(1) Consider the circuit shown in Figure 1. The switch has been in position “a” for a very long time and is switched to position “b” at $t = 0$.

(a) Develop and solve the differential equation for $v(t)$. Do not use the step-by-step method.

(b) Sketch the voltage $v(t)$ vs $t$ approximately to scale.

\[ \begin{align*}
\text{For } t &< 0 \\
\text{The capacitors change to the source voltage of } 40 \text{ V.} \\
V(0^-) &= 40 \text{ V}
\end{align*} \]

\[ \begin{align*}
\text{For } t > 0 \\
V(0^+) &= V(0^-) = 40 \text{ V}
\end{align*} \]

\[ \begin{align*}
R \frac{dv}{dt} + v(t) &= 0 \\
6 \text{ mA} \\
i &= \frac{dv}{dt} \\
RC \frac{dv}{dt} + v(t) &= 0 \\
\frac{\Delta v(t)}{dt} + \frac{v(t)}{RC} &= 0
\end{align*} \]
The solution to the 1st order differential equation is of the form

\[ V(t) = V_p(t) + V_c(t) \]

\[ V_p(t) = 0 \] because there is no forcing function. We know that \( V_c(t) \) is of the form

\[ V_c(t) = KE^{st} \]

\[ s + \frac{1}{RC} = 0 \]

\[ s = -\frac{1}{1 \times 10^4 \times 25 \times 10^{-6}} \]

\[ s = -0.4 \]

\[ T = RC = 2.5 \text{ sec} \]

\[ V = V_c = KE^{-0.4t} \]

We know that \( V(t=0) = 40 \) \( = KE \)

\[ \Rightarrow \]

\[ V(t) = 40e^{-0.4t} \]

\[ H0e^{-1} = 14.7 \]

\[ 40 \]

\[ 30 \]

\[ 20 \]

\[ 10 \]

\[ 14.7 \]

\[ 7.5 \text{ sec} \]
(2) You are given the AC circuit of Figure 2. Use a cosine reference. Use mesh analysis to solve for the phasor currents \( I_1 \) and \( I_2 \). Give their values in polar form.

\[
\begin{align*}
&100 \cos(80t + 20^\circ) V \\
&80 \sin(80t + 40^\circ) V
\end{align*}
\]

\[
\begin{align*}
\text{As a phasor, } &80 L_{-50} \\
\text{Mesh #1} &
\end{align*}
\]

\[
\begin{align*}
-100 L_{20} + 200 I_1 - j50 I_1 + j50 I_2 + 60 L_{-30} &= 0 \\
(200 - j50) I_1 + j50 I_2 &= 100 L_{20} - 60 L_{-30}
\end{align*}
\]

\[
\begin{align*}
\text{Mesh #2} &
\end{align*}
\]

\[
\begin{align*}
-60 L_{-30} - j50 (I_2 - I_1) + j50 I_2 + 50 I_2 &= 80 L_{-50} \\
j50 I_1 + (50 + j40) I_2 &= 60 L_{-30} + 80 L_{-50}
\end{align*}
\]

\[
\begin{bmatrix}
[200-j50] & [10+j50] \\
[0+j50] & [50+j40]
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
= 
\begin{bmatrix}
[100L_{20}-60L_{-30}] \\
[60L_{-30}+80L_{-50}]
\end{bmatrix}
\]

\[
\begin{align*}
I_1 &= 0.325 L_{162.4^\circ} A \\
I_2 &= 2.06 L_{-73.6^\circ} A
\end{align*}
\]
(3) You are given the RLC circuit of Figure 3.

(a) Develop the differential equation that can be used to solve for \(v(t)\).
Show your steps.

(b) Give (determine) the values of \(v(0^+)\) and \(\frac{dv(0^+)}{dt}\).

(c) Determine which form below should be used to solve for \(v(t)\).
Do not solve the equation. Explain your answer.

- \(v(t) = 50 + (A_1 + A_2 t)e^{-st} u(t), \ V\)
- \(v(t) = 50 + e^{-st}[A_1 \cos(8.66t) + A_2 \sin(8.66t)] u(t), \ V\)
- \(v(t) = 50 + A_1 e^{-st} + A_2 e^{-st} u(t), \ V\)

For \(t > 0\):
\[
R\frac{di(t)}{dt} + L\frac{di}{dt} + v(t) = V_s
\]

But \(i = C \frac{dv}{dt}\)
\[
RC \frac{dv}{dt} + L\frac{dv}{dt} + v = V_s
\]
\[
\frac{d^2v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{V(t)}{LC} = \frac{V_s}{LC}
\]

