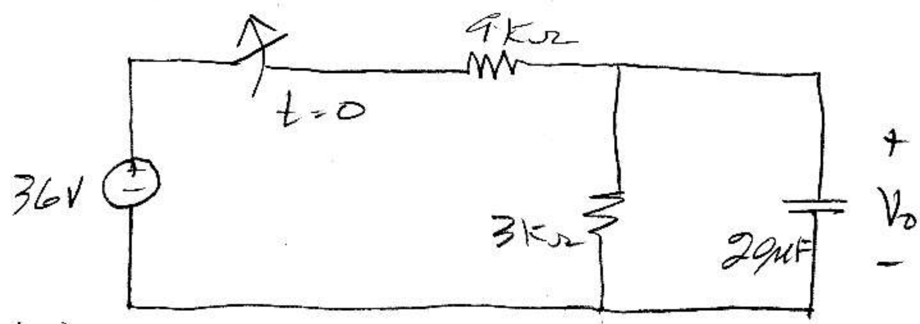
ECE 300
HW # 6
Fall 2007

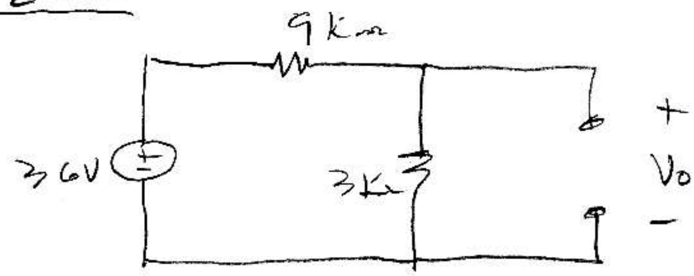
7.10 RC unforced:

(a) Develop the differential equation in $v_o(t)$ and solve in order to find $v_o(t)$.
(b) Use the step-by-step method to find $v_o(t)$.
Answers:

$v_o(t) = 9e^{-16.67t}$ V u(t); $t_{decay} = 65.9$ msec



1(a)
FOR t < 0

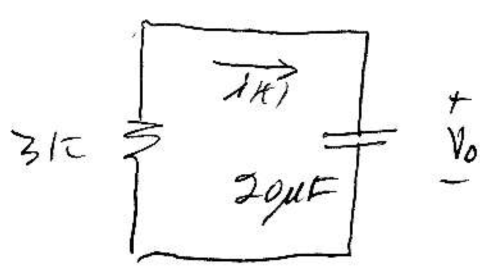


capacitor looks like an open circuit

$$V_o = \frac{36 \times 3K}{3K + 9K} = 9V$$ This is $V_o(0^-)$

Now $V_c(0^+) = V_c(0^-)$ because the voltage across the capacitor cannot change instantaneously.

FOR t > 0



7.10 continued

We write

$$Ri + V_o = 0$$

$$i = C\frac{dV_o}{dt}, \quad \text{so}$$

$$RC\frac{dV_o}{dt} + V_o = 0$$

$$\frac{dV_o}{dt} + \frac{V_o}{RC} = 0$$

$$RC = 3\times 10^3 \times 20\times 10^{-6} = 60\times 10^{-3} = .06$$

Then

$$V_o = K e^{-\frac{t}{.06}}$$

At $t = 0^+$, $V_o(0^+) = 9$, giving

$$\boxed{V_o(t) = 9e^{-16.67t}\,V \quad u(t)}$$

Determine the time required for $V_o(t)$ to go to $\frac{1}{3}V_o(0) = \frac{1}{3}\times 9 = 3$

