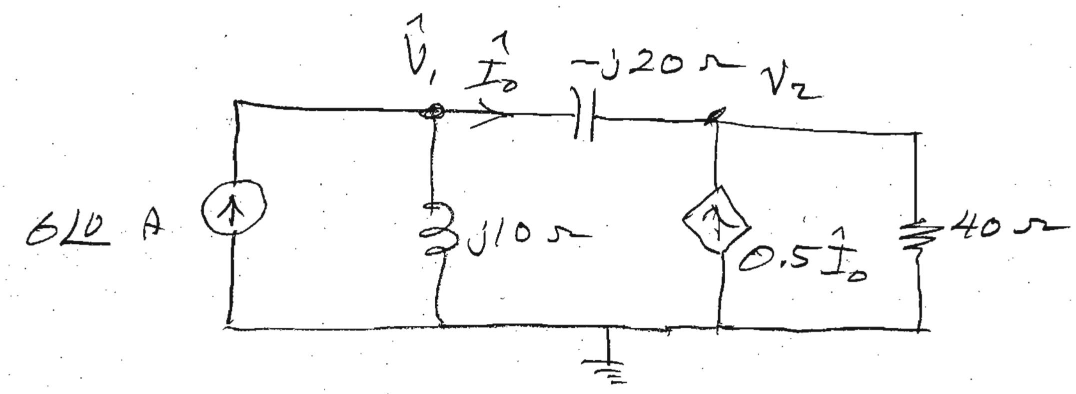
$$= \frac{8 \times 1.68}{2} \cos(-40 + 25.4)$$

$$P_{source} = \underline{6.5\ W}$$

$P_{source} = P_1 + P_2$ (within calculator roundoff)

$$\left.\begin{array}{l} P_{coil} = 0 \\ P_{cap} = 0 \end{array}\right]$$ We have proven this

(11.8) In the following circuit, determine the average power absorbed by the $40\,\Omega$ resistor.



By nodal analysis

At Node $\hat{V}_1$

$$\frac{\hat{V}_1}{j10} + \frac{\hat{V}_1 - \hat{V}_2}{-j20} = 6\angle 0$$

$$-j0.1\hat{V}_1 + j0.05\hat{V}_1 - j0.05\hat{V}_2 = 6\angle 0$$

$$\boxed{(0 - j0.05)\hat{V}_1 + (0 - j0.05)\hat{V}_2 = 6\angle 0}$$

At Node $\hat{V}_2$

$$\frac{V_2 - V_1}{-j20} - 0.5 I_o + \frac{V_2}{40} = 0$$

but $I_o = \dfrac{V_1 - V_2}{-j20}$

40

$$\frac{V_2 - V_1}{-j20} - \frac{0.5(V_1 - V_2)}{-j20} + \frac{V_2}{40} = 0$$

(11.8) continued

$$j0.05V_2 - j0.05V_1 - j0.025V_1 + j0.025V_2 + 0.025V_2 = 0$$

$$\boxed{(0 - j0.075)V_1 + (0.025 + j0.075)V_2 = 0}$$

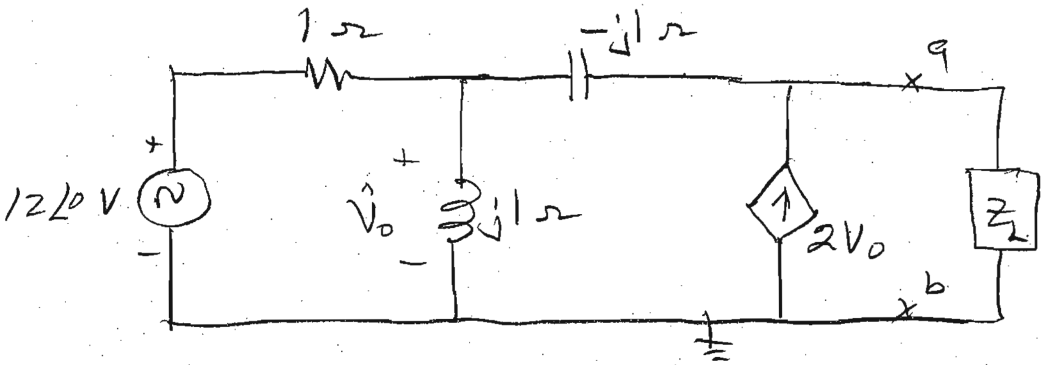
$$\begin{bmatrix} (0 - j0.05) & (0 - j0.05) \\ (0 - j0.075) & (0.025 + j0.075) \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} = \begin{bmatrix} 6\angle 0 \\ 0 \end{bmatrix}$$

