8.16

$T<0$ 10Ω 60Ω $V_C(0^-) = 24V$

$I_L(0^-) = 0A$

$30V$ $24V$ 40Ω

$I_C = C\,dV/dT$

$V_L = L\,dI/dT$

① $40\Omega \cdot i + 60\Omega \cdot i + L\,di/dT + V_C = 0$

$R = 40\Omega + 60\Omega$ $V_C = \frac{1}{C}\int_{-\infty}^{T} i\,dT$

$Ri + L\,di/dT + \frac{1}{C}\int_{-\infty}^{T} i\,dT = 0$

$d^2i/dT^2 + \frac{R}{L}\,di/dT + \frac{i}{LC} = 0 \implies s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$S_1, S_2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \implies \dfrac{-\frac{R}{L} \pm \sqrt{(R/L)^2 - 4(\frac{1}{LC})}}{2}$

However, easier to do this

$\frac{1}{2}s^2 + (\frac{R}{2L})s + \frac{1}{2 \cdot LC} = 0$ divide both sides by 2

$S_1, S_2 = -(\frac{R}{2L}) \pm \sqrt{(\frac{R}{2L})^2 - \cancel{4}(\frac{1}{LC})(\cancel{\frac{1}{4}})}$ which gives

$S_1, S_2 = -\underset{\alpha}{(\frac{R}{2L})} \pm \sqrt{(\frac{R}{2L})^2 - (\frac{1}{\sqrt{LC}})^2} \to w_0$

$\alpha = \frac{R}{2L} = \frac{100}{5} = 20$ $w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times10^{-3} \cdot 2.5}} = 20$

$S_1 = S_2$

$I(T) = (A + BT)e^{-\alpha T} \implies (A + BT)e^{-20T}\,u(T)$ A

$I(0) = (A + B \cdot 0) \cdot 1 = A = 0$

$dI/dT = -20BTe^{-20T} + Be^{-20T}$

Using ①
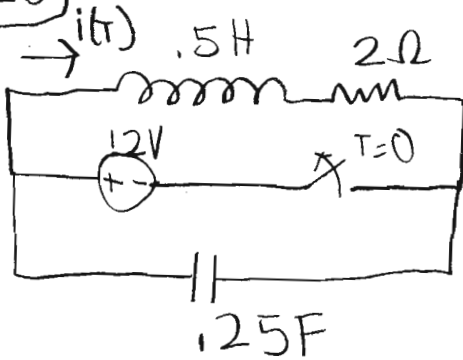
$di/dT = \frac{1}{L}(-Ri - V_C) = \frac{1}{2.5}(-100(0) - 24) = -9.6$

$-9.6 = -20B \cdot 0 \cdot 1 + B \cdot 1 \implies B = -9.6$

$I(T) = (-9.6\,T)e^{-20T}\,u(T)$ A

8.20

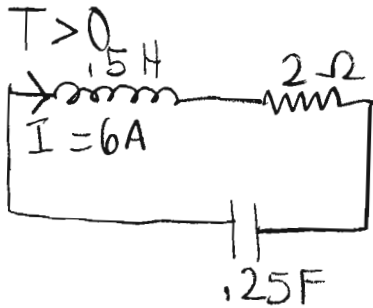$$V_L = L \frac{di}{dt} \qquad I_{C_0} = 0A$$
$$I_C = C \frac{dV}{dT} \qquad V_{L_0} = 0V$$
$$I_0 = \frac{12V}{2\Omega} = 6A \qquad V_{C_0} = 12V$$

$$Ri + L\frac{di}{dt} + V_C = 0$$

$$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$$

$$\alpha = \frac{2\Omega}{2 \cdot .5H} = 2 \qquad W_0 = \frac{1}{\sqrt{.25 \cdot .5}} = 2.828$$