$$3 = 9e^{-16.67t}$$

$$-16.67t \ln e = \ln\left(\tfrac{3}{9}\right) = -1.099$$

$$16.67t = 1.099$$

$$\boxed{t = 65.9 \text{ msec}}$$

7.10 Continued

(b) Step-By-Step

$$V_o = V_o(\infty) + [V_o(0^+) - V_o(\infty)]e^{-\frac{t}{\tau}}$$

$V_o(\infty) = 0$

$V_o(0^+) = 9$

$\tau = .06$

$\therefore$ $\boxed{V_o = 9e^{-16.67t} \text{ V}, \; u(t)}$

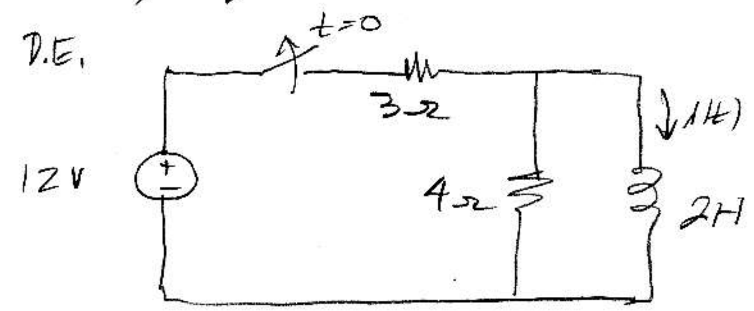
7.12 RL unforced:

(a) Develop the differential equation in i(t) and solve in order to find i(t).
(b) Use the step-by-step method to find i(t).
Answer:

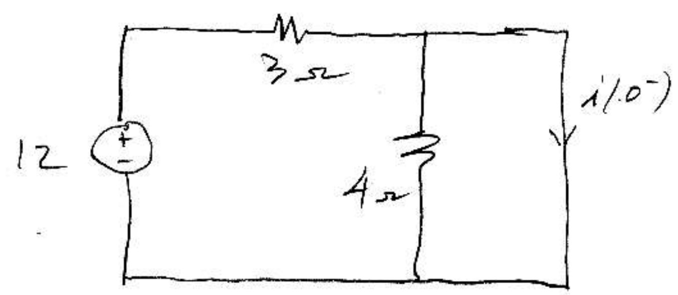
$$i(t) = 4e^{-2t}\ \text{A}\ \ u(t)$$

Switch closed for a very long time, opened at $t = 0$. Find $i(t)$, $t > 0$.

(a) D.E.



For $t < 0$:



The 4Ω resistor is shorted. Thus,

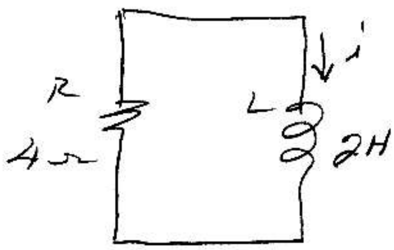
$$i(0^-) = \frac{12}{3} = 4\,A$$

Now $i(0^+) = i(0^-)$ because the current through the inductor cannot change instantaneously.

$$\boxed{i(0^+) = 4\,A}$$

7.12 continued

For $t > 0$



We write

$$Ri + L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L} i(t) = 0$$

$$\frac{R}{L} = \frac{1}{\tau} = \frac{4}{2} = 2 \sec^{-1}$$

$$\frac{di}{dt} + 2i(t) = 0$$

$$i(t) = k e^{-2t}\ A \quad u(t)$$

$$i(0^+) = 4 = k$$

$$\boxed{i(t) = 4e^{-2t}\ A, \quad u(t)}$$

(b) $i(0^+) = 4, \quad i(\infty) = 0, \quad \tau = \frac{1}{4}$

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$$

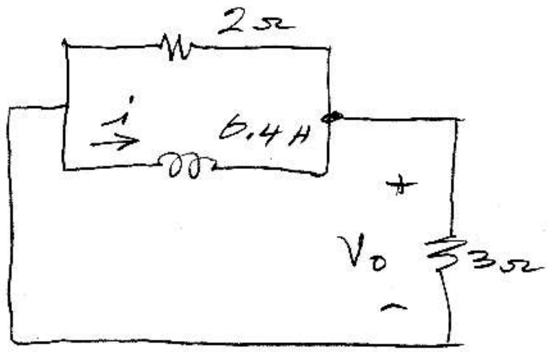
$$\boxed{i(t) = 4e^{-2t}\ A \quad u(t)}$$

step-by-step

7.18 RL unforced:

Work using the step-by-step method. Answer: $v_o(t) = 1.2e^{-3t}$ V u(t)

