Work the exam on your own engineering paper. Work on one side of your paper only. Attach your work to the back of this exam sheet and staple in the top left hand corner. Each problem 20%.

(1) You are given the circuit of Figure 1.

(a) Use mesh analysis to find the currents $i_1$, $i_2$, $i_3$ and $i_4$.
(b) How much power is supplied by the 24 V source?
(c) How much power is dissipated by the 3 $\Omega$ resistor?
Test B

\[ A^T A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \]

\[ A^T A = \begin{bmatrix} 6 & 0 & -2 & -2 \\ 0 & -2 & -1 & 7 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

\[ \begin{bmatrix} 6 & 0 & -2 & -2 \\ 0 & -2 & -1 & 7 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 24 \\ -9 \\ 4 \\ -2 \end{bmatrix} \]

\[ \lambda_1 = -0.425 \text{ A} \]

\[ \lambda_2 = 3.58 \text{ A} \]

\[ \lambda_3 = -2 \text{ A} \]

\[ \lambda_4 = -0.55 \text{ A} \]
(2) Use any method of your choice to answer the following questions about the circuit of Figure 2.
   (a) Determine the voltage $V_o$ as shown in Figure 2.
   (b) How much power is absorbed by the 10 Ω resistor shown in Figure 2.

First use nodal; make $V_x$ the unknown node voltage, write KCL at $V_x$.

$$\begin{align*}
3 \left( \frac{V_x - 2V_o + V_x}{6} + \frac{V_x - 12}{3} \right) &= 0 \\
5V_x - 10V_o + 3V_x + 10V_x - 120 &= 0 \\
6V_x + V_o - 12 &= 0 \\
V_o &= 12 - V_x \\
5V_x - 10(12 - V_x) + 3V_x + 10V_x &= 120 \\
28V_x &= 120 + 120 \\
V_x &= 8.57 V \\
V_o &= 12 - 8.57 = 3.43 V \\
V_o &= 3.43 V \\
\end{align*}$$

(a) $V_x = 8.57 V$

$$V_o = 12 - 8.57 = 3.43 V$$

$$V_o = 3.43 V$$

(b) $P_{10} = \frac{V_x^2}{10} = \frac{8.57^2}{10} = 7.34 W$
By mesh analysis

At A, \( \sum E = 0 \)
\[-2V_0 + 6i_1 + 10(i_1 - i_2) = 0 \]

\[16i_1 - 10i_2 = 2V_0 \]
But \( V_0 = -3i_2 \)

\[16i_1 - 10i_2 = 2(-3i_2) \]

\[16i_1 - 4i_2 = 0 \]

At B, \( \sum E = 0 \)
\[10(i_2 - i_1) + 3i_2 + 12 = 0 \]
\[-10i_1 + 13i_2 = -12 \]

\[
\begin{bmatrix}
16 & -4 \\
-10 & 13
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
-12
\end{bmatrix}
\]

(k)
\( i_1 = 0.2857 \) A, \( i_2 = -1.143 \) A
\( V_0 = -3i_2 = 3.433 \text{ V} \) OK

(1b) \[
P_{10} = (i_2 - i_1)^2 \times 10 = 7.35 \text{ W} \]
(3) Use any method to find the voltage $V_{ab}$ in the circuit of Figure 3.

Use nodal analysis to find $V_x$

\[
100 \left( \frac{V_x - 30}{20} + \frac{V_x - 10}{10} + \frac{V_x + 20}{50} = 0 \right)
\]

\[
5V_x - 150 + 10V_x - 100 + 2V_x + 40 = 0
\]

\[
17V_x = 210
\]

\[V_x = 12.35 \text{ V}\]

Apply KVL; \( \Sigma V \) rises = 0, clockwise

\[V_{ab} + 20 + V_x = 0\]

\[V_{ab} = -20 - V_x = -32.35 \text{ V}\]

\[\overline{V_{ab}} = -32.35 \text{ V}\]
(3) Use any method to find the voltage $V_{ab}$ in the circuit of Figure 3.

Using mesh analysis

By inspection

$$\begin{bmatrix} 30 & -10 \\ -10 & 40 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -30 \end{bmatrix}$$

$i_1 = 0.882 \, A$  \hspace{1cm} $i_2 = 0.647 \, A$

$V_{ab} = 50 \, i_2 = 0$

$V_{ab} = -32.75 \, V$  \hspace{1cm} \text{check}
(4) Find $R_{eq}$ for the circuit of Figure 4.

Starting at the right:

\[ \frac{12}{11} \div 6 = \frac{12 \times 6}{12 + 6} = \frac{12 \times 6}{18} = 4 \Omega \]

The \( 8 \div 4 = 12 \)

\[ 12 \div \frac{8}{2} \div \frac{4}{2} = 12 \Omega \]

\[ \frac{12}{11} \div 24 = 4 \Omega \]

\[ 8 \div 4 = 12 \Omega \]

Then

\[ 12 \div 24 = 8 \]

And

\[ 8 \div 8 = 4 \Omega \]

\[ R_{eq} = 4 \Omega \]
(5) You are given the circuit of Figure 5. Use any method to find the voltage $v_2$.

Write KCL using node $V_2$

\[
\frac{v_2 - 40}{10} + \frac{v_2}{5} - 0.8v_2 = 0
\]

\[v_2 - 40 + 2v_2 - 8v_2 = 0\]

\[-5v_2 = 40\]

\[v_2 = -8\text{ V}\]
(6) This problem is for extra credit. Part (a) is for 1 point. Part (b) is for 1 point. There is no partial credit given. In each case, the answer is either correct or not correct.

(a) Find the voltage $V_{ab}$ indicated in the diagram of Figure 6 (a)

$$V_{ab} = \frac{15 \times 10}{10 + 5} = 10 \text{ V}$$

(b) Give the value of the current $I_x$ in the 12 Ω resistor, up to the second significant decimal place, for the circuit shown Figure 6b.

$$I_x = \frac{0.1 \times 5}{4 \text{ MΩ}} = \frac{0.05}{4} \times 10^{-6}$$

$${I_x} \approx 0.00 \ldots \text{ to 2 significant figures}$$