Desk Copy

ECE 300
Spring Semester, 2005
HW Set #9

Due: April 4, 2005 Name Green
wlg AM PM Print(last, first)

Revised: version 2

Use Engineering Paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. <u>Be sure to show how you got your answers.</u> Each problem counts 5 points.**

9.23 (a) $v(t) = 40\cos(wt - 60^\circ)$ V, (b) $v(t) = 38.4\cos(wt + 96.8^\circ)$ V;
(c) $i(t) = 6\cos(wt + 80^\circ)$ A, (d) $i(t) = 11.5\cos(wt - 52.1^\circ)$ A

9.36 $i(t) = 26.6\cos(200t - 3.9^\circ)$ mA

9.42 $v_o(t) = 17.14\cos(200t)$ V

10.13 $V_X = 29.4\angle 62.9^\circ$ V

10.15 $I = 7.91\angle 43.5^\circ$ A ------------------- I got $I = 1.374\angle 104.04^\circ$ A

10.25 $i_o = 1.414\cos(2t + 45^\circ)$ A

10.29 $I_1 = 4.67\angle -20.17$ A, $I_2 = 1.79\angle 37.35^\circ$ A

10.57 $Z_N = Z_{th} = 21.6\angle -33.7^\circ\ \Omega$, V_{th} $107.3\angle 146.6^\circ$ V, $I_N = 4.96\angle -179.7^\circ$ A

10.62 $Z_{th} = 2.29\angle -103.24^\circ\ \Omega$, $V_{th} = 3.073\angle 140.18$ V, $V_o = 2.3\angle -163.3^\circ$ V

10.68 $v_{th} = 11.52\sin(10t - 50.2^\circ)$ V, $Z_{th} = 1.92\angle -50.19\ \Omega$

9.23

Give a single sinusoid for each of the following.

(a) $\hat{V} = 40\angle{-60^\circ}\ V$

$$\boxed{v(t) = 40\cos(\omega t - 60^\circ)\ V}$$

(b) $\hat{V} = -30\angle{10^\circ} + 50\angle{60^\circ}\ V$

$$\hat{V} = (-4544 + j\,0.918) = 38.36\angle{96.8^\circ}\ V$$

$$\boxed{v(t) = 38.4\cos(\omega t + 96.8^\circ)\ V}$$

(c) $\hat{I} = j6e^{-j10^\circ}\ A = 6e^{-j10} \cdot e^{j90}$

$$\hat{I} = 6e^{j80}\ A$$

$$\boxed{i(t) = 6\cos(\omega t + 80^\circ)\ A}$$

(d) $\hat{I} = \frac{2}{j} + 10\angle{-45}$

$$\hat{I} = -j2 + 10\angle{-45} = 2\angle{-90} + 10\angle{-45}$$

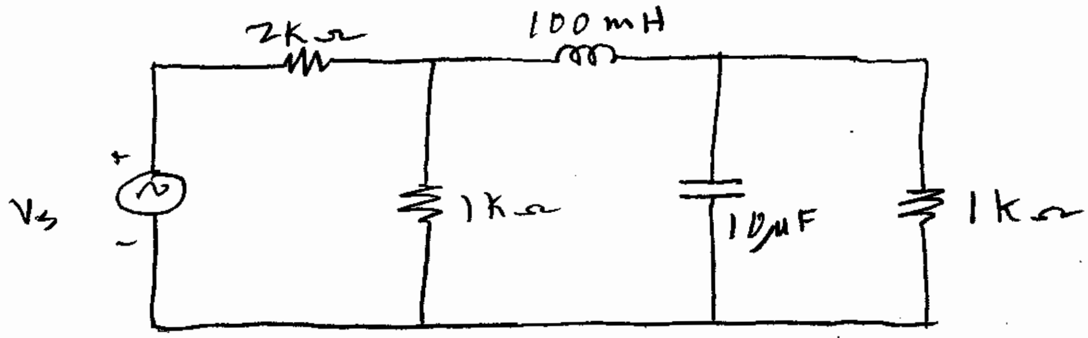
$$\hat{I} = 7.07 - j\,9.071 = 11.5\angle{-52.1}$$

$$\boxed{i(t) = 11.5\cos(\omega t - 52.1)\ A}$$

9.36

In the circuit below, determine i.