$\vec{V}_1 = 62.4\angle 81^\circ$ V $\qquad \vec{V}_2 = 59.2\angle 99.5^\circ$ V

$$P_{40} = \frac{|\vec{V}_2|^2}{2\times 40} = \frac{(59.2)^2}{2\times 40}$$

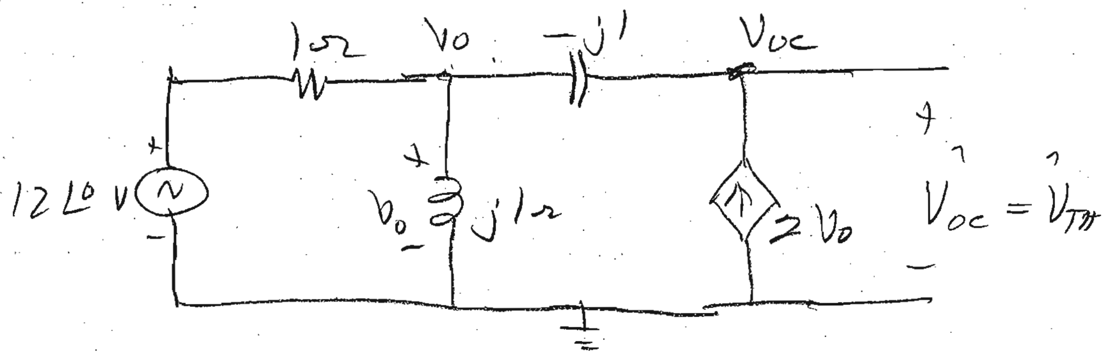
$$P_{40} = 43.8 \text{ W}$$

(11.15) For the following circuit, find the value of Z_L that will absorb the maximum power and the value of the maximum power.



$$Z_L = Z_{ab}^* = Z_{TH}^*$$

Find the open circuit voltage below (V_{oc})



$$\frac{V_o - 12}{1} + \frac{V_o}{j1} + \frac{V_o - V_{oc}}{-j1} = 0$$

$$V_o - 12 - j\cancel{V_o} + j\cancel{V_o} - jV_{oc} = 0$$

$$\boxed{+V_o + (0-j)\,V_{oc} = 12}$$

(11.15) continued

At V_{oc} node

$$\frac{V_{oc} - V_o}{-j1} - 2V_o = 0$$

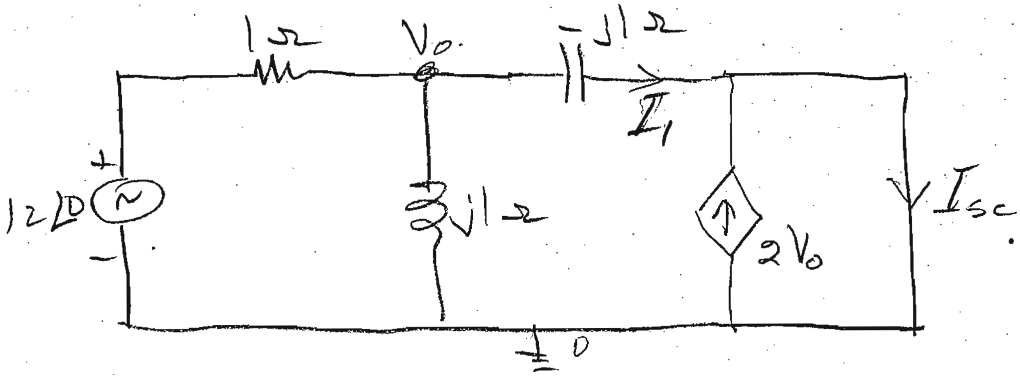
$$jV_{oc} - jV_o - 2V_o = 0$$

$$(-2-j)V_o + (0+j)V_{oc} = 0$$

$$\begin{bmatrix} (1) & (0-j) \\ (-2-j) & (0+j) \end{bmatrix} \begin{bmatrix} V_o \\ V_{oc} \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

$$V_o = 8.49\angle 135^\circ\,V \;;\quad V_{oc} = V_{TH} = 18.97\angle 71.56^\circ\,V$$