Determine $v_0(t)$ when $i(0) = 1$ A.
use step-by-step



$V_0(\infty) = 0$; $V_0(0^+) = \frac{1\times 2}{5}\times 3 = \frac{6}{5} = 1.2V$

$\tau = \frac{L}{R_T} = \frac{.4}{2\|3} = \frac{.4\times 5}{6} = \frac{1}{3}$ sec

∴

$$V_0(t) = V_0(\infty) + [V_0(0^+) - V_0(\infty)]e^{-\frac{t}{\tau}}$$

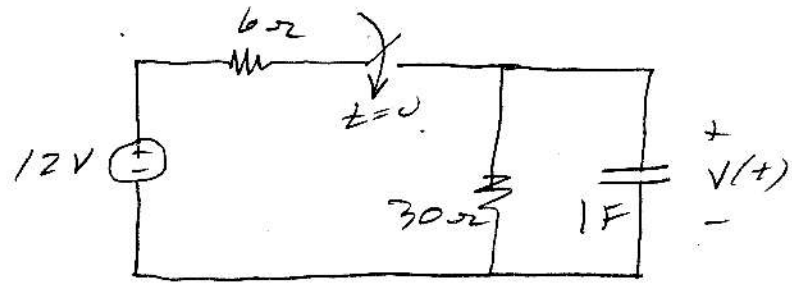
$$\boxed{V_0(t) = 1.2e^{-3t}\ V,\ u(t)}$$

step-by-step

7.41 RC forced:

Use the step-by-step method to solve for v(t). Answer: $v(t) = 10(1 - e^{-0.2t})$ V u(t)

Find $v(t)$ for $t > 0$.

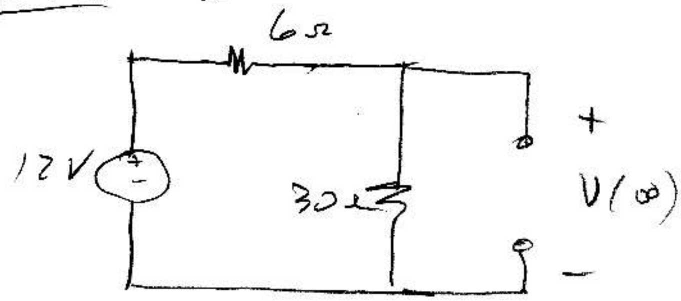


For $t < 0$

switch open: $V(0^-) = 0$

$\therefore V(0^+) = 0$

For $t > 0$ $t = \infty$



$$V(\infty) = \frac{12 \times 30}{30 + 6} = 10\ V$$

$$\tau = R_{eq} C = (30 /\!/ 6) 1 = 5$$

$\frac{180}{36}$

$$\therefore V(t) = V(\infty) + [V(0^+) - V(\infty)] e^{-\frac{t}{\tau}}$$

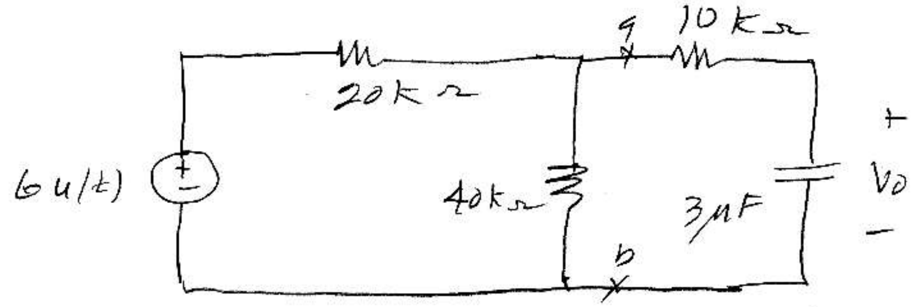
OR

$$\boxed{V(t) = 10 - 10 e^{-0.2t}\ V \quad u(t)}$$

7.45 RC forced:

Use the differential equation method to develop the differential equation whose solution gives $v_o(t)$.
Answer: $v_o(t) = 4 - 3e^{-14.3t}$ V u(t)