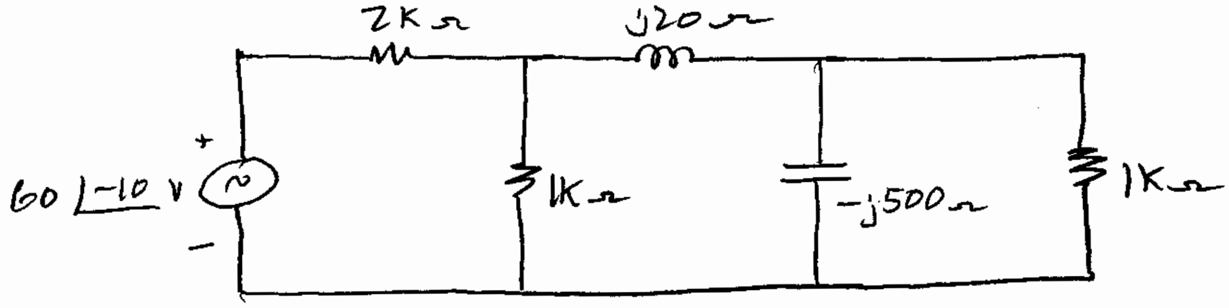
Let $V_s = 60\cos(200t - 10°)$ V



Prepare the phasor circuit

$100\text{mH} \rightarrow j\omega L \rightarrow j200 \times 0.1 \rightarrow j20\,\Omega$

$10\mu F \rightarrow \dfrac{-j}{200 \times 10 \times 10^{-6}} \rightarrow \dfrac{-j}{2\times10^{-3}} \rightarrow -j500\,\Omega$



$1000 \parallel (-j500) = \dfrac{1000(-j500)}{1000 - j500} = 200 - j400$

Add the $j20$ to $200 - j400 = 200 - j380$

$1000 \parallel (200 - j380) = \dfrac{1000(200 - j380)}{1000 + 200 - j380} = 242.6 - j239.8$

$Z_{IN} = 2000 + 242 - j239.8 = 2242 - j239.8$

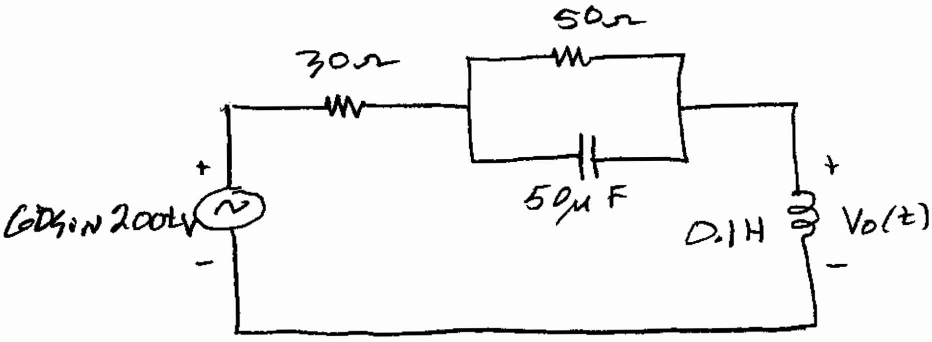
Answer book error

$\hat{I} = \dfrac{60\angle{-10}}{(2242 - j239.8)} = .0266\angle{-3.895}$ A

$$\boxed{i(t) = 26.61\cos(200t - 3.89°) \text{ A}}$$

9.42

Calculate $V_o(t)$ in the following ckt.

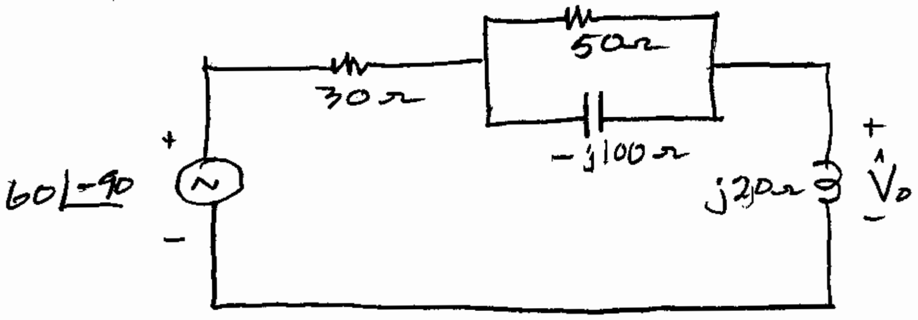


Change to phasor circuit

$0.1\,H \rightarrow j200 \times .1 = j20\,\Omega;$

$50\mu F \rightarrow \dfrac{-j}{200 \times 50 \times 10^{-6}} \rightarrow \dfrac{-j}{10 \times 10^{-3}} \rightarrow -j100\,\Omega$

$60 \sin 200t \rightarrow 60 \cos(200t - 90)$



$$50 \parallel (-j100) = \frac{50(-j100)}{50 - j100} = 40 - j20$$

Using voltage division;

