ECE 300 Desk Copy
Spring Semester, 2005
HW Set #8

Due: March 29, 2005 Name green
wlg AM PM Print(last, first)

Use Engineering Paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers. Each problem counts 5 points.**

8.6 (a) $v_R(0^+) = 0$ V, $v_L(0^+) = 0$ V

(b) $\frac{dv_R(o^+)}{dt} = 0V/s \quad \frac{dv_L(0^+)}{dt} = \frac{V_s}{(CR_s)} V/s$

(c) $v_L(\infty) = 0V, \quad v_R(\infty) = \frac{v_s R}{(R+R_s)} \quad V$

8.8 $i(t) = 4te^{-3t}$ **A**

8.13 R = 120 ohms

8.17 $v(t) = [64.53e^{-2.68t} - 4.641e^{-37.32t}]$ **V**

8.19 $v(t) = 24\sin(0.5t)$ **V**

8.23 C = 40 mF

8.25 $v_0(t) = e^{-0.25t}[24\cos(1.984t) + 3.024\sin(1.984t)]$ **V**

8.26 $i(t) = [2 + 2e^{-t}\sin(2t) +]$ **A**

8.41 $i(t) = e^{-2t}[0.7275\sin(4.583t)]$ **A**

8.49 $i(t) = \{ 3 + [e^{-2t}(3 + 6t)] \}$ **A**

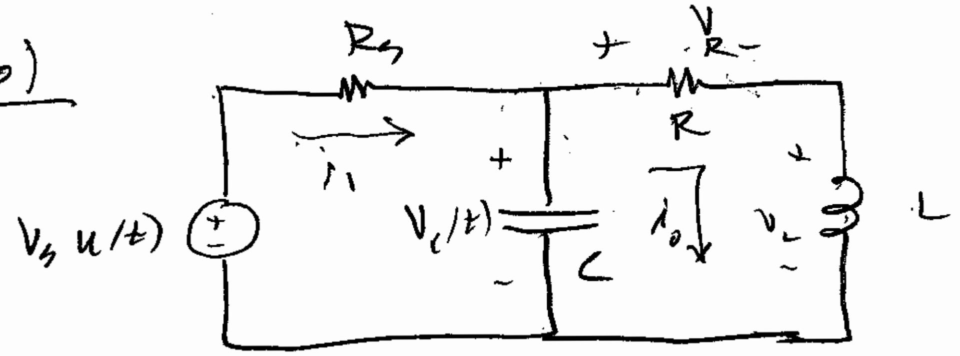
The following is a good web site to see how other students (at Univ. of North Florida in Jacksonville, FL) have been working with the MSF430 project. See Summer 2003 in particular.

http://www.unf.edu/~gmerckel/

Check the following site for free samples of MSP430F449 chip (up to 3). Please order 3 to replace the ones in the kit. Thanks.

http://focus.ti.com/docs/prod/productfolder.jhtml?genericPartNumber=MSP430F449&pfsection=samples

(8.6)



