Desk Copy

ECE 300
Test #3
Spring Semester, 2005

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Name _____
Print (last, first)

PM Section AM Section

This is a closed book, closed notes test. Use engineering paper for your work and work only on one side of your paper. Do not place any work on the exam sheet. Show how you obtain your answers.

(1) You are given the circuit shown in Figure 1. Asssume that the switch has been in position A for a long time. At t = 0 the switch is moved to position B.

(a) Develop the first order differential equation for finding v(t).
(b) Solve the differential equation to give the expression for v(t), t > 0.
(c) What is the time constant for the circuit for t > 0? Give a numerical value.
(d) Sketch the waveform for v(t), -1 < t < 1.

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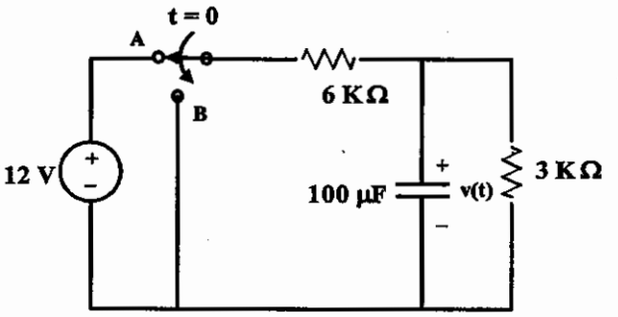

Figure 1: Circuit for problem 1.

(2) You are given the circuit of Figure 2. The switch has been closed for a very long time. At t = 0 the switch is opened.

(a) Find i(t) for t < 0.
(b) Develop the differential equation for i(t), t > 0.
(c) Solve the differential equation and find i(t) for t > 0.

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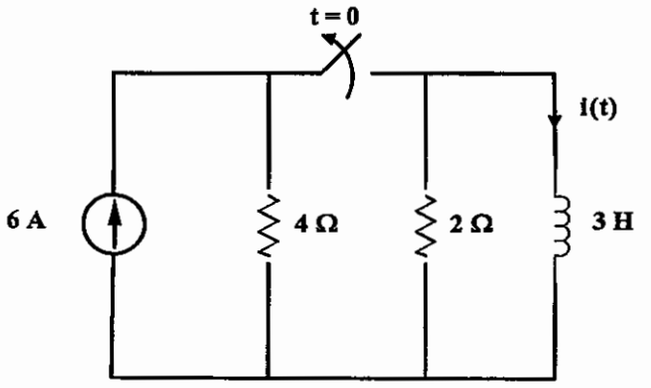

Figure 2: Circuit for problem 2.

(3) You are given the circuit of Figure 3. Find the time constant; express in seconds.

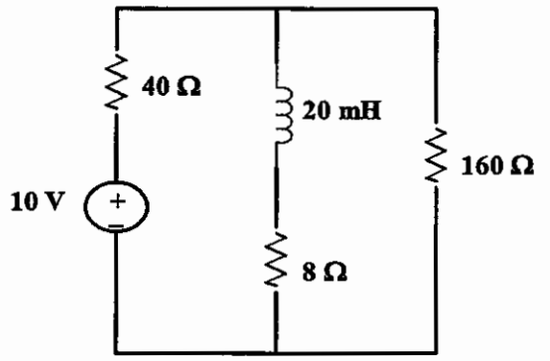

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Figure 3: Circuit for problem 3.

(4) You are given the circuit of Figure 4.

(a) Develop the differential equation of the circuit that can be used to directly solve for $v_c(t)$.
(b) Give the characteristic equation of the circuit.
(c) Give the undamped natural resonant frequency of the circuit, ω_o.
(d) Solve the differential equation of (a) for $v_c(t)$, $t > 0$.

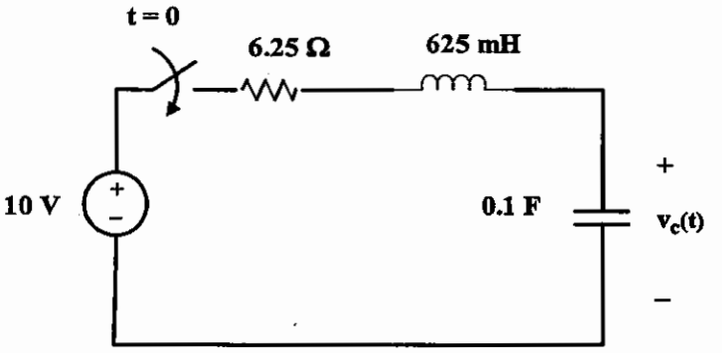

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Figure 4: Circuit for problem 4.