Now find V_{sc} in the following



$$\frac{V_o - 12}{1} + \cancel{\frac{V_o}{j1}} + \cancel{\frac{V_o}{-j}} = 0$$

$$V_o = 12\ V$$

(11.15) continued

Now $I_1 + 2V_o = I_{sc}$

$$I_1 = \frac{V_o}{-j} = \frac{12}{-j}$$

so

$$\frac{12}{-j} + 2 \times 12 = I_{sc}$$

so

$$I_{sc} = (24 + j12)$$

$\therefore$

$$Z_{TH} = \frac{V_{TH}}{I_{sc}} = \frac{18.97\angle 71.6}{(24+j12)}$$

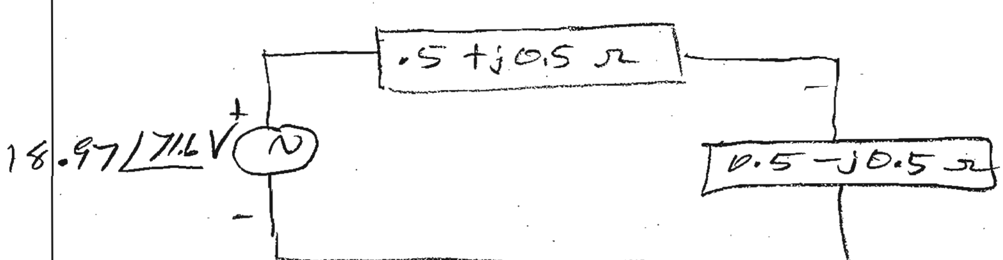
$$Z_{TH} = 0.707\angle 45^\circ = 0.5 + j0.5$$

This means that

$$\boxed{Z_L = 0.5 - j0.5}$$

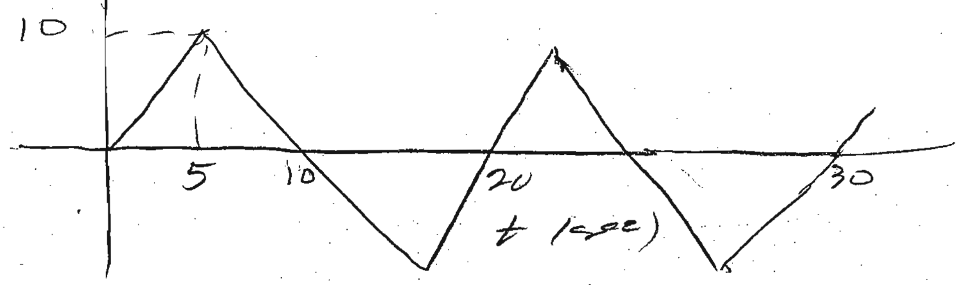
$$P_L = \left(\frac{|V_{TH}|}{1}\right)^2 \times \frac{0.5}{2} = \underline{89.96\,W \doteq 90\,W}$$

Thevenin Circuit plus load



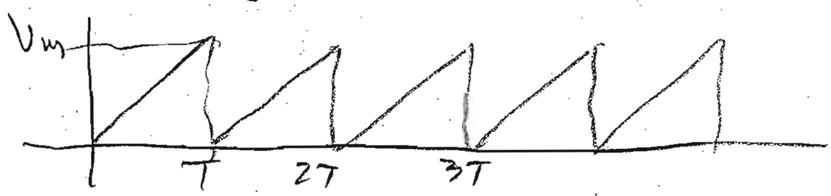
(11.29)

Find the Rms Value of the following.



By inspection, the Rms value of the above wave form with a period of 10sec. is the same as the Rms value of the wave form with a period of 20 sec.

We have shown in the lab exercise that the Rms value of the following



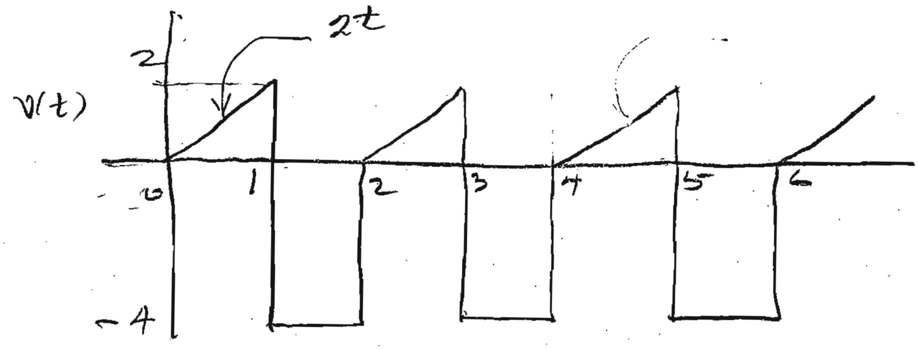
$$V_{rms} = \frac{V_m}{T}$$

Therefore the Rms value of the wave form at the top of this sheet is

$$\boxed{V_{rms} = \frac{10}{\sqrt{3}} = 5.774\ V}$$

(11.31)