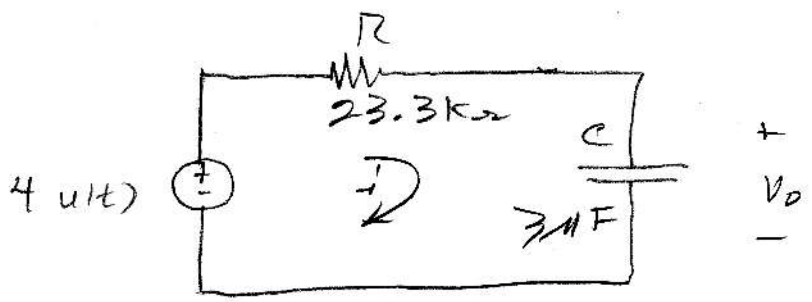
Find $V_o(t)$ when $V_s = 6u(t)$,
Assume $V_o(0^-) = 1V$



Using Thevenin to the left of a-b:

$$R_{TH} = \frac{(20k)(40k)}{20k + 40k} = \frac{800k}{60} = 13.33k\Omega$$

$$V_{TH} = \frac{6 \times 40k}{20k + 40k} = 4V$$



$$\tau = R_{eq}C = 23.3 \times 10^{3} \times 3 \times 10^{-6} = 69.9 \times 10^{-3}$$

$$\frac{1}{\tau} = 14.3 \text{ sec}^{-1}$$

We write

$$Ri + V_o = 4$$

$$i = C\frac{dV_o}{dt}$$

7.45 continued

$$RC\frac{dV_o}{dt} + V_o(t) = 4$$

$$\frac{dV_o}{dt} + \frac{V_o(t)}{RC} = \frac{4}{RC}$$

$$\frac{dV_o}{dt} + 14.3\,V_o(t) = 14.3 \times 4$$

$$V_o = V_{op} + V_{oc}$$

$$V_{op} = K_p$$

leads to

$$14.3\,K_p = 14.3 \times 4$$

$$K_p = 4$$

$$V_o = 4 + K_c e^{-14.3t}$$

$$V_o(0^+) = 1$$

$$1 = 4 + K$$

$$K = -3$$

$$\therefore \boxed{V_o(t) = 4 - 3e^{-14.3t} \; u(t)}$$

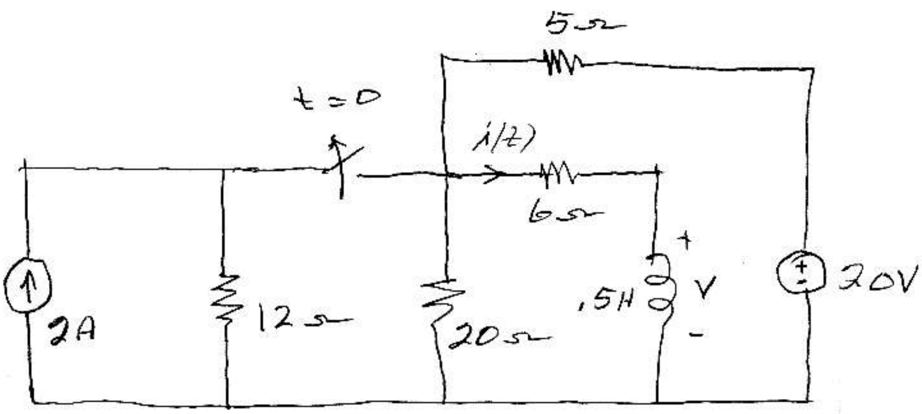
7.56 RL forced:

Use the differential equation method to develop the differential equation whose solution gives v(t)..

