7.10 RC unforced:

(a) Develop the differential equation in \( v_o(t) \) and solve in order to find \( v_e(t) \).

(b) Use the step-by-step method to find \( v_e(t) \).

Answers:

\[
v_o(t) = 9e^{-16.67 t} V u(t); \quad t_{\text{decay}} = 65.9 \text{ msec}
\]
7.10 continued

We write

\[ V_1 + V_0 = 0 \]
\[ i = C \frac{dV_0}{dt}, \quad v_0 \]
\[ RC \frac{dV_0}{dt} + V_0 = 0 \]
\[ \frac{dV_0}{dt} + \frac{V_0}{RC} = 0 \]
\[ RC = -3 \times 10^3 \times 20 \times 10^{-6} = 60 \times 10^{-3} = 0.06 \]

Then

\[ V_0 = k e^{-\frac{t}{0.06}} \]

At \( t = 0^+ \), \( V_0(0^+) = 7 \), giving

\[ V_{0(t)} = 7 e^{-16.67t} \text{ V mV t} \]

Determine the time required for \( V_{0(t)} \) to go to \( \frac{1}{3} \) \( V_{0(0)} = \frac{1}{3} \times 7 = 3 \)

\[ 3 = 7 e^{-16.67t} \]

\[ -16.67t \ln e = \ln \left( \frac{3}{7} \right) = -1.099 \]

\[ 16.67t = 1.099 \]

\[ t = 65.9 \text{ m sec} \]
10) Step By Step

\[ V_0 = V_0(\infty) + \left[ V(0^+) - V(\infty) \right] e^{-\frac{t}{\tau}} \]

\[ V_0(\infty) = 0 \]
\[ V_0(0^+) = 9 \]
\[ \tau = 0.06 \]

\[ V_0 = 9e^{-16.67t} \quad \text{for} \quad t \geq 18 \]
7.12 RL unforced:

(a) Develop the differential equation in \( i(t) \) and solve in order to find \( i(t) \).

(b) Use the step-by-step method to find \( i(t) \).

Answer:

\[
i(t) = 4e^{-2t} \ \text{A} \ u(t)
\]

Switch closed for a very long time, opened at \( t = 0 \). Find \( i(t) \), \( t > 0 \).

For \( t < 0 \):

The 4 \( \Omega \) resistor is shorted. Thus,

\[
i(0^-) = \frac{12}{3} = 4 \ \text{A}
\]

Now \( i(0^+) = i(0^-) \) because the current through the inductor cannot change instantaneously.

\[
i(0^+) = 4 \ \text{A}
\]
2.12 continued

For $t > 0$

We write:

\[
\begin{align*}
R \frac{d^2i}{dt^2} + L \frac{di}{dt} + \frac{R}{L} i(t) &= 0 \\
\frac{d^2i}{dt^2} + \frac{R}{L} i(t) &= 0 \\
\frac{R}{L} i(t) &= \frac{1}{\gamma} = \frac{4}{2} = 2 e^{-2t} \\
\frac{d^2i}{dt^2} + 2(i(t)) &= 0 \\
\phi(t) &= k e^{-2t} u(t) \\
i(0^+) &= 4 = k \\
i(t) &= 4 e^{-2t} A, u(t) \\
\end{align*}
\]

(b) \[i(0^+) = 4, \quad i(\infty) = 0, \quad \gamma = \frac{1}{4}, \quad i(t) = \phi(t) + \left[ i(0^+) - i(\infty) \right] e^{-\gamma t} \]

\[i(t) = 4 e^{-2t} A, u(t) \]

Step-by-step
7.18 RL unforced:

Work using the step-by-step method. Answer: \( v_o(t) = 1.2e^{-3t} \ V \ u(t) \)

Determine \( v_0(t) \) when \( i(0) = 1 \ A \).

\[ \begin{align*}
7 & \quad 2\pi \\
4.4 \ H & \\
V_0 & = 3 \pi \nonumber \\
\end{align*} \]

\[ V_{o1}(\infty) = 0 \] \[ V_{o1}(0^+) = \frac{1 \times 2^2}{5} \times 3 = \frac{6}{5} = 1.2 \ V \]

\[ \gamma = \frac{L}{RC} = \frac{4}{0.13} = \frac{4 \times 5}{6} = \frac{1}{3} < 1 \]

\[ V_0(t) = V_{o1}(\infty) + \left[ V_{o1}(0^+) - V_{o1}(\infty) \right] e^{-\frac{t}{\tau}} \]

\[ V_0(t) = 1.2 e^{-3t} \ V \ u(t) \]

Step-by-step
7.41 RC forced:

Use the step-by-step method to solve for \( v(t) \). Answer: \( v(t) = 10(1 - e^{-0.2t}) \) V u(t)

**For \( t > 0 \)**

Find \( v(t) \) for \( t > 0 \).