Find the rms value of the following.



$$V_{rms} = \sqrt{\frac{1}{2}\left[\int_0^1 4t^2\,dt + \int_1^2 16\,dt\right]}$$

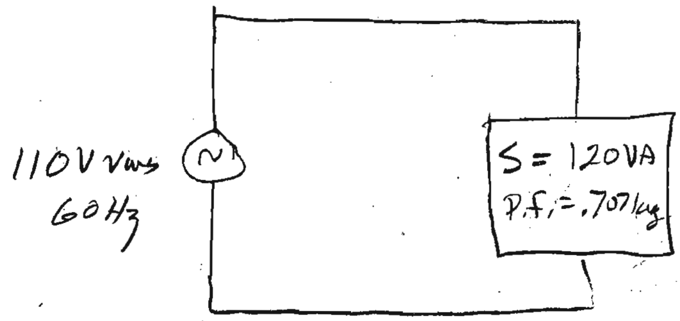
$$V_{rms} = \sqrt{\frac{1}{2}\times\frac{4t^3}{3}\Big|_0^1 + \frac{1}{2}\times 16t\Big|_1^2}$$

$$V_{rms} = \sqrt{\frac{4}{6} + \frac{48}{6}}$$

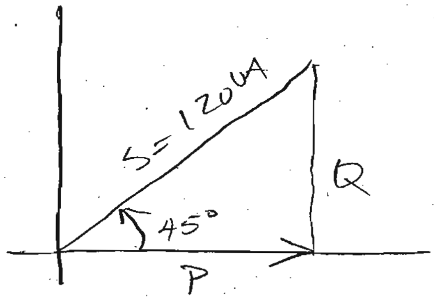
$$V_{rms} = \sqrt{\frac{52}{6}}$$

$$V_{rms} = 2.94\ V$$

(11.42)



Draw the power triangle



$\cos\theta = .707$

$\theta = 45°$

(a) Calculate the complex power

$\vec{S} = 120\angle 45° \text{ VA}$

$\vec{S} = 120\cos 45 + j\,120\sin 45$

$\vec{S} = (84.85 + j\,84.85)\text{ VA}$

(b) Find the rms current supplied to the load

$|\vec{S}| = 120\text{ VA} = |V_{source}| \times |I_{source}|$

$I_{SOURCE} = I_{LOAD} = \dfrac{120}{110} = 1.09\text{ A rms}$

(11.42) continued

(c) Determine $\vec{Z}$

$$\vec{S} = |I_{LOAD}|^2 \times \vec{Z}$$

$$\vec{Z} = \frac{120\angle 45}{(1.09)^2}$$

$$\tilde{Z} = 101\angle 45^\circ\ \Omega$$

$$Z = (71.4 + j71.4)\ \Omega$$

(d) Assume $Z = R + j\omega L$
find R and L

$$Z = 71.4 + j71.4$$

$$Z = R + j\omega L$$

$$\boxed{R = 71.4\ \Omega}$$

$$\omega L = 71.4$$

$$L = \frac{71.4}{2\pi \times 60}$$

$$\boxed{L = 0.189\ H}$$

(11.47)

For each of the following cases, find
- complex power, $\vec{S}$
- average power, P
- reactive power, Q