(a) $V_R(0^+)$; $V_R(0^-) = 0 \therefore V_R(0^+) = 0$

$\boxed{i_o(0^-) = 0 \text{ and } i_o(0^+) = 0}$

$V_L(0^+)$: $V_L(t) = L\frac{di}{dt}$

$V_L(0^-) = 0$

$V_c(0^-) = 0$; Voltage across the capacitor cannot change inst; $\therefore$

$V_c(0^+) - V_R(0^+) - V_L(0^+) = 0$

$V_L(0^+) = V_c(0^+) - V_R(0^+) = 0 - 0 = 0$

(b) $\frac{dV_R(0^+)}{dt}$; $\frac{dV_L(0^+)}{dt}$

Now $V_L(0^+) = L\frac{di_o(0^+)}{dt}$ but $V_L(0^+) = 0$

$\therefore \frac{di_o}{dt} = 0$ Note $\frac{dV_R}{dt} = R\frac{di_o}{dt}$

$\therefore \boxed{\frac{dV_R(0^+)}{dt} = 0}$

(c) $V_c - V_R - V_L = 0$ OR

$\frac{dV_c}{dt} - \frac{dV_R}{dt} - \frac{dV_L}{dt} = 0$

(8,6),

but we know

$$\frac{dV_R(0^+)}{dt} = 0$$

We need $\frac{dV_C(0^+)}{dt}$,

Now $i(0^+) = \frac{V_s}{R_s} = C\frac{dV_C(0^+)}{dt}$

so

$$\frac{dV_C(0^+)}{dt} = \frac{V_s}{R_s \times C}$$

$$\frac{dV_L(0^+)}{dt} = \frac{dV_C(0^+)}{dt} = \frac{V_s}{R_s C}$$

(c) $V_R(\infty) = \frac{V_s \times R}{R + R_s}$

$$V_L(\infty) = L\frac{di(\infty)}{dt} = 0$$

8.8 Given

$$\frac{d^2 i}{dt^2} + 6\frac{di}{dt} + 9i = 0$$

$i(0) = 0, \quad \frac{di(0)}{dt} = 4$

Obtain the char. Eq. & solve for $i(t)$.

Char. Eq is

$$s^2 + 6s + 9 = 0$$

Roots:

$$(s+3)(s+3) = 0 \quad \text{repeated roots}$$

$$i(t) = (A_1 + A_2 t)e^{-3t}$$

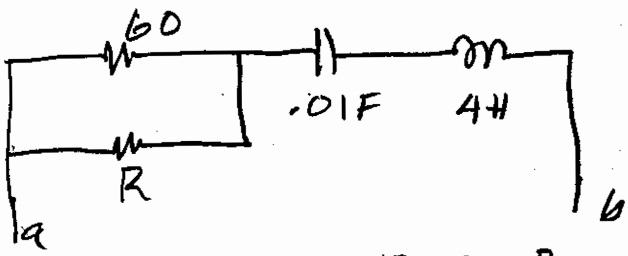
$$i(0) = \boxed{0 = A_1}$$

$$\left.\frac{di}{dt}\right|_{t=0} = \left.-3A_2 t e^{-3t} + A_2 e^{-3t}\right|_{t=0}$$

$$4 = A_2$$

$$\boxed{i(t) = 4te^{-3t} \text{ A}}$$

8.13



$$R_{eq} = \frac{60R}{60+R}$$

Find R so that the Char Eq has critically damped roots

It is sufficient to write KVL

$$L\frac{di}{dt} + R_{eq}\, i + V_c = 0$$

$$i = C\frac{dV_c}{dt} = .01\frac{dV_c}{dt}$$

$$\overset{L}{4} \times \overset{C}{.01} \times \frac{d^2V_c}{dt^2} + \frac{.6R}{60+R}\frac{dV_c}{dt} + V_c = V_{in}$$