$$S_1, S_2 = \alpha \pm \sqrt{\alpha^2 - W_0^2}$$

$$S_1 = 2 + j2$$
$$S_2 = 2 - j2$$

$$i(t) = (A_1 \cos 2t + A_2 \sin 2t) e^{-2t} A \, u(t)$$

$$A_1 = i(0) = 6A$$

$$\frac{di}{dt} = -\frac{1}{L}(-Ri - V_C) = -2(-12 + 12) = 0$$

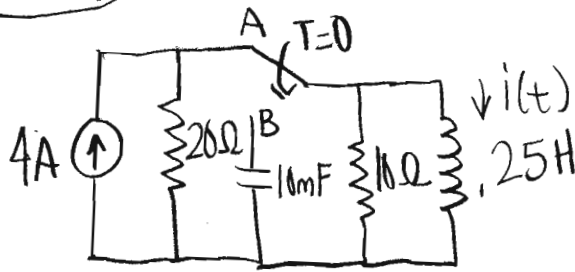$$\frac{di}{dT} = -2(A_1 \cos 2T + A_2 \sin 2T)e^{-2T} + (-2A_1 \sin 2T + 2A_2 \cos 2T) e^{-2T}$$

$$\frac{di}{dT} = -2A_1 + 2A_2$$

$$2A_1 = 2A_2 \qquad A_2 = 6A$$
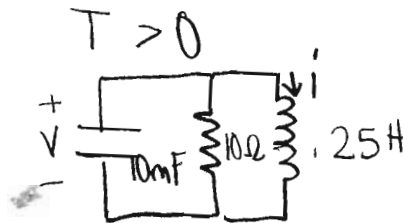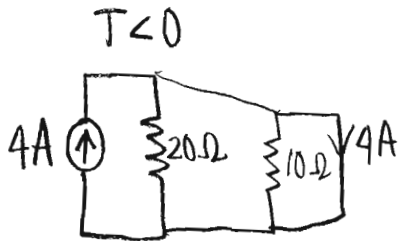
$$i(T) = [6\cos 2T + 6\sin 2T]e^{-2T} A \, u(T)$$

(8.24)

$V_L = L \, di/dT$ $V_{C_0} = 0 \, V$

$I_C = C \, dV/dT$ $V_{L_0} = 0 \, V$

$I_0 = 4 \, A$

T<0

T>0

$V = L \, di/dT$

$\dfrac{V}{R} + I_L + C \, dV/dt$

$$\dfrac{L \, di/dT}{R} + i + CL \, d^2i/dT^2 \implies \dfrac{d^2i}{dT^2} + \dfrac{1}{RC}\dfrac{di}{dT} + \dfrac{i}{LC} = 0$$

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0 \implies S_1, S_2 = -\dfrac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$\alpha = \frac{1}{2RC} = \frac{1}{2} \cdot 10\,\Omega \cdot 10\times10^{-3}F = 5$ $W_0 = \frac{1}{\sqrt{.25 \cdot 10^{-3}}} = 20$

$S_1 = 5 + j\,19.4$

$S_2 = 5 - j\,19.4$

$i(T) = e^{-5T}(A_1 \cos 19.4T + B \sin 19.4T) \, A \; u(t)$

$i(0) = 4 = A_1$

$di/dT = (-5A_1 \cos 19.4T^{\nearrow 0} - 5B \sin 19.4T^{\nearrow 0} - 19.4 A \sin 19.4T^{\nearrow 0} + 19.4 B \cos 19.4T) \, e^{-5T}$
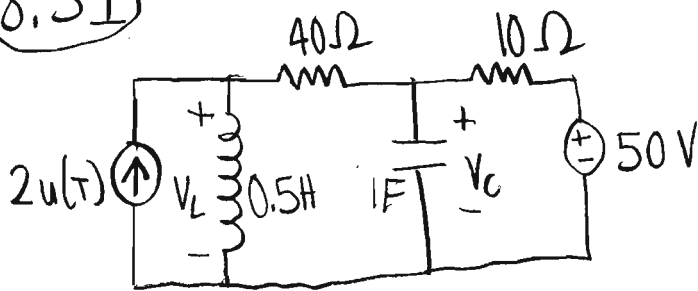
$\dfrac{di(0)}{dT} = \dfrac{V}{L} = \dfrac{0}{.25H} = 0$