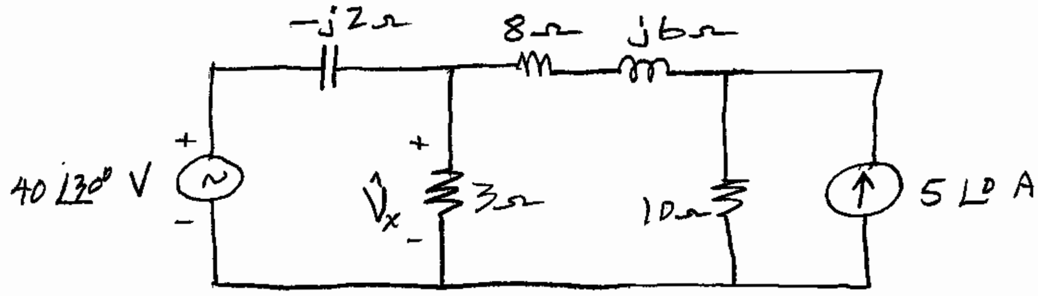
$$\hat{V}_o = \frac{(60\angle{-90})(20\angle{90})}{30 + 40 \cancel{-j20} + \cancel{j20}} = \frac{1200\angle 0}{70}$$

$$\hat{V}_o = 17.14\angle 0$$

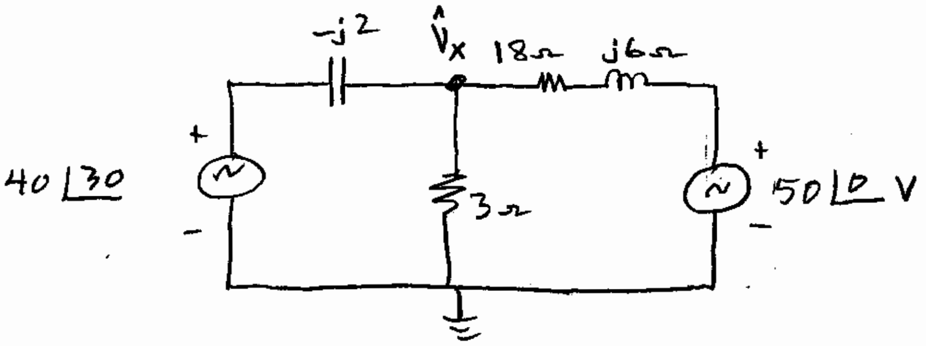
$$\boxed{V_o(t) = 17.14 \cos(200t)\ \text{V}}$$

10.13

Determine $\hat{V}_x$ in the following circuit using any method of your choice.



Use source transformation on the right side



$$\frac{\hat{V}_x - 40\angle 30}{2\angle -90} + \frac{\hat{V}_x}{3} + \frac{\hat{V}_x - 50}{18+j6} = 0$$

$$\frac{\hat{V}_x}{2\angle -90} + \frac{\hat{V}_x}{3} + \frac{\hat{V}_x}{18+j6} = 20\angle 120 + \frac{50}{18+j6}$$

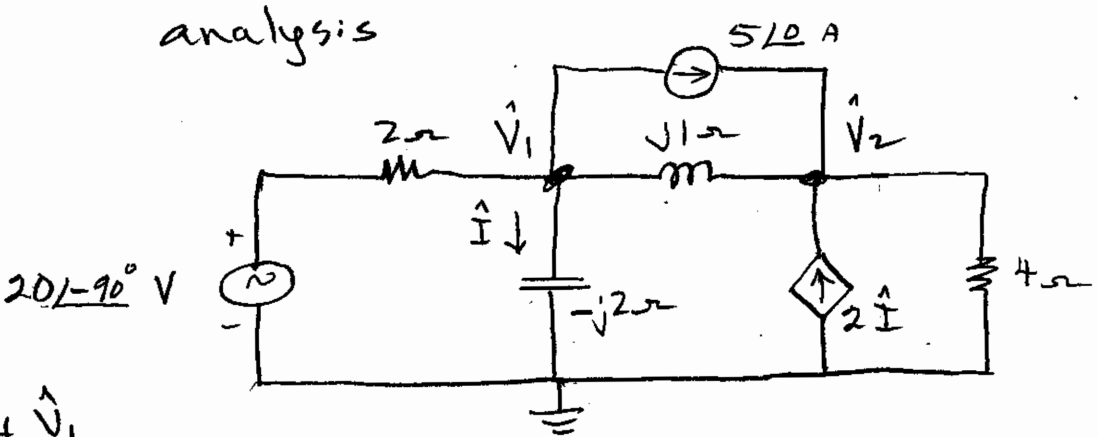
$$\hat{V}_x\left[.5\angle 90 + .333 + .05 - j.0167\right] = 20\angle 120 + 2.64\angle -18.4$$