(5) You are given the circuit of Figure 5. Use nodal analysis to find the phasor voltage $\mathbf{V}_o$. Use a cosine reference.

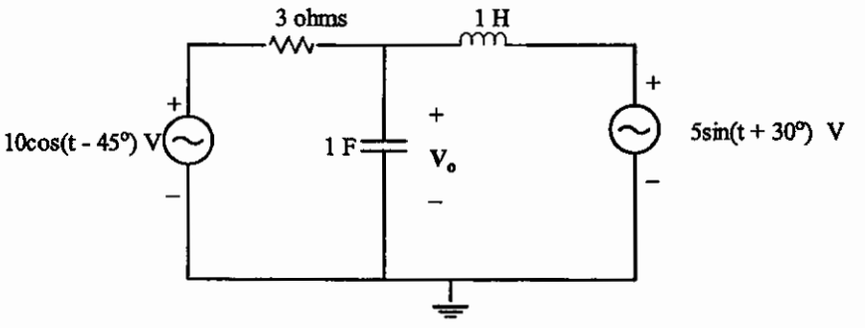

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Figure 5: Circuit for problem 5.

(6) You are given the circuit of Figure 6.

(a) Find $\mathbf{V_{TH}}$ and $\mathbf{Z_{TH}}$ (where $\mathbf{V_{TH}}$ is a phasor voltage) as seen looking into terminals a-b.
(b) Draw the Thevenin equivalent circuit seen looking into terminals a-b.

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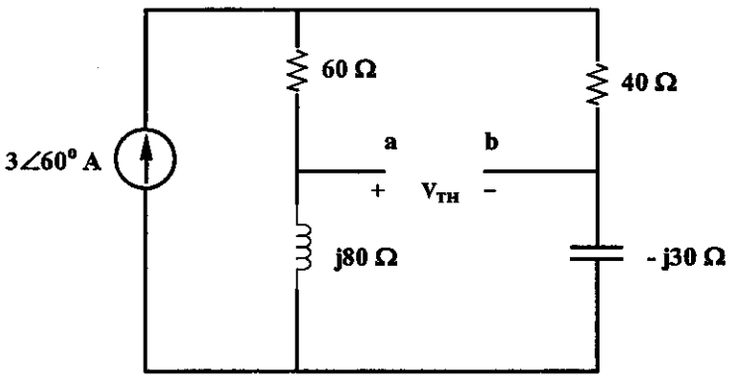

Figure 6: Circuit for problem 6.

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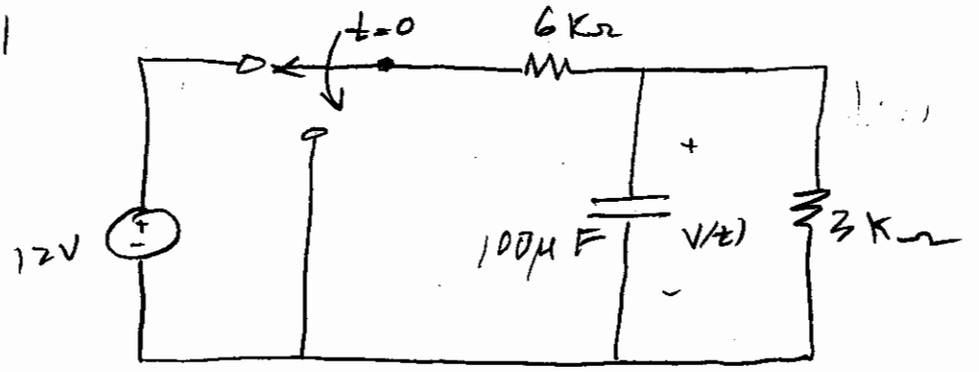