$$\frac{d^2V_c}{dt^2} + \frac{15R}{R+60}\frac{dV_c}{dt} + 25V_c = 0$$

$(s+5)(s+5) = 0$ critically damped

so

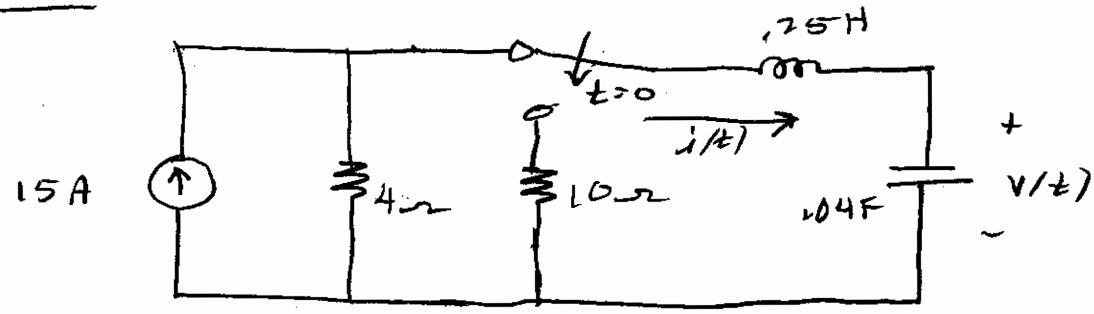
$$\frac{15R}{R+60} = 10$$

$$15R = 10R + 600$$

$$5R = 600$$

$$\boxed{R = 120\ \Omega}$$

8.17



Find $v(t)$, $t \geq 0$

t<0

$$i(0^-) = 0$$

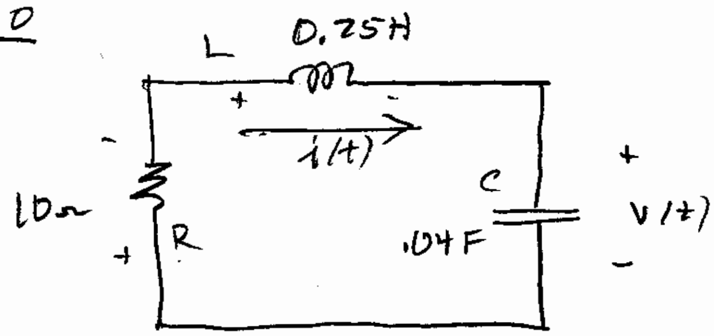
$$V(0^-) = 15 \times 4 = 60V$$

We know

$$\boxed{V(0^+) = V(0^-) = 60V}$$

$$\boxed{i(0^+) = i(0^-) = 0}$$

t>0



$$Ri + L\frac{di}{dt} + V = 0 \qquad (1)$$

Use $i = C\frac{dV}{dt}$ (1a)

$$RC\frac{dV}{dt} + LC\frac{d^2V}{dt^2} + V = 0 \qquad (2)$$

$$\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{V}{LC} = 0 \qquad (3)$$

8.17

$$\frac{d^2 v}{dt^2} + \frac{10}{.25}\frac{dv}{dt} + \frac{v}{.25 \times .04} = 0 \qquad (4)$$

$$\frac{d^2 v}{dt^2} + 40\frac{dv}{dt} + 100v = 0 \qquad (5)$$

char. Eq.

$$s^2 + 40s + 100 = 0$$

$$(s + 2.68)(s + 37.3) \qquad (6)$$

Now

$V(\infty) = V_{ss} = 0$ by

inspection of the circuit and of

$$\frac{d^2 v}{dt^2} + 40\frac{dv}{dt} + 100v = 0 \qquad (67)$$

The solution to the above is

$$v(t) = A_1 e^{-2.68t} + A_2 e^{-37.3t}$$

solve for A_1 & A_2 using $V(0^+)$

$\frac{dV(0^+)}{dt}$. From (1a)