$$\hat{V}_x\left[.3833 + j0.4833\right] = 18.11\angle 114.4^\circ$$

$$\boxed{\hat{V}_x = 29.4\angle 62.9^\circ \text{ V}}$$

10.15

Solve for the current $\hat{I}$ using nodal analysis



At $\hat{V}_1$

$$\frac{\hat{V}_1 - 20\angle -90}{2} - \frac{\hat{V}_1}{j2} + \frac{\hat{V}_1 - \hat{V}_2}{j1} + 5\angle 0 = 0$$

$$V_1(0.5 + j0.5 - j) + j V_2 = -5 - j10$$

$$\boxed{(0.5 - j0.5)\hat{V}_1 + j\hat{V}_2 = -5 - j} \quad (1)$$

At $\hat{V}_2$

$$\frac{\hat{V}_2 - \hat{V}_1}{j1} + \frac{\hat{V}_2}{4} - 2\hat{I} = 5$$

$$\hat{I} = \frac{\hat{V}_1}{-j2} = j0.5V_1$$

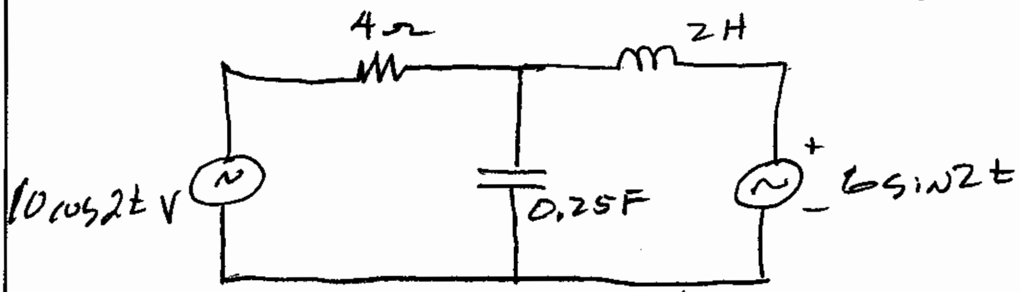
$$-j\hat{V}_2 + \cancel{j\hat{V}_1} + 0.25\hat{V}_2 - \cancel{j\hat{V}_1} = 5$$

$$\boxed{0\hat{V}_1 - j0.75 V_2 = 5}$$

$$\hat{V}_1 = 2.749\angle 14.04 \text{ V}$$

$$\boxed{\hat{I} = \frac{\hat{V}_1}{-j2} = 1.374\angle 104.04^\circ \text{ A}}$$

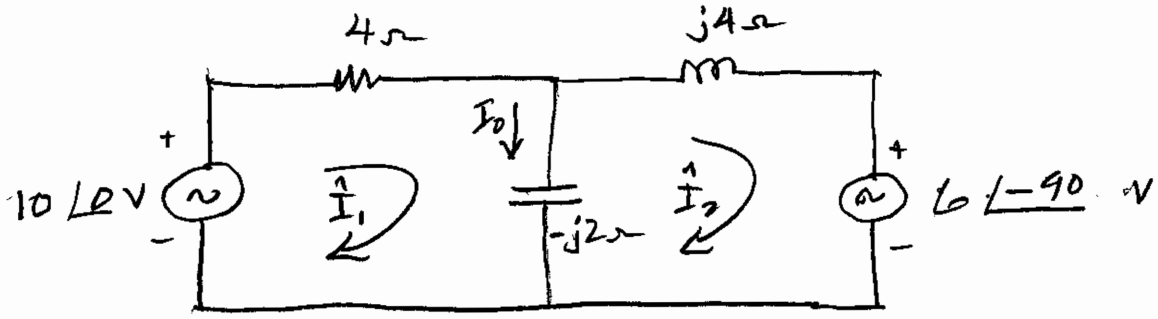
10.25 Solve for i_0 below using mesh analysis



Go to phasor circuit

$2H \rightarrow j \times 2 \times 2 = j4\,\Omega$

$.25F \Rightarrow \dfrac{-j}{2 \times .25} = -j2\,\Omega$



$$\begin{bmatrix} 4-j2 & j2 \\ j2 & j2 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 6\angle 90 \\ -6\angle -90 \end{bmatrix}$$

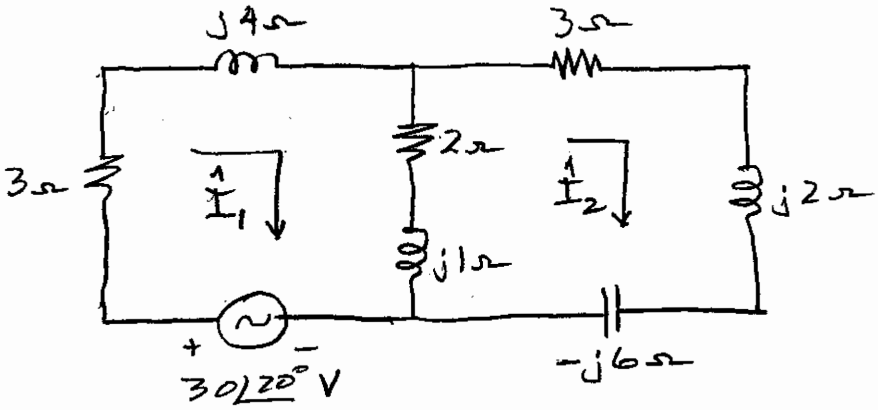
$\hat{I}_1 = 2 + j0.5, \quad \hat{I}_2 = 1 - j0.5$