**For \( t < 0 \)**

Switch open: \( v(0^-) = 0 \)

\( v(0^-) = 0 \)

**For \( t > 0 \)**

\( t = \infty \)

\[ V(\infty) = \frac{12 \times 30}{30 + 6} = 10 \text{ V} \]

\( \gamma = \frac{1}{RC} = \frac{1}{30 \times 6} = 5 \)

\[ v(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-\frac{t}{\gamma}} \]

or

\[ v(t) = 10 - 10e^{-0.2t} \text{ V u}(t) \]
7.45 RC forced:

Use the differential equation method to develop the differential equation whose solution gives \( v_o(t) \).

Answer: \( v_o(t) = 4 - 3e^{-14.3t} \cdot V \ u(t) \)

Find \( v_o(t) \) when \( v_o = 6 \ u(t) \).

Assume \( v_o(0) = 1 \ V \)

\[
\begin{align*}
\text{Finding } v_o(t) & \text{ when } v_o = 6 \ u(t) \\
\text{Assume } v_o(0) & = 1 \ V
\end{align*}
\]

Using Thévenin to the left of a-b:

\[
R_{Th} = \frac{(60 \ \Omega)(40 \ \Omega)}{20 \ \Omega + 40 \ \Omega} = \frac{60 \ \Omega}{60} = 1 \ \Omega
\]

\[
V_{Th} = \frac{6 \times 40 \ \Omega}{20 \ \Omega + 40 \ \Omega} = 4 \ V
\]

\[
\begin{align*}
7 & = R \frac{1}{C} = 23.3 \times 10^{-3} \times 3 \times 10^{-6} = 69.9 \times 10^{-3} \\
\frac{1}{7} & = 14.3 \ \text{sec}^{-1}
\end{align*}
\]

We write

\[
R i + v_o = 4
\]

\[
i = C \frac{dv_o}{dt}
\]
\[ R C \frac{dV_o}{dt} + \frac{V_o}{t} = 4 \]

\[ \frac{dV_o}{dt} + \frac{V_o (t)}{RC} = \frac{4}{RC} \]

\[ \frac{dV_o}{dt} + 14.3 V_o (t) = 14.3 \times 4 \]

\[ V_o = V_{0p} + V_{0c} \]

\[ V_{0p} = k_p \]

\[ k_p = \frac{1}{e^{k_p t} + 0} \]

\[ 14.3 k_p = 14.3 \times 4 \]

\[ k_p = 4 \]

\[ V_o = 4 + k_c e^{-14.3 t} \]

\[ V_o (t) = 4 - 3 e^{-14.3 t} \]

\[ V_o (t) = 4 - 3 e^{-14.3 t} \]
7.56 RL forced:

Use the differential equation method to develop the differential equation whose solution gives \( v(t) \).

Answers:

\( i(0) = 2 \) needed as an initial condition along the way.

\[ v(t) = -4e^{-20t} \ \text{V} \ \text{u}(t) \]

\[ u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases} \]

\[ \begin{cases} \frac{5}{2} & \text{at } t = 0 \\ \frac{1}{2} \end{cases} \]

\[ 20 \text{V} \]

\[ 2A \]

\[ 12 \Omega \]

\[ 20 \Omega \]

\[ 5 \Omega \]

\[ 3 \text{A} \]

\[ 6 \Omega \]

\[ 5 \text{V} \]

\[ 5 \text{V} \]

\[ 3 \text{V} \]

\[ 12 \text{V} \]

\[ 10 \text{V} \]

\[ 360 \]

\[ 30 \text{V} \]

\[ V_x = 12 \text{V} \]

\[ i(10^{-4}) = \frac{12}{6} = 2 \text{A} \]

\[ i(10^{-4}) = 2 \text{A} \]
For $t > 0$

Find the Thevenin to the left of $a-b$.

\[ R_{TH} = 6 + 20/15 = 10 \text{ ohm} \]

\[ V_{TH} = \frac{20 \times 20}{20 + 5} = 16 \text{ V} \]

**Figure A:**

\[ 10\alpha + V = 16 \]

But \[ \alpha = \frac{1}{10} \left( \int_{0}^{t} (v(t)) dt + 2/100 \right) \]

\[ \left[ 10 \left( \frac{2}{10} \int_{0}^{t} v(t) dt + 2/100 \right) \right] + V = 16 \quad (1) \]