$$C\frac{dv}{dt} = i \qquad \text{but } i(0^+) = 0$$

so $\boxed{\frac{dV(0^+)}{dt} = 0}$

$$\boxed{60 = A_1 + A_2}$$

$$\frac{dv}{dt} = -2.68 A_1 e^{-2.68t} - 37.3 A_2 e^{-37.3t}$$

evaluate at $t = 0^+$

8.17

$$\frac{dv(0^+)}{dt} = 0 = -2.68A_1 - 37.3A_2$$

so we have

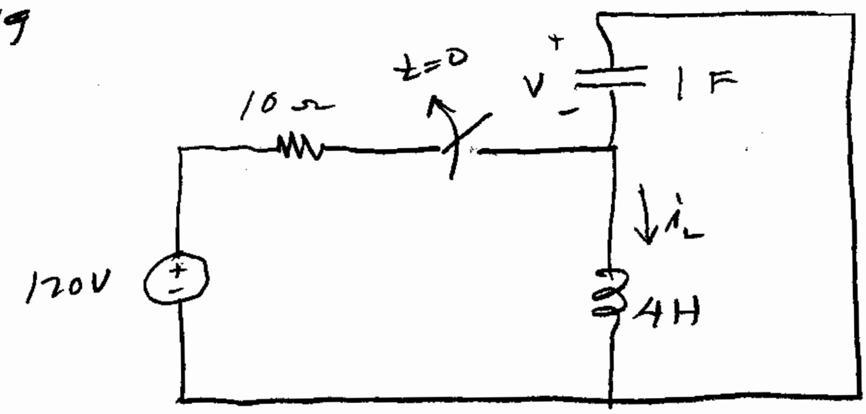
$$A_1 + A_2 = 60$$

$$-2.68A_1 - 37.3A_2 = 0$$

$$A_1 = 64.6, \quad A_2 = -4.64$$

$$v(t) = \left[64.6e^{-2.68t} - 4.64e^{-37.3t}\right] V$$

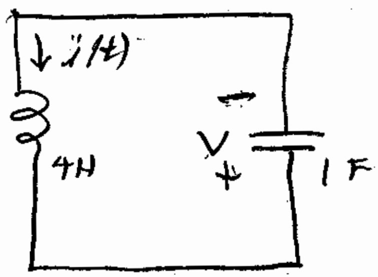
8.19



$i_L(0^-) = i_L(0^+) = \frac{120}{10} = 12\ A$

$V(0^-) = 0$

$t \geq 0$



$L\frac{di}{dt} + V = 0$ $\qquad \frac{di(0)}{dt}$

$LC\frac{d^2V}{dt^2} + V = 0$

$\frac{d^2V}{dt^2} + \frac{V}{LC} = 0$

$i(0^+) = 12 = C\frac{dV(0^+)}{dt}$

$\boxed{\frac{dV(0^+)}{dt} = -12}$

$\boxed{\frac{d^2V(0^+)}{dt^2} = \frac{-V(0^+)}{LC} = 0}$

8.19 back to

$$\frac{d^2v}{dt^2} + \frac{v}{LC} = 0$$

Char. Eq

$$s^2 + \frac{1}{LC} = 0$$

$$s^2 + 0.25 = 0$$

$$(s + j0.5)(s - j0.5) = 0$$

$$v(t) = A_1 e^{j0.5t} + A_2 e^{-j0.5t}$$

$$\frac{dv}{dt} = j0.5 A_1 e^{j0.5t} - j0.5 A_2 e^{-j0.5t}$$

$$\boxed{-12 = j0.5 A_1 - j0.5 A_2}$$

$$\frac{d^2v}{dt^2} = -0.25 A_1 e^{j0.5t} - .25 A_2 e^{-j0.5t}$$

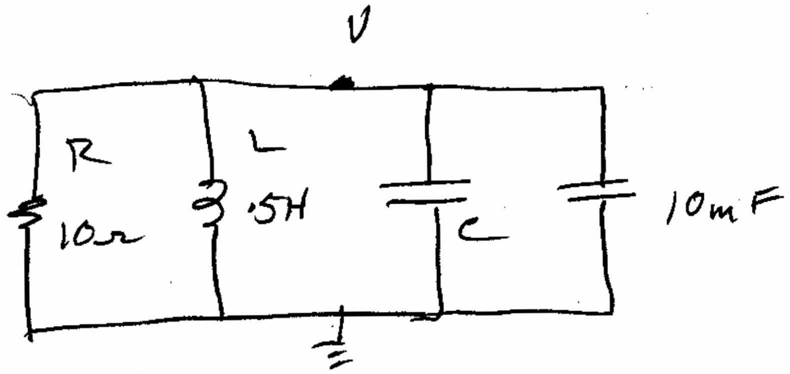
$$0 = A_1 + A_2$$

$$A_1 = -j12, \quad A_2 = +j12$$

$$v(t) = -j12 \left[e^{j0.5t} - e^{-j0.5t} \right]$$

$$\boxed{v(t) = +24 \sin 0.5t \quad V}$$

8.23



Assume unforced: want only C.E.