$\hat{I}_0 = \hat{I}_1 - \hat{I}_2 = 2 + j0.5 - 1 + j0.5$

$\hat{I}_0 = 1 + j1 = 1.414\angle 45^\circ$

$$\boxed{i_0(t) = 1.414\cos(2t + 45^\circ)}$$

10.29 Use mesh analysis to find $\hat{I}_1$ and $\hat{I}_2$ in the following circuit.



By inspection

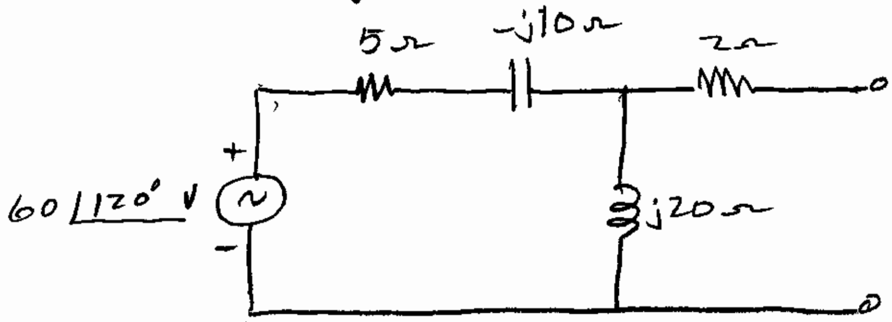
$$\begin{bmatrix} 5+j5 & -2-j1 \\ -2-j1 & 5-j3 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 30\angle 20 \\ 0 \end{bmatrix}$$

Hand calculator solution:

$\hat{I}_1 = 4.67 \angle -20.17^\circ$ A $\qquad \hat{I}_2 = 1.79 \angle 37.4^\circ$ A

10.57

Find the Thevenin equivalent & Norton equivalent circuits for the following:



$$Z_{TH} = 2 + \frac{(j20)(5-j10)}{5-j10+j20}$$

$$Z_{TH} = 2 + \frac{(20\angle 90)(5-j10)}{(5+j10)} = 2+16-j12$$

$$\boxed{Z_{TH} = 18-j12 = 21.6\angle -33.7^\circ\ \Omega}$$

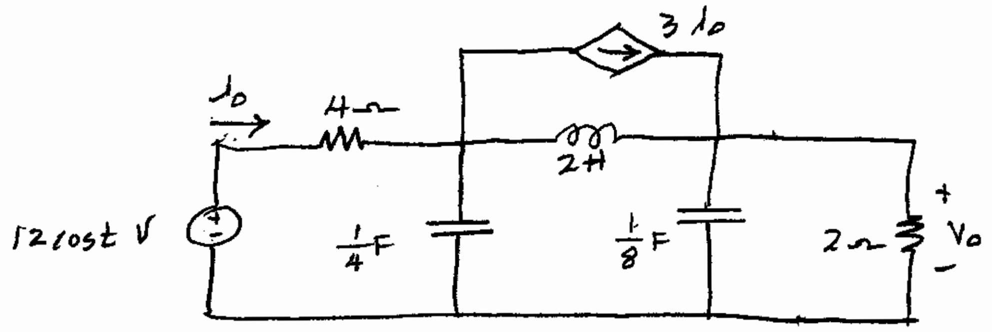
$$\hat{V}_{TH} = \frac{(60\angle 120)(20\angle 90)}{5+j10}$$

$$\boxed{\hat{V}_{TH} = 107.33\angle 146.5}$$

$$\hat{I}_N = \frac{\hat{V}_{TH}}{Z_{TH}} = \frac{107\angle 146.5}{21.6\angle -33.7}$$