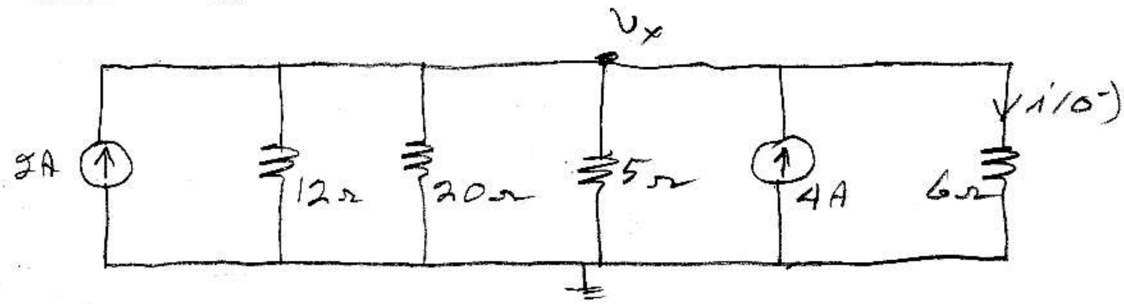
Answers:

$i(0) = 2A$ needed as an initial condition along the way.

$v(t) = -4e^{-20t}$ V u(t)



FOR $t < 0$



$$60\left(\frac{V_x}{12} + \frac{V_x}{20} + \frac{V_x}{5} + \frac{V_x}{6} = 6\right)$$

$$5V_x + 3V_x + 12V_x + 10V_x = 360$$

$$30V_x = 360$$

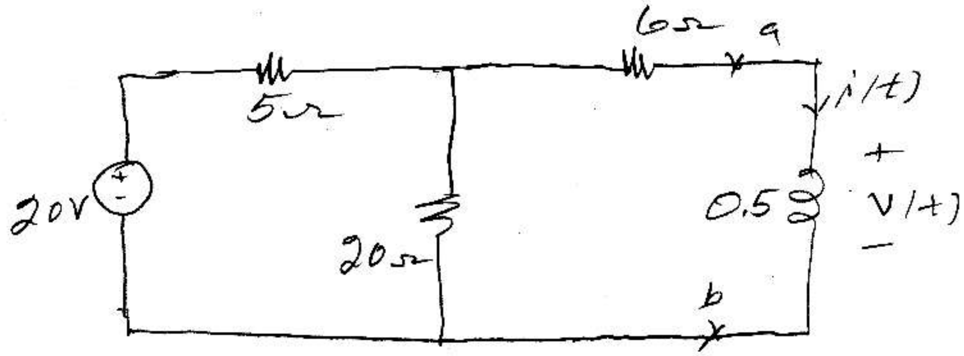
$$V_x = 12 \text{ V}$$

$$i(0^-) = \frac{12}{6} = 2A$$

$$\boxed{\therefore\ i(0^+) = 2A}$$

7.56

For $t > 0$



Find the Thevenin to the left of a-b.

$$R_{TH} = 6 + 20 \| 5 = 10\Omega$$

$$V_{TH} = \frac{20 \times 20}{20 + 5} = 16V$$

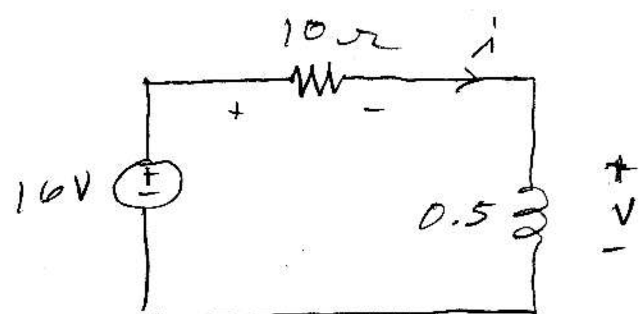


Figure A:

$$10i + V = 16$$

but $i = \frac{1}{L} \int_0^t v(t)dt + i(0^+)$

$$\left[10\left[2\int_0^t v(t)\,dt + i(0)\right]\right] + V = 16 \qquad (1)$$

Take the derivative of (1) wrt

7.56

$$20\,v(t) + \frac{dv}{dt} = 0$$

$$\frac{dv}{dt} + 20\,v(t) = 0$$

$$v(t) = K\,e^{-20t}$$

with $i(0^+) = 2A$; $V_R = 20V$

$\therefore\ V(0^+) = -4V$

$$\boxed{v(t) = -4\,e^{-20t}\ V}$$

Another way;