(a) $v(t) = 112\cos(\omega t + 10°)\,V$

$i(t) = 4\cos(\omega t - 50°)\,A$

Now

$$\vec{S} = \frac{\vec{V} \times \vec{I}^*}{2} = \frac{(112\angle 10)(4\angle 50)}{2}$$

$$\vec{S} = 224\angle 60 = 224\cos 60 + j\,224\sin 60$$

$$\vec{S} = (112 + j\,194)\ VA$$

$$\vec{S} = P + jQ$$

$\boxed{P = 112\,W}$; $\boxed{Q = 194\ VARs}$

(b) $v(t) = 160\cos 377t$; $i(t) = 4\cos(377t + 45°)$

Use same procedure as above

$$\vec{S} = \frac{\vec{V} \times \vec{I}^*}{2} = \frac{(160\angle 0)(4\angle -45)}{2}$$

$$\vec{S} = 320\angle -45 = 226.3 - j\,226.3$$

$P = 226.3\ W$

$Q = -226.3\ VAR$ or 226.3 capacitive

(11.47) CONTINUED

(c) $\vec{V} = 80\angle 60$ rms $Z = 50\angle 30^\circ\ \Omega$

Now

$$\vec{S} = \frac{|V_{rms}|^2}{Z^*}$$

$$\vec{S} = \frac{(80)^2}{50\angle -30} = 128\angle 30 \ \text{VA}$$

$$\vec{S} = (110.85 + j\,64)\,\text{VA}$$

so

$\boxed{P = 110.85\ \text{W}}$; $\boxed{Q = 64\ \text{VAR}}$

(d) $\vec{I} = 10\angle 60$ rms, $Z = 100\angle 45^\circ\ \Omega$

Now

$$\vec{S} = |I_{rms}|^2 \vec{Z} = (10)^2 \times 100\angle 45$$

$$\vec{S} = (10\angle 45)\ \text{kVA}$$

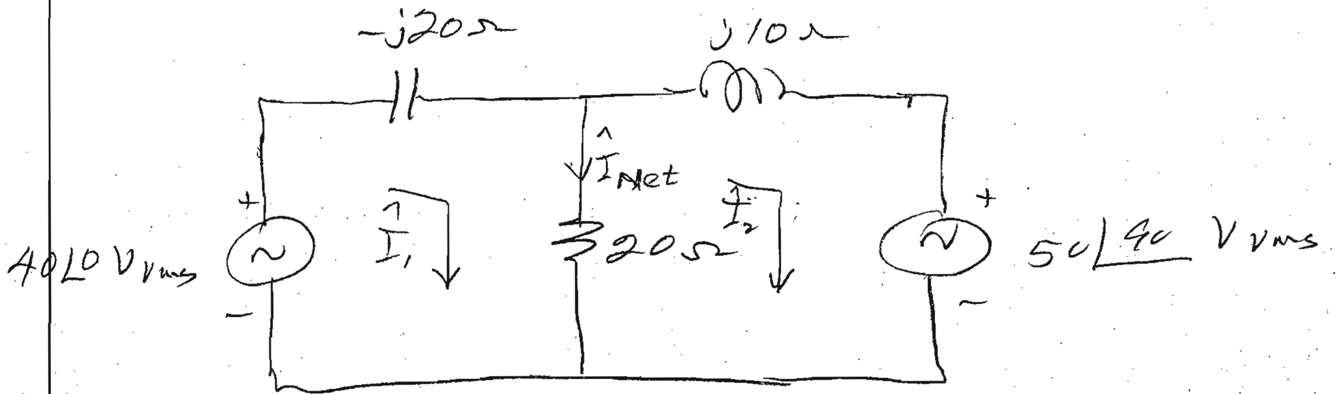
$$\vec{S} = (7.07 + j7.07)\ \text{kVA}$$

$$P = 7.07\ \text{kW}$$

$$Q = 7.07\ \text{VARs}$$

(11.55)

Find the complex power absorbed by each of the 5 elements in the following circuit.



$$\begin{bmatrix} (20-j20) & -20 \\ -20 & 20+j10 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 40\angle 0 \\ -j50 \end{bmatrix}$$

$\vec{I}_1 = 3.54 \angle 8.13$ A rms

$\vec{I}_2 = 3.61 \angle -56.3$ A rms

40 V source

$\vec{S}_{ABSORBED} = (-40)(\vec{I}_1^{*}) = 141.6 \angle 171.9°$

$S_{ABSORBED} = (-140.2 + j19.96)$ VA

Capacitor

$\vec{S} = |\vec{I}_1|^2 \times (-j20) = -j\,250.6$ VAR or VA

resistor

$\vec{S} = |I_{NET}|^2 \times 20$

(11.55)