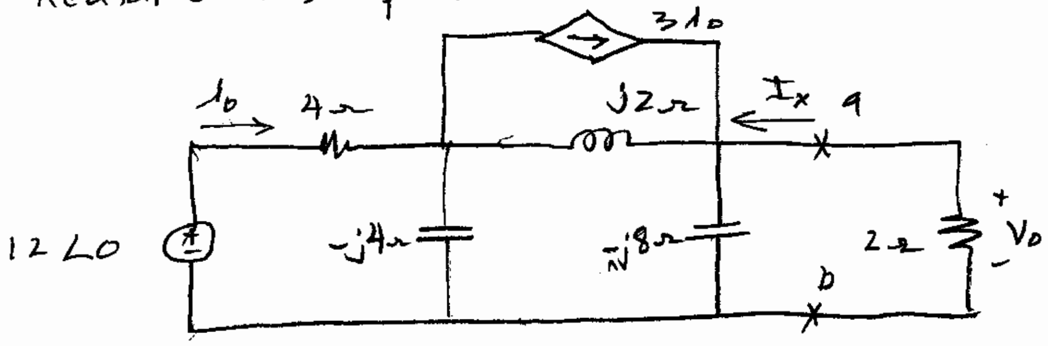
$$\boxed{\hat{I}_N = 4.96\angle -179.8\ \text{A}}$$

10.62 Use Thevenin's theorem to find $V_o(t)$ in the following circuit.

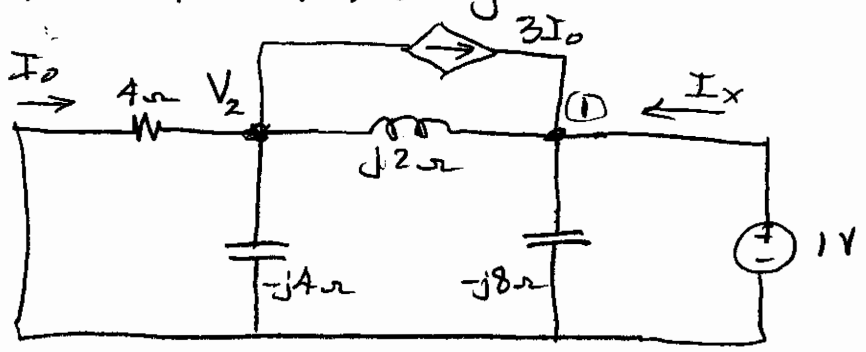


$\omega = 1$; $\frac{1}{4}F \rightarrow -j4\Omega$; $\frac{1}{8}F \rightarrow -j8\Omega$; $2H \rightarrow j2\Omega$

Redraw as phasor circuit



Find Z_{TH} looking into ab with 2Ω resistor removed, apply a 1V source and find the resulting current.



$$\frac{V_2}{4} - \frac{V_2}{j4} + \frac{V_2 - 1}{j2} + 3I_o = 0 \qquad (1)$$

$$I_o = \frac{-V_2}{4} \qquad (2)$$

substitute (2) into (1).

10.62

$$\frac{V_2}{4} - \frac{V_2}{j4} + \frac{V_2 - 1}{j2} - \frac{3V_2}{4} = 0$$

multiply through by $j4$

$$jV_2 - V_2 + 2V_2 - 2 - j3V_2 = 0$$

$$(1 - j2)V_2 = 2$$

$$\boxed{V_2 = 0.4 + j0.8 \ \text{V}}$$

At ①

$$-\frac{1}{j8} + \frac{1 - V_2}{j2} - 3I_o = I_x$$

substituting in for V_2 and I_o

$$-\frac{1}{j8} + \frac{0.6 - j0.8}{j2} + \frac{(1.2 + j2.4)}{4} = I_x$$

multiply through by $j8$

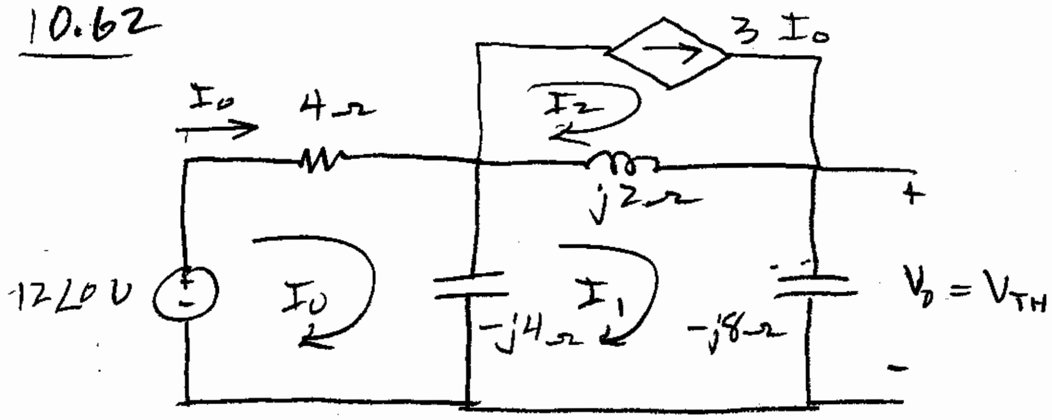
$$-1 + 2.4 - j3.2 + j2.4 - 4.8 = j8$$

$$I_x = \frac{-3.4 - j0.8}{j8} = -0.1 + j0.425 \ \text{A}$$

$$\therefore \ Z_{TH} = \frac{1}{I_x} = 2.29 \angle -103.24^\circ \ \Omega$$

To find V_{TH} we go back to the 2 circuit with the $2\,\Omega$ resistor removed and find V_o as indicated in the following circuit.