$$\frac{V}{R} + C_{eq}\frac{dV}{dt} + \frac{1}{L}\int_0^t V\,dt + i(0) = 0$$

$$C_{eq}\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{V}{L} = 0$$

$$\frac{d^2V}{dt^2} + \frac{1}{RC_{eq}}\frac{dV}{dt} + \frac{V}{LC_{eq}} = 0$$

underdamped; $\alpha = 1$

$$s^2 + 2\alpha + \omega_0^2$$

$$\frac{1}{RC_{eq}} = 2$$

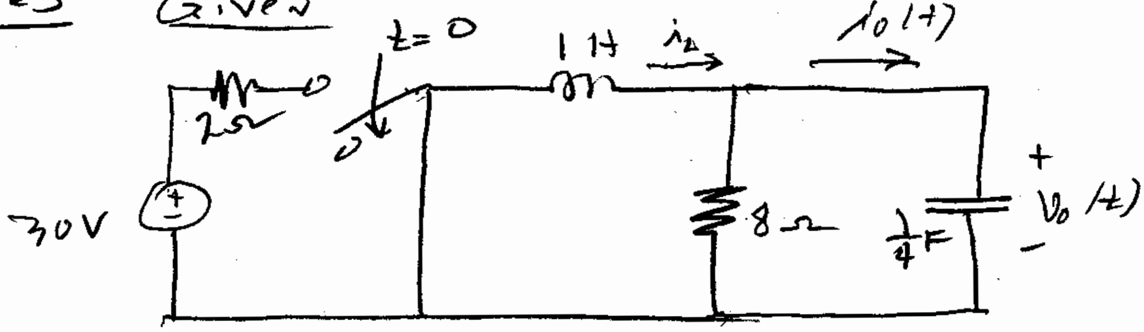
$$C_{eq} = \frac{1}{2R} = \frac{1}{20} = .05$$

$$10mF + C = .05$$

$$C = .05 - .01 = .04$$

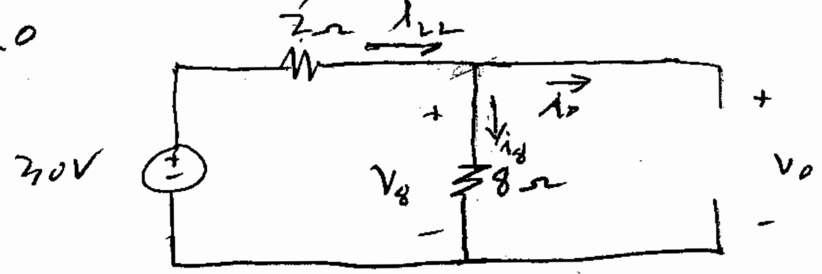
$$\boxed{C = 40\,mF}$$

8.25 Given



Find $i_0(t)$, $V_0(t)$ $t \geq 0$

t<0

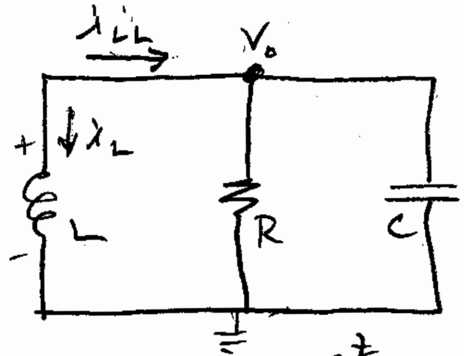


$V_8(0^-) = 24V$, $i_8(0^-) = 3A$,

$i_{LL}(0^-) = i_{LL}(0^+) = 3$

$V_0(0^-) = V_0(0^+) = 24$

t≥0



$i_L = -i_{LL}$

$i_L(0^+) = -3$

$$C\frac{dV_0}{dt} + \frac{V_0}{R} + \frac{1}{L}\int_0^t V_0\,dt + i_L(0^+) = 0$$

$$C\frac{d^2V_0}{dt^2} + \frac{1}{R}\frac{dV_0}{dt} + \frac{V_0}{L} = 0$$

$$\frac{d^2V_0}{dt^2} + \frac{1}{RC}\frac{dV_0}{dt} + \frac{V_0}{LC} = 0$$

8.25

$$\frac{d^2 V_o}{dt^2} + \frac{4}{8}\frac{dV_o}{dt} + \frac{V_o\,4}{1} = 0$$

Char. Eq.

$$s^2 + 0.5s + 4 = 0 \qquad (1)$$

$$(s + .25 + j1.9843)(s + .25 - j1.9843) = 0$$

Compare (1) to

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

$$2\alpha = 0.5$$

$$\boxed{\alpha = 0.25}$$

by inspection $\omega_d = 1.98$

$$\sqrt{\omega_o^2 - \alpha^2} = 1.98$$

$$V_o(t) = e^{-.25t}\left[A_1 \cos 1.98t + A_2 \sin 1.98t\right]$$

Know

$$V_o(0^+) = 24$$

To get $\frac{dV_o(0^+)}{dt}$

$$C\frac{dV_o(0^+)}{dt} + \frac{V_o}{R} + \frac{1}{L}\int_0^{0^+} V_o dt + i_L(0^+) = 0.$$

$$\frac{1}{4}\frac{dV_o(0^+)}{dt} = -\frac{V_o}{R} - i_L(0^+)$$

$$\frac{1}{4}\frac{dV_o(0^+)}{dt} = \left[-\frac{24}{8} - (-3)\right] = -3 + 3 = 0$$