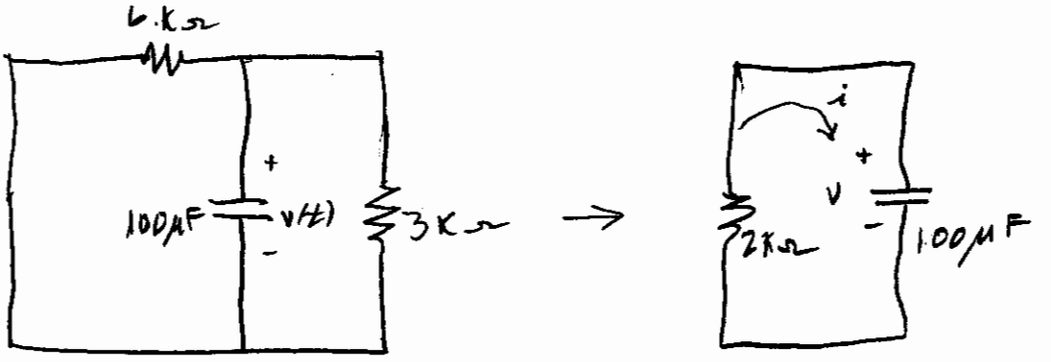
(a) Determine the first order d.e. for the voltage, V(t)

$t < 0$

$$i(t) = \frac{12}{9} = A$$

$$V(0^-) = \frac{12}{9} \times 3 = \boxed{4V = V(0^+)}$$

$t > 0$



$$\frac{3 \times 6}{9} = 2K\Omega$$

$$2K\,i(t) + V = 0 \qquad i = C\frac{dV}{dt}$$

$$2\times10^3 \times .1\times10^{-3}\frac{dV}{dt} + V(t) = 0$$

$$\boxed{\frac{dV}{dt} + 5V(t) = 0}$$

(b) $V(t) = Ke^{-5t}$

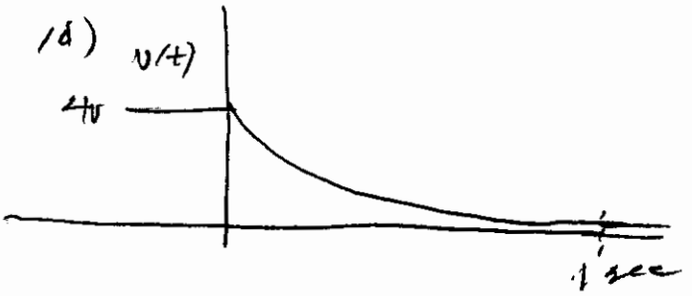
$V(0) = 4 = K$

#1

$$v(t) = 4e^{-5t} \text{ V}$$

1c) $\tau = RC = 0.2$ sec

1d)

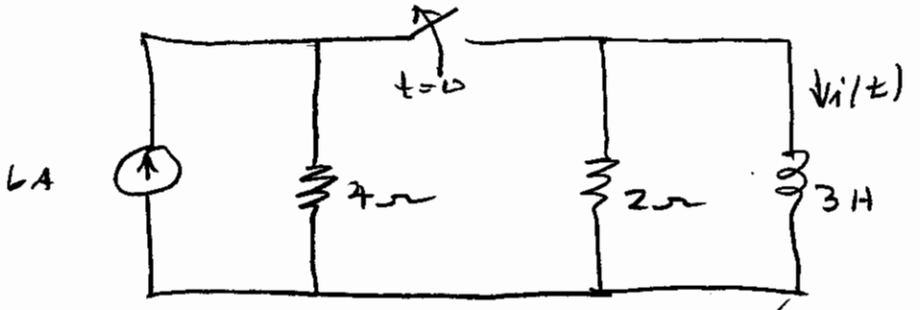


(2) For the following circuit;

(a) Find $i(t)$, $t<0$

(b) Develop the diff. eq. for $i(t)$, $t>0$.

(c) Solve for $i(t)$ from the diff. eq.