10.62



We have;

$$(4-j4)I_o + j4 I_1 + 0 I_2 = 12\angle 0$$

$$4j I_o - j10 I_1 - j2 I_2 = 0$$

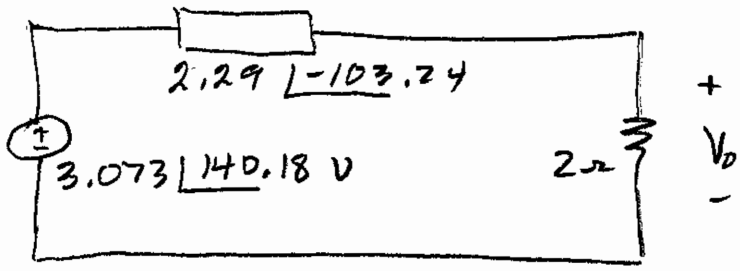
$$3 I_o + 0 I_1 - I_2 = 0$$

$I_o = (1.23 + j1.48)\,A$; $I_1 = (-0.246 - j0.295)$

$I_2 = (3.69 + j\,4.43)\,A$

$$V_{TH} = (-j8)(I_1)$$

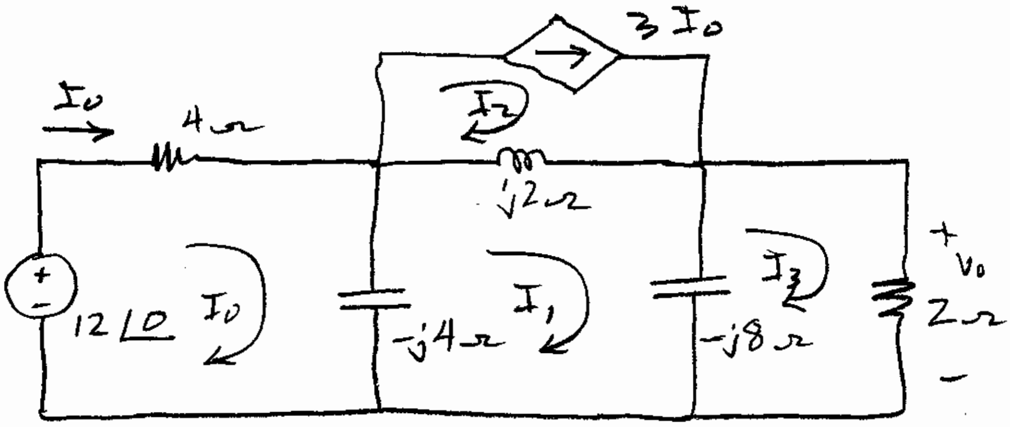
$$V_{TH} = 3.073\angle 140.18^\circ\ V$$



$$V_o = \frac{(3.073\angle 140.18)\,2}{2 + 2.29\angle -103.24} = 2.3\angle -163.3^\circ\ V$$

10.62

By Mixed mesh analysis



$$(4-j4)I_o + j4I_1 + 0I_2 + 0I_3 = 12$$

$$j4I_o - j10I_1 - j2I_2 + j8I_3 = 0$$

$$3I_o + 0I_1 - I_2 + 0I_3 = 0$$

$$0I_o + j8I_1 + 0I_2 + (2-j8)I_3 = 0$$

under matAA.m

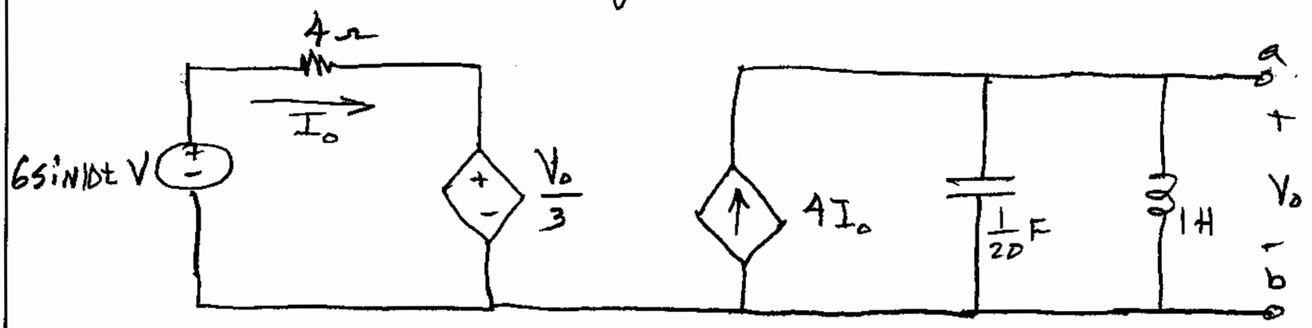
$$I_3 = (-1.101 - j0.3303) = 1.149\angle -163.3$$