$$\frac{dV_o(0^+)}{dt} = 0$$

8.25

$$V_0(0^+) = 24, \quad \frac{dV_0(0^+)}{dt} = 0$$

$$\boxed{24 = A_1}$$

$$\frac{dV_0}{dt} = -.25e^{-.25t}\left[24\cos 1.98t + A_2 \sin 1.98t\right]$$

$$+ e^{-.25t}\left[-47.52\sin 1.98t + 1.98A_2\cos 1.98t\right]$$

Evaluate at 0^+

$$0 = -.25\left[24\right] + 1.98A_2]$$

$$A_2 = \frac{+6}{1.98} = 3.03$$

$$V_0(t) = e^{-.25t}\left[24\cos 1.98t + 3.03\sin 1.98t\right]$$

Then

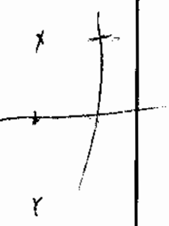
$$i_0(t) = C\frac{dV_0}{dt}$$

8.26 Step response of RLC ckt given by

$$\frac{d^2 i}{dt^2} + 2\frac{di}{dt} + 5i = 10$$

Given $i(0^+) = 2$, $\frac{di(0^+)}{dt} = 4$ find $i(t)$

Characteristic Eq is;



$$s^2 + 2s + 5 = 0$$

$$(s+1+j2)(s+1-j2) = 0$$

Form of the solution will be
(first note $i_{ss} = 2$)

$$i(t) = 2 + e^{-\alpha t}\left[A_1 \cos\omega_d t + A_2 \sin\omega_d t\right]$$

where
$$s^2 + 2\alpha s + \omega_0^2 = 0$$

comparing with
$$s^2 + 2s + 5$$

$$2\alpha = 2 \; ; \; \alpha = 1 \; ; \quad \omega_0^2 = 5$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{5-1} = 2$$

$$i(t) = 2 + e^{-t}\left[A_1 \cos 2t + A_2 \sin 2t\right]$$

$$i(0) = 2 = 2 + A_1 \qquad A_1 = 0$$

$$i(t) = 2 + e^{-t} A_2 \sin 2t$$

8.26

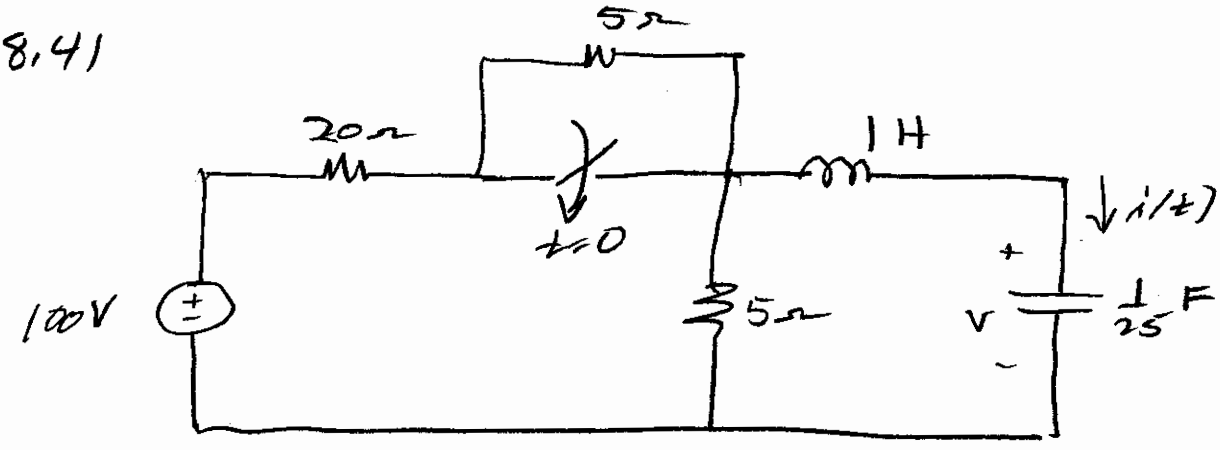
$$\left.\frac{di}{dt}\right|_{t=0} = e^{-t} A_2 [2\cos 2t - A_2 e^{-t} \sin 2t \Big|_{t=0}$$