$5 \cdot 4 = 19.4 \, B$ $B = 1.03$

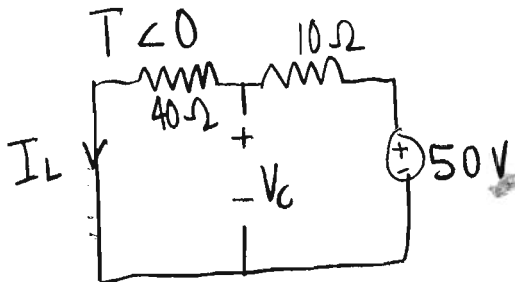$i(T) = (4 \cdot \cos 19.4T + 1.03 \sin 19.4T) e^{-5T} \, u(t) \, A$

HW 7

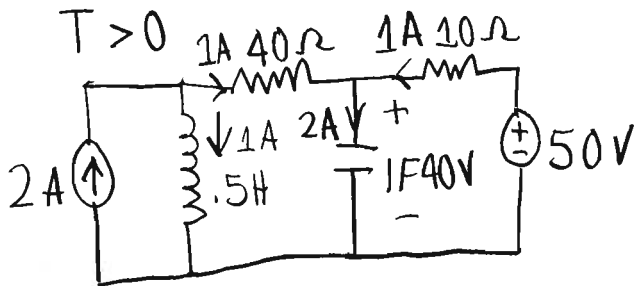## 8.31

$$I_C = C \frac{dV}{dT}$$
$$V_L = L \frac{di}{dT}$$

$T < 0$

$I_L(0^-) = 1A$
$V_C(0^-) = 40V$

$V_L(0^-) = 0V$
$I_C(0^-) = 0A$

$T > 0$

$V_C(0^-) = V_C(0^+)$
$V_C(0^+) = 40V$

$I_C(0^+) = 2A$

$$I_L(0^-) = I_L(0^+) = 1A$$

$$V_L = 40\Omega \cdot 1A + 40V = 80V$$

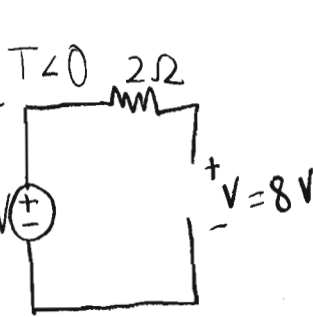$V_L(0^+) = 80V \qquad I_L(0^+) = 1A$

$V_C(0^+) = 40V \qquad I_C(0^+) = 2A$

HW 7

8.35

$I_C = C \frac{dV}{dT}$
$V_L = L \frac{di}{dT}$

$T < 0$

$I_{L_{0^-}} = 0A$
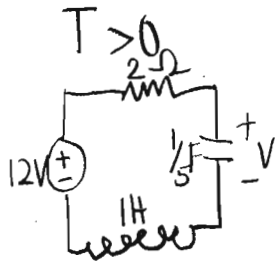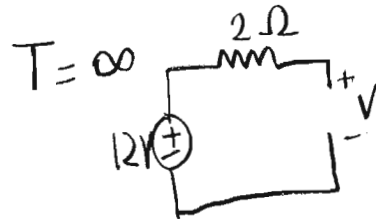$V_{0^-} = 8V$
$V_{L_0} = 0V$
$I_{C_0} = 0A$

$T > 0$

$\frac{dV(0)}{dT} = 0$

$T = \infty$

$-12V + 2\Omega \cdot I + V + L \frac{dI}{dT}$

$$\frac{d^2V}{dT^2} + \frac{R}{L}\frac{dV}{dT} + \frac{V}{LC} = \frac{12V}{LC} \Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\alpha = \frac{R}{2L} = \frac{2\Omega}{2H} = 1 \qquad W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \cdot \frac{1}{5}}} = 2.236$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - W_0^2} = -1 + j2$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - W_0^2} = -1 - j2$$

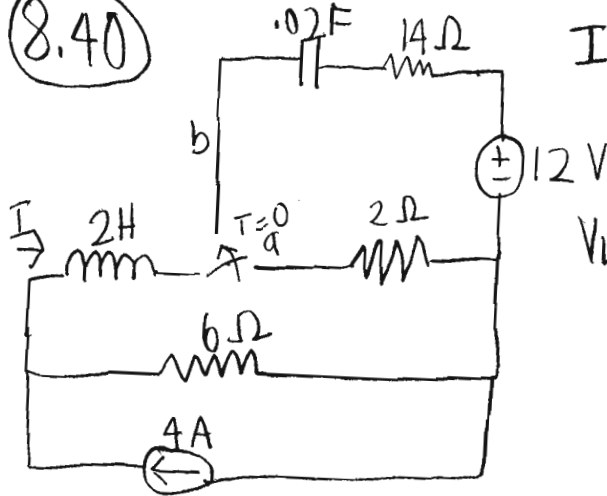$$V(T) = \underline{12} - (A\cos 2T + B\sin 2T)e^{-T} u(T) \, V$$