From Figure A, page 2

$$Ri + L\frac{di}{dt} = 16$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{16}{L}$$

$$\frac{di}{dt} + 20i = 32$$

$$i = i_p + i_c$$

$$i_p = 1.6\ A$$

$$i_c = K_c\,e^{-20t}$$

$$i = 1.6 + K_c\,e^{-20t}$$

$$i(0^+) = 2 = 1.6 + K_c$$

$$K_c = 0.4$$

7.56 cont.

$$i(t) = 1.6 + 0.4e^{-20t}$$

$$v = L\frac{di}{dt}$$

$$v = .5\left[-20 \times .4e^{-20t}\right]$$

$$\boxed{v(t) = -4e^{-20t}}$$

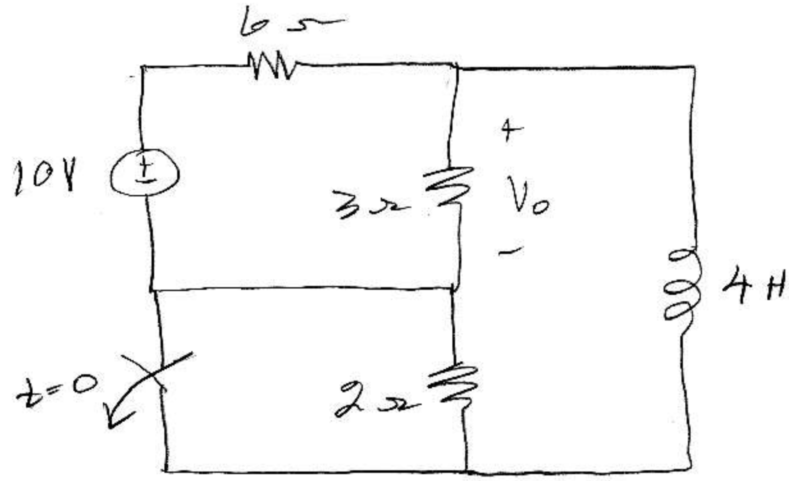
same as before

7.64 RL forced:

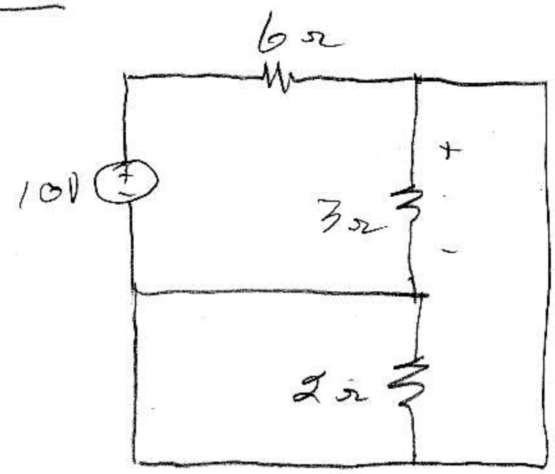
Use the step-by-step method to find i(t): Then use v(t) = Ldi/dt.

Also, on this problem, use MATLAB to plot v(t). On your plot, use graphical methods to show the one second time constant. This involves drawing a horizontal line along the voltage and then an appropriate vertical line to intersect the time axis at the time constant value. This is illustrated in the text and the lab manual. Run your MATLAB program out to 5 time constants. Include your MATLAB program and your plot with your homework.