$4 = A_2 \times 2 \qquad A_2 = 2$

$\therefore$

$$\boxed{i(t) = 2 + 2e^{-t}\sin 2t}$$

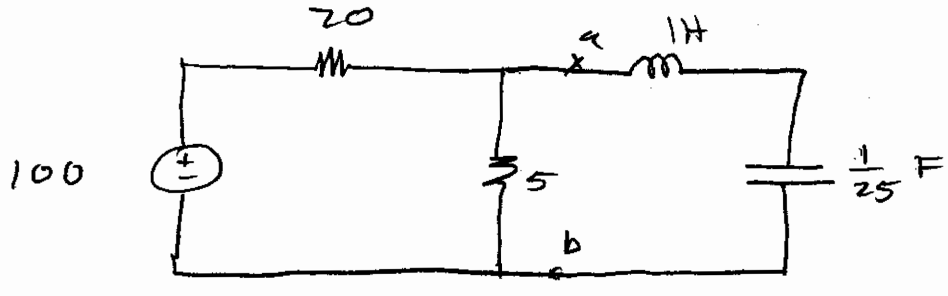
8.41



$i(0^-) = i(0^+) = 0$

$$V(0^-) = V(0^+) = \frac{100 \times 5}{30} = \frac{50}{3}$$

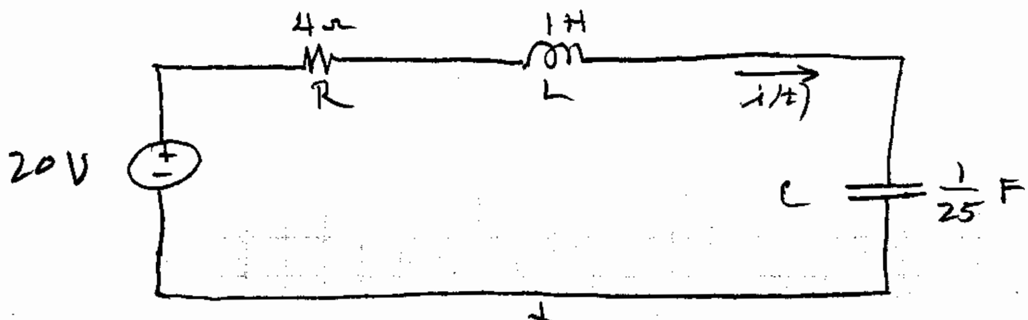
$t \geq 0$



Thevenin to the left of a-b

$$V_{TH} = \frac{5 \times 100}{25} = 20\,V$$

$$R_{TH} = \frac{5 \times 20}{25} = 4\,\Omega$$



$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + V_c(0) = 20 \qquad (1)$$

8.41 $\boxed{i(0^+) = 0}$

$$L\frac{di(0^+)}{dt} = 20 - V_c(0) - R\,\cancel{i(0^+)}^{\,0}$$

$$\boxed{\frac{di(0^+)}{dt} = \frac{20 - \frac{50}{3}}{1H} = \frac{10}{3}}$$

Differentiate Eq #1

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$\frac{d^2i}{dt^2} + 4\frac{di}{dt} + 25i = 0$$

$$s^2 + 4s + 25 = 0$$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$(s + 2 + j4.58)(s + 2 - j4.58) = 0$$