$$V(0^+) = 8V = 12 - A \qquad A = 4$$

$$\frac{dV}{dT} = (-A\cos 2T - B\sin 2T - 2A\sin 2T + 2B\cos 2T)e^{-T}$$
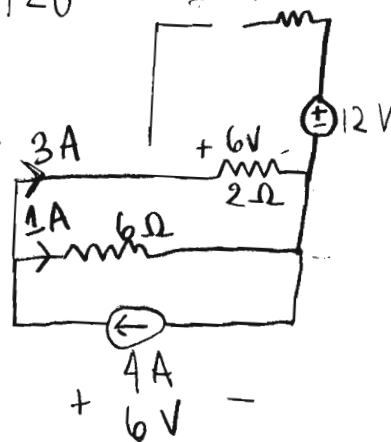
$$\frac{dV}{dT} = -A + 2B \qquad B = 2$$

$$V(T) = 12 - (4\cos 2T + 2\sin 2T)e^{-T} u(T) \, V$$

(8.40)

$I_C = C \frac{dV}{dT}$

$T < 0$

$V_C(0^-) = 0V$
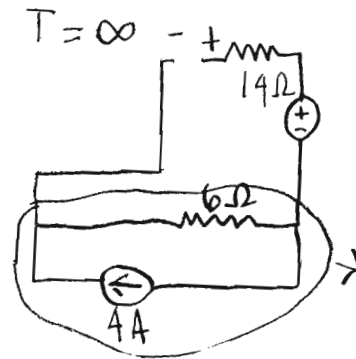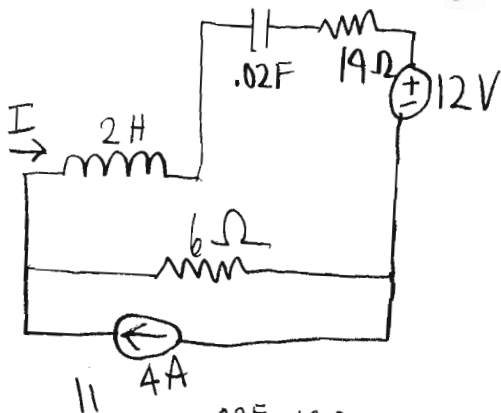
$V_L = L \frac{di}{dT}$ 3A

$I_L(0^-) = 3A$

$V_L(0^-) = 0V$

$I_C(0^-) = 0A$

.02F 14Ω

b

12 V

I 2H T=0 2Ω

+6V 12V

1A 6Ω

4A

+ 6V −

T > 0

T = ∞

$I(\infty) = 0$

.02F 14Ω 12V

14Ω

I 2H

6Ω

6Ω

4A

11 4A

6Ω 24V

$\frac{dI}{dT} = \frac{1}{L}(-20 \cdot 3A + 12V)$

$\frac{dI}{dT} = -24$

.02F 14Ω

I 2H

12V

+ −

6Ω

24V

$-24V + 6\Omega \cdot I + L\frac{dI}{dT} - V_C + 14\Omega \cdot I + 12V = 0 \Rightarrow \frac{d^2 I}{dT^2} + \frac{R}{L}\frac{dI}{dT} + \frac{1}{LC} = \frac{12V}{LC}$

$\alpha = \frac{R}{2L} = \frac{6\Omega + 14\Omega}{2 \cdot 2H} = 5$ $W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{.02 \cdot 2}} = 5$

$I(T) = (A + BT)e^{-5T} u(T) A$

$I(0) = A = 3$

$\frac{dI}{dT} = -5BTe^{-5T} - 5Ae^{-5T} + Be^{-5T} \Rightarrow \frac{dI}{dT} = -5A + B$

$B = -24 + 15 = 9$

$I(T) = (3 - 9T)e^{-5T} u(T) A$