$$\boxed{V_o = 2 \times I_3 = 2.3\angle -163.3 \text{ A}}$$

Check

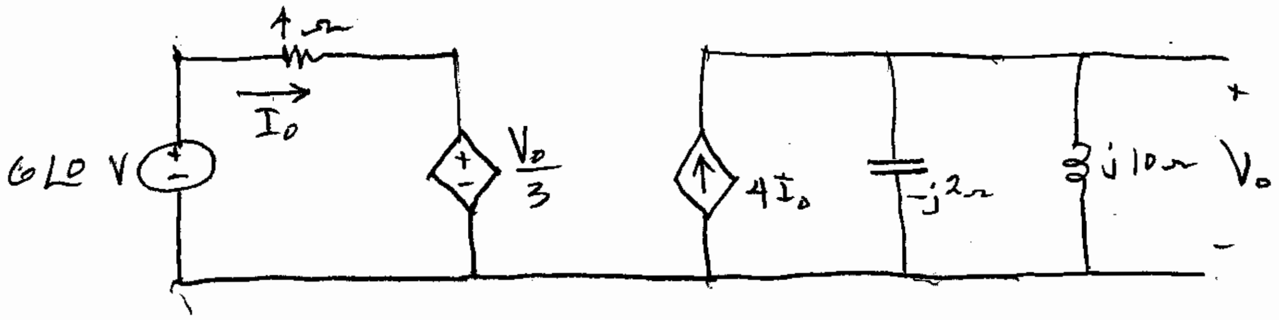
10.68

Find the Thevenin equivalent at terminals a-b



Prepare the phasor circuit:

$$\frac{1}{20}\text{ F} \rightarrow \frac{1}{j\omega C} = \frac{20}{j\,10} = -j2\ \Omega \qquad 1\text{ H} \rightarrow j\omega L = j10\ \Omega$$



$$V_o = V_{TH} = \left[\frac{(4I_o)(2\angle -90)}{(j10 - j2)}\right] 10\angle 90$$

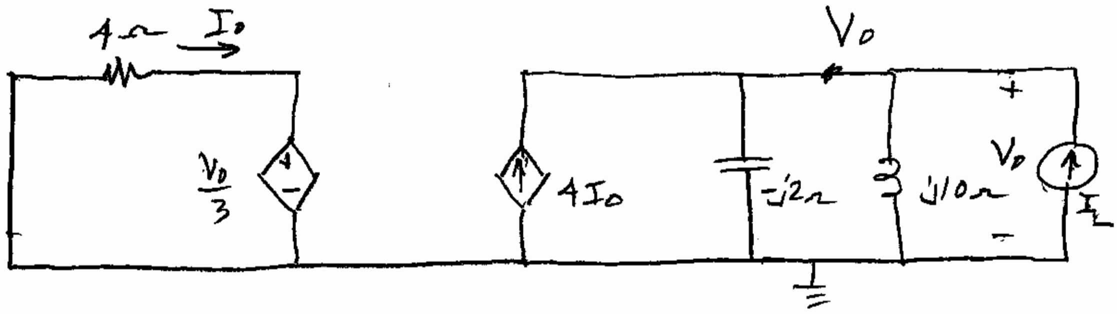
$$V_o = -j10 I_o \quad \text{but } 4I_o = 6 - \frac{V_o}{3}$$

$$4I_o = 6 + \frac{j10}{3} I_o$$

$$I_o = \frac{6}{4 - j\frac{10}{3}} = 0.89 + j.74 = 1.15\angle 39.8^\circ \text{ A}$$

$$V_o = -j10 I_o = 11.5\angle -50.2 = V_{TH}$$

10.68



$$\frac{V_o}{j10} - \frac{V_o}{j2} - 4I_o = I_L$$

$$I_o = -\frac{V_o}{12}$$

$$\frac{V_o}{j10} - \frac{V_o}{j2} + \frac{V_o}{3} = I_L$$

mult by $j30$

$$3V_o - 15V_o + j10V_o = j30 I_L$$

$$[-12 + j10]V_o = j30 I_L$$

$$Z_{TH} = \frac{V_o}{I_L} = \frac{j30}{(-12+j10)}$$

$$Z_{TH} = 1.92\angle -50.19 \ \Omega$$