$$i(t) = i(\infty) + e^{-2t}[A_1\cos 4.58t + A_2 \sin 4.58t)$$

$$i(\infty) = 0$$

$$i(t) = e^{-2t}[A_1\cos 4.58t + A_2\sin 4.58t]$$

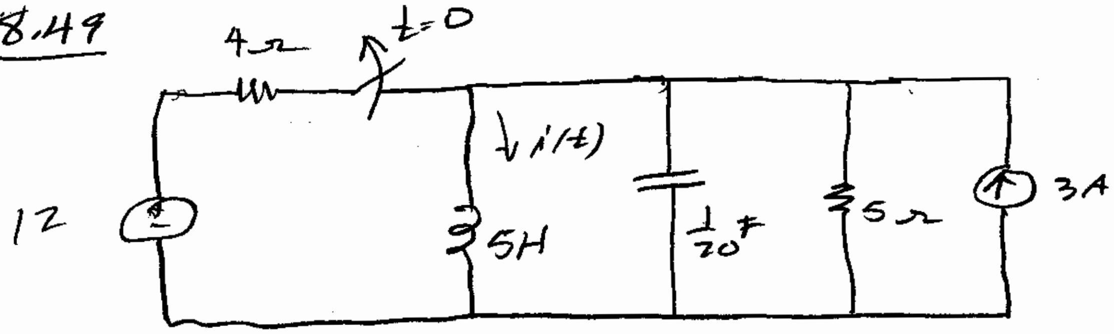
$$0 = A_1$$

$$\frac{di}{dt} = -2e^{-2t}[A_2 \sin 4.58t] + e^{-2t}[4.58 A_2 \cos 4.58t]$$

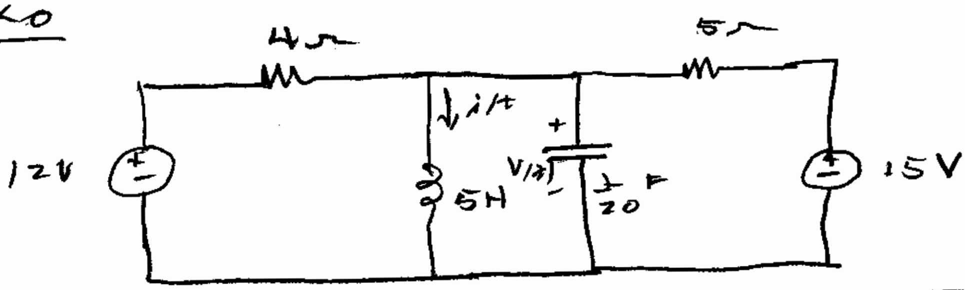
$$\frac{10}{3} = 4.58\,A_2 \qquad A_2 = 0.73$$

$$\boxed{i(t) = 0.73 e^{-2t}\sin 4.58t \quad A}$$

8.49



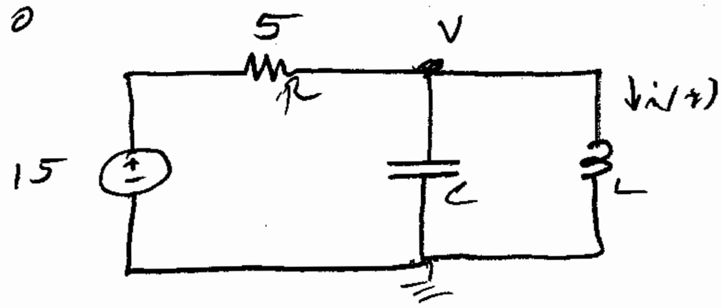
t < 0



$$i(0^-) = \frac{12}{4} + \frac{15}{5} = \boxed{6\,A = i(0^+)}$$

$$V(0^-) = 0 = V(0^+)$$

t > 0



$$\frac{V-15}{R} + C\frac{dV}{dt} + i(t) = 0 \qquad V = L\frac{di}{dt}$$

$$\frac{V}{R} + C\frac{dV}{dt} + i = \frac{15}{R}$$

$$\frac{L}{R}\frac{di}{dt} + LC\frac{d^2 i}{dt^2} + i = \frac{15}{R}$$

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{15}{RLC}$$

8.49

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 4i = 12$$

$$s^2 + 4s + 4 = 0$$

compare

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$\alpha = 2, \quad \omega_0 = 2$$

critically damped

$$i(t) = i(\infty) + (A_1 + A_2 t)e^{-2t}$$

$$i(\infty) = 3$$

$$i(t) = 3 + (A_1 + A_2 t)e^{-2t}$$

$$i(0^+) = 6$$

$$6 = 3 + A_1 \qquad A_1 = 3$$

$$V_L = L\frac{di}{dt} \qquad V_L(0^+) = 0$$

$$\frac{di(0^+)}{dt} = 0$$

$$\frac{di}{dt} = -2(3 + A_2 t)e^{-2t} + e^{-2t}(A_2) =$$

$$0 = -6 + A_2 \qquad A_2 = 6$$

$$i(t) = \left\{3 + e^{-2t}[3 + 6t]\right\} A$$