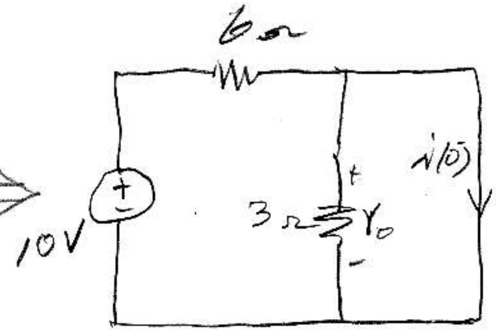
Answers:
$$i(t) = \frac{5}{6}(1+e^{-t}) \;\; A \quad u(t)$$
$$v(t) = 1.67(1-e^{-t}) \;\; V \quad u(t)$$



$t < 0$



⇒



The 2Ω resistor is shorted out

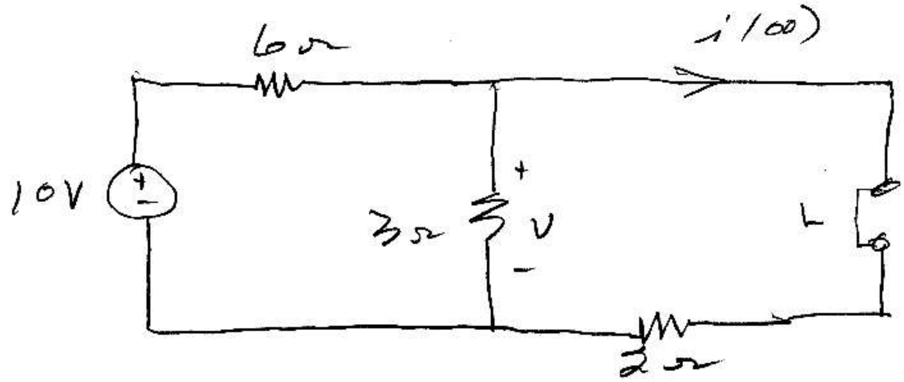
The 3Ω resistor is shorted out

②

7.64 cont,

$i(0^+) = i(0^-) = \frac{10}{6} = 1.667\ A$

For $t \to \infty$



L looks like a short circuit as $t \to \infty$.