Take the derivative of (1) w.r.t
\[ V(t) + \frac{dv}{dt} = 0 \]

\[ \frac{dv}{dt} + 30v(t) = 0 \]

\[ v(t) = K e^{-20t} \]

with \( v(0) = \Theta \); \( V_r = 30V \)

\[ V(0) = -4V \]

\[ V(t) = -4 e^{-20t} V \]

Another way:

From Figure A, page 2

\[ R_i + L \frac{di}{dt} = 10 \]

\[ \frac{di}{dt} + \frac{R}{L} i = \frac{10}{L} \]

\[ \frac{di}{dt} + 20i = 32 \]

\[ i = i_p + i_c \]

\[ i_p = 1.6 + \]

\[ i_c = K e^{-20t} \]

\[ i = 1.6 + K e^{-20t} \]

\[ i(0) = 2 = 1.6 + Kc \]

\[ Kc = 0.4 \]
\[ f(t) = 1.6 + 0.4 e^{-20t} \]

\[ V = \frac{L \frac{df}{dt}}{20t} \]

\[ V = 15 \left[ -20 \times 4e^{-20t} \right] \]

\[ V(t) = -4e^{-20t} \]

Same as before
Use the step-by-step method to find $i(t)$: Then use $v(t) = L\frac{di}{dt}$.

Also, on this problem, use MATLAB to plot $v(t)$. On your plot, use graphical methods to show the one second time constant. This involves drawing a horizontal line along the voltage and then an appropriate vertical line to intersect the time axis at the time constant value. This is illustrated in the text and the lab manual. Run your MATLAB program out to 5 time constants. Include your MATLAB program and your plot with your homework.

Answers:

$$i(t) = \frac{5}{6}(1 + e^{-t}) \ A \ u(t)$$
$$v(t) = 1.67(1 - e^{-t}) \ V \ u(t)$$

![Diagrams showing circuit analysis and MATLAB plotting instructions.]
7.64 (cont.,)

\[ i'(10^4) = i'(10^7) = \frac{10}{6} = 1.667 \text{ A} \]

For \( t \to \infty \)

\[ i'(10^6) \]

\[ \text{L looks like a short circuit at } t = \infty \]

\[ i'(10^6) = \frac{V'(\infty)}{2} \]

by no S.R.

\[ \frac{V'(\infty) - 10 + \frac{V'(\infty)}{3} + \frac{V'(\infty)}{2}}{6} = 0 \]

\[ V'(\infty) - 10 = 2V'(\infty) + 3V'(\infty) = 0 \]

\[ 6V'(\infty) = 10 \]

\[ V'(\infty) = \frac{10}{6} = \frac{5}{3} \text{ V} \]

\[ i'(10^6) = \frac{5/3}{2} = \frac{5}{6} \text{ A} \]

\[ R_{eq} = 2 + 3 \frac{6}{6} = 2 + 2 = 4 \text{ R} \]

\[ L = 4 \text{ H} \]

\[ \frac{L}{\mu} = 1 \text{ mH} \]
\[ s(t) = v(\infty) + \left( v(10^4) - v(\infty) \right) e^{-\frac{t}{10^4}} \]

\[ v(\infty) = \frac{5}{6}, \quad v(10^4) = \frac{10}{6} \]

\[ v(t) = \frac{5}{6} + \left( \frac{10}{6} - \frac{5}{6} \right) e^{-\frac{t}{10^4}} \]

\[ v(t) = \frac{5}{6} + \frac{5}{6} e^{-\frac{t}{10^4}} \]

\[ \frac{d}{dt} \left[ \frac{\partial^2 s}{\partial t^2} \right] = \frac{10}{6} - \frac{10}{6} e^{-\frac{t}{10^4}} \]

\[ \frac{d}{dt} \left[ \frac{\partial^2 s}{\partial t^2} \right] = \frac{5}{3} \left( 1 - e^{-\frac{t}{10^4}} \right) v(t) \]

**Now do MATLAB:**

**Run for \( t = 5 \)**

**\( \Delta t = 0.005 \)**

**Plot**
Look for

\[(1 - e^{-1}) \frac{5}{2} = 1.052\]
History: This program is written as part of HW #7, P7.64 ECE 300, F2007
This is a simulation of the voltage across the specified resistor.

5/3(1-exp(-t)) V.
The program is written on my office computer. Written by wlg 10/21/07
Program name. plot7_64.m

t = 0:.005:6;

v = (5/3)*(1 - exp(-t));

plot(t,v)
grid
ylabel('Voltage (volts)')
xlabel('time (sec)')
title('Voltage Response for Program plot7_underscore64')