With numbers
\[
\frac{d^2v}{dt^2} + 10\frac{dv}{dt} + 100v(t) = 50
\]
Characteristic Equation

\[ s^2 + 10s + 100 = 0 \]

\[ s + 5 + j8.66)(s + 5 - j8.66) = 0 \]

So

\[ V(t) = 50 + e^{-5t} \left[ A_1 \cos 8.66t + A_2 \sin 8.66t \right] \]

Initial conditions

\[ i(0^-) = 0 = i(0^+) = 0 \]

\[ V(0^-) = 0 = V(0^+) = 0 \]

\[ i(0^-) = 0 \]

\[ i(t) = \frac{dV}{dt} \]

\[ \frac{dV(t)}{dt} = \frac{i(t)}{C} = C \]

\[ \frac{dV(t)}{dt} = 0 \]
(4) You are given the circuit configuration of Figure 4. Assume \( V \) is the phasor voltage for 
\( 200\cos(100t + 20^\circ) \) V and \( I \) is the phasor current for \( 4\cos(100t - 10^\circ) \) A.
(a) Prepare the phasor diagram showing phasors \( V \) and \( I \).
(b) Use inspection to determine if the load is inductive or capacitive. Explain your answer.

\[ v(t) = 200\cos(100t + 20^\circ) \text{ V} \quad i(t) = 4\cos(100t - 10^\circ) \text{ A} \]

10%

(b) Since \( V \) leads \( I \) the load is inductive. \( \frac{E}{L} \)

Another view
\[ Z = \frac{V}{I} = \frac{200\sqrt{20}}{4\sqrt{10}} = 50\sqrt{20} \]

Since \( Z \) has a positive \( \angle \) point the load is inductive.
(5) You are given the circuit shown in Figure 5. Assume a cosine reference.

(a) Use nodal analysis to find the phasor voltages $V_1$ and $V_2$ and express these voltages in polar form.
(b) Give the expression for $v_1(t)$.
(c) Prepare a phasor diagram for phasor $V_S$, $V_1$ and $V_2$.
(d) For the phasors $V_S$ and $V_2$ which is leading and by how much angle?

\[
\begin{align*}
\text{(a) } & \quad A \hat{V}_1
\end{align*}
\]
\[
\begin{align*}
\frac{\hat{V}_1 - 100}{10} + \frac{\hat{V}_1}{-j10} + \frac{V_1 - V_2}{j5} = 0 \\
0.1V_1 - 10 + j0.1V_1 - j0.2V_1 + j0.2V_2 = 0 \\
(0.1 - j0.1)V_1 + (0 + 0.2)V_2 = 10
\end{align*}
\]
\[
\begin{align*}
\text{(a) } & \quad A \hat{V}_2
\end{align*}
\]
\[
\begin{align*}
\frac{\hat{V}_2}{20} + \frac{V_2 - V_1}{j5} = 10 \\
0.05V_2 - j0.2V_2 + j0.2V_1 = 10 \\
(0 + 0.2)V_1 + (0.05 - j0.2)V_2 = 10
\end{align*}
\]
\[
\begin{align*}
\begin{bmatrix}
0.1 - j0.1, & 0 + 0.2 \\
-0.05 - j0.2, & 0.22
\end{bmatrix}
\begin{bmatrix}
\hat{V}_1 \\
\hat{V}_2
\end{bmatrix} =
\begin{bmatrix}
10 \\
10
\end{bmatrix}
\end{align*}
\]
\[
\begin{align*}
\hat{V}_1 = 11.4\angle-37.8^\circ & \quad V_2 = 8.9\angle-26.6^\circ
\end{align*}
\]
(5)

(6)

\[
V_{1/2} = 64.7 \cos (\theta + 6.3^\circ) \ V
\]

(6)

(d) \ \vec{v}_5 \ \text{ending} \ \vec{v}_2 \ \text{by} \ 26.6^\circ
(6) You are given the circuit of Figure 6A. The switch has been open for a very long time. The switch is closed at $t = 0$ and the resulting voltage, $v(t)$, is shown in Figure 6B.

(a) Determine the time constant for the circuit.
(b) If $R = 10$ ohms, what is the value of $L$?

![Circuit Diagram](image)

Figure 6B: Response from circuit of Figure 6A.

(a) $i = 5e^{-\frac{t}{\tau}} A$, $\tau = \frac{L}{R}$

At $t = 7$,

$v(t) = 5e^{-\frac{t}{\tau}} = 1.84 A$

At 1.84, $t = \sqrt{7} = \frac{7}{2}$

(b) $\frac{L}{R} = 2 \Rightarrow \sqrt{L} = 20$