$\vec{I}_{NET} = \vec{I}_1 - \vec{I}_2 = 3.81 \angle 66.8^\circ$ A

$\vec{S} = (3.81)^2 \times 20 = 290.3$ W or VA

inductor

$\vec{S} = |I_2|^2 j10 = j36.1$ VAR or VA

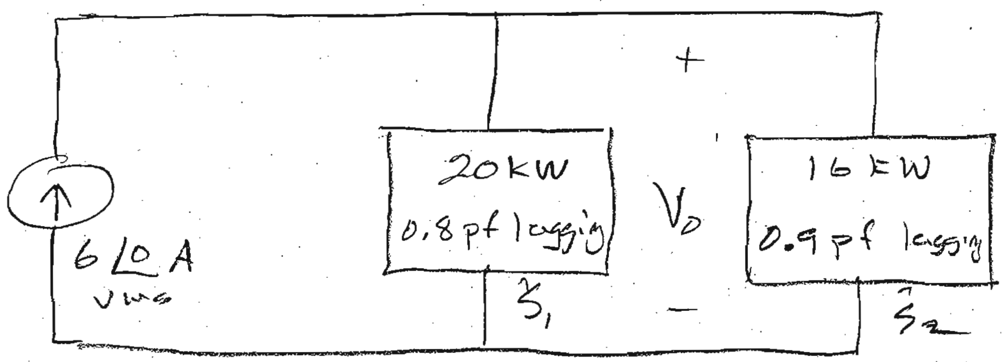
50 volt source

$\vec{S} = ((50\angle 90)(3.61\angle 56.3))$

$\vec{S}_{Absorbed} = 180.5 \angle 146.3$

$S_{ABSORBED} = (-150.17 + j100.15)$ VA

11.60 For the circuit below, find V_o and the input power factor



First find the combined complex power load; $\vec{S} = \vec{S}_1 + \vec{S}_2$

<u>For $\vec{S}_1$</u>

$\cos\theta_1 = .8 = \frac{20k}{S_1}$ $\qquad \theta_1 = \cos^{-1}(.8) = 36.9°$

$S_1 = \frac{20k}{.8} = 25$ KVA

$\vec{S}_1 = 25\angle 36.9°$ KVA

<u>For $\vec{S}_2$</u>

$\cos\theta_2 = .9 = \frac{16K}{S_2}$ $\qquad \theta_2 = \cos^{-1}(.9) = 25.8°$

$S_2 = \frac{16K}{.9} = 17.78$ KVA

$\vec{S}_2 = 17.78\angle 25.8°$ KVA

$\vec{S} = \vec{S}_1 + \vec{S}_2 = 42.59\angle 32.29°$ KVA

11.60 continued

Now

$$\vec{S} = \hat{V}_{rms} I_{rms}^{*}$$

so

$$\hat{V}_{o_{rms}} = \frac{\hat{S}}{I_{rms}^{*}} = \frac{42.59 \angle 32.29}{(6 \angle 0)}$$

$$\boxed{\vec{V}_{o_{rms}} = 7.098 \angle 32.29^{\circ} \text{ V}}$$

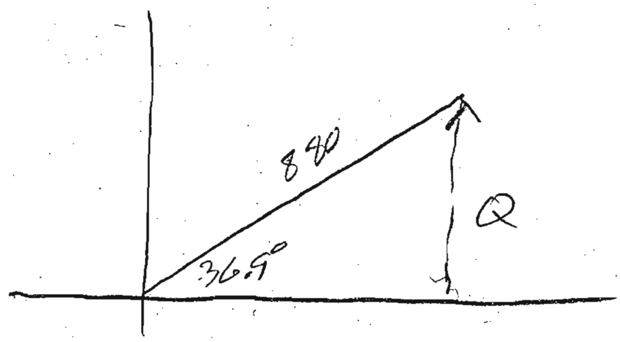
The power factor angle is the same angle as the composite complex power, $\vec{S}$.

$\therefore$ $\boxed{\text{p.f.} = \cos(32.29) = 0.8454 \text{ lagging}}$

11.70

An 880-VA, 220-V, 50Hz load has a pf of 0.8 lagging. What value of parallel capacitance will correct the load power factor to unity.

Power Triangle



$\theta = \cos^{-1} .8 = 36.9°$

$$\frac{Q}{880} = \sin 36.9 = 0.6$$

$$Q = 880 \times 0.6 = 528 \text{ VARs}$$

A capacitor that will render this number (but in the opposite direction) is

$$\boxed{C = \frac{528}{\left(\frac{220}{\sqrt{2}}\right)^2 \times 2\pi \times 50} = 69.45\mu F}$$

note that the voltage is not given as Rms