$i(\infty) = \frac{V(\infty)}{2}$

by nodal

$\frac{V(\infty) - 10}{6} + \frac{V(\infty)}{3} + \frac{V(\infty)}{2} = 0$

$V(\infty) - 10 + 2V(\infty) + 3V(\infty) = 0$

$6V(\infty) = 10$

$V(\infty) = \frac{10}{6} = \frac{5}{3}$

$\boxed{i(\infty) = \frac{5/3}{2} = \frac{5}{6}\ A}$

$R_{eq} = 2 + 3 \parallel 6 = 2 + 2 = 4\ \Omega$

$L = 4\ H$

$\frac{L}{R} = 1$ sec

7.64 cont

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$$

$$i(\infty) = \frac{5}{6}, \quad i(0^+) = \frac{10}{6} \text{ A}$$

$$\tau = 1$$

$$i(t) = \frac{5}{6} + \left[\frac{10}{6} - \frac{5}{6}\right]e^{-t}$$

$$\boxed{i(t) = \frac{5}{6} + \frac{5}{6}e^{-t}}$$

$$v(t) = L\frac{di}{dt} + 2i(t)$$

$$= 4\left[-\frac{5}{6}e^{-t}\right] + \frac{10}{6} + \frac{10}{6}e^{-t}$$

$$= \frac{10}{6} - \frac{10}{6}e^{-t}$$

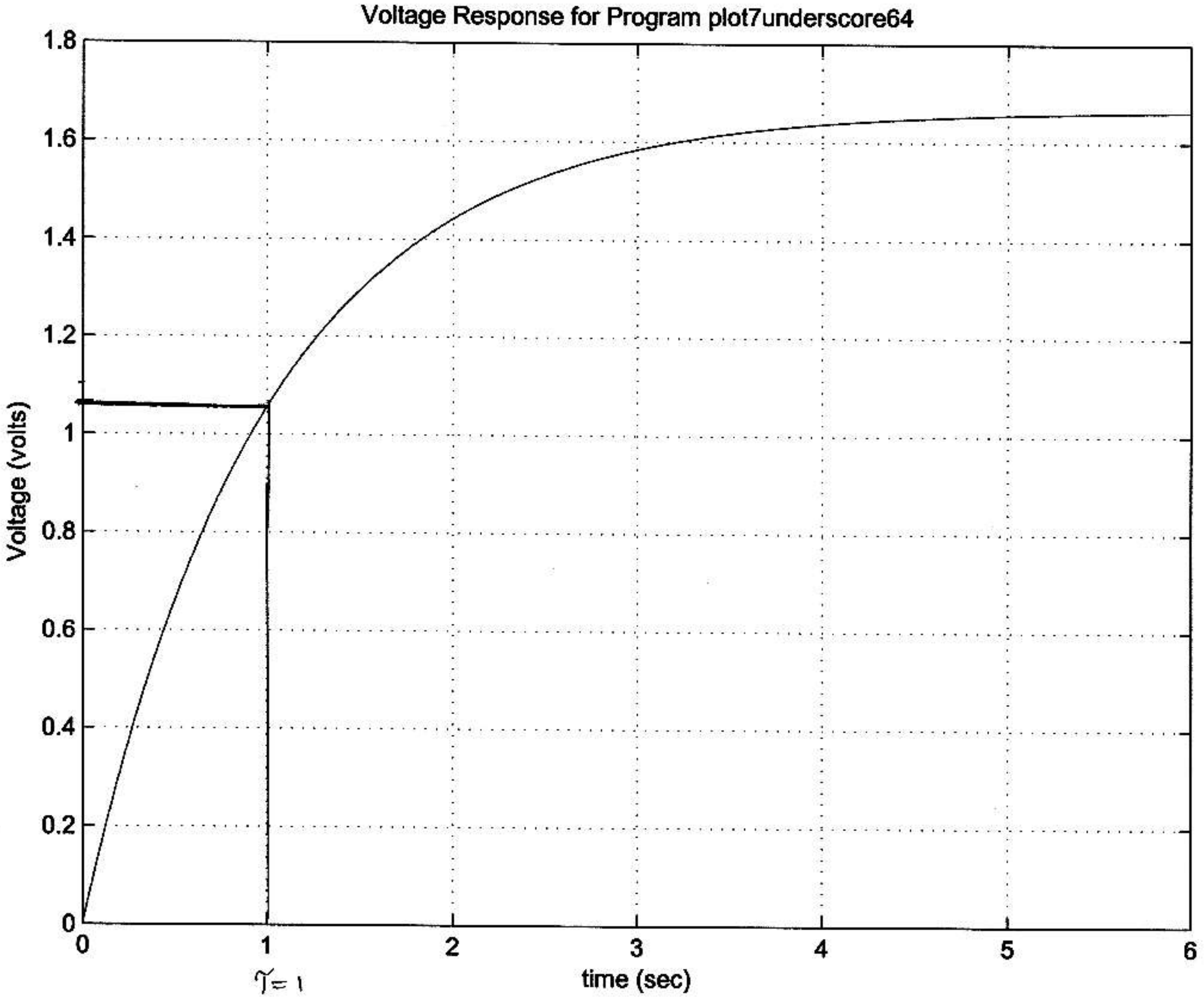
$$\boxed{V(t) = \frac{5}{3}(1 - e^{-t}) \text{ V} \quad u(t)}$$

Now do MATLAB;

Run for t=5

$\Delta t = .005$

Plot



Look for

$$(1-e^{-1})\frac{5}{3} = 1.053$$

```
% History:  This program is written as part of HW #7, P7.64 ECE 300, F2007
% This is a simulation of the voltage accross the specified resistor.
%
%    5/3(1-exp(-t)) V.
% The program is written on my office computer.  Written by wlg 10/21/07
% Program name.  plot7_64.m

t = 0:.005:6;

v = (5/3)*(1 - exp(-t));

plot(t,v)
grid
ylabel('Voltage (volts)')
xlabel('time (sec)')
title('Voltage Response for Program plot7underscore64')
```