(a) $t<0$; $\boxed{i(t) = 6A}$

(b) $t>0$

$$Ri(t) + L\frac{di}{dt} = 0$$

$$\boxed{\frac{di}{dt} + \frac{R}{L} i(t) = 0}$$

(c) $i = Ke^{-t/\tau}$ $\qquad \tau = \frac{L}{R} = \frac{3}{2} = 1.5$

$i(0^-) = i(0^+) = 6 = K$

$$\boxed{i(t) = 6e^{-\frac{2}{3}t} \ A}$$

(3) Find the time constant for the circuit below.

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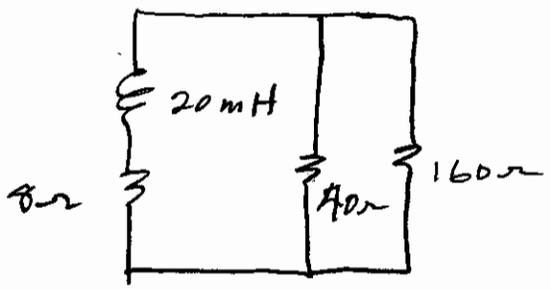


$$40 \parallel 80 = \frac{40 \times 160}{200} = \frac{160}{5}$$

$$R_{eq} = \frac{160}{5} + 8 = \frac{160 + 40}{5} = 40\,\Omega$$

$$\frac{L}{R} = \frac{20\ mH}{40} = 0.5\ msec$$

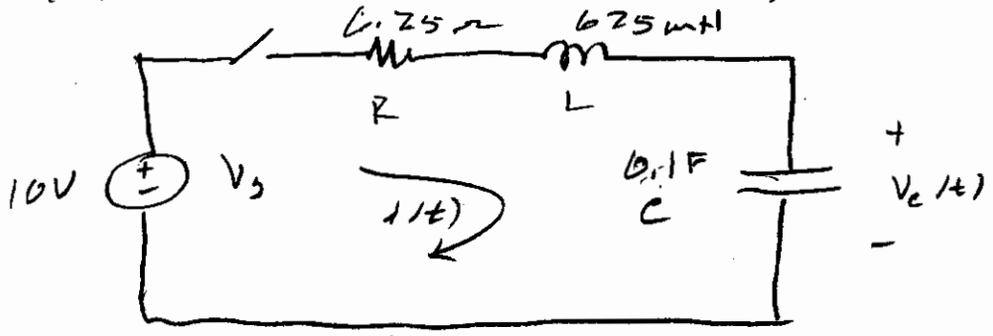
$$\tau = \frac{L}{R} = 0.0005\ seconds$$

(4) 1a) Develop the d.e. that can be solved for $v_c(t)$

1b) Give the C.E.

1c) Give the UNdamped natural resonant frequency, ω_0,

1d) Solve the d.e. for $v_c(t)$



$$R\,i(t) + L\frac{di}{dt} + v_c(t) = V_s$$

$$i(t) = C\frac{dv_c(t)}{dt}$$

$$RC\frac{dv_c(t)}{dt} + LC\frac{d^2 v_c(t)}{dt^2} + v_c(t) = V_s$$

$$\frac{d^2 v_c(t)}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{v_c(t)}{LC} = \frac{V_s}{LC}$$

$$\frac{d^2 v_c}{dt^2} + 10\frac{dv_c(t)}{dt} + 16\,v_c(t) = 160$$

(b) $$s^2 + 10s + 16 = 0$$

$$(s+2)(s+8) = 0$$

(4) (c) $\omega_0 = \sqrt{16} = 4$ rad/sec

(d) $V_c = V_{ct} + V_{css}$

$V_{css} = K$

~~$16K = 160$~~

$\boxed{K = 10 = V_{css}}$

with $s^2 + 10s + 16 = (s+2)(s+8)$

$V_{ct} = K_1 e^{-2t} + K_2 e^{-8t}$

$V_c(t) = 10 + K_1 e^{-2t} + K_2 e^{-8t}$

$0 = 10 + K_1 + K_2$

$\boxed{K_1 + K_2 = -10}$

since $i(0^+) = C \frac{dV_c(0^+)}{dt}$ and $i(0^+) = 0$

then $\frac{dV_c(0^+)}{dt} = 0$

$\frac{dV_c(t)}{dt} = -2K_1 e^{-2t} - 8K_2 e^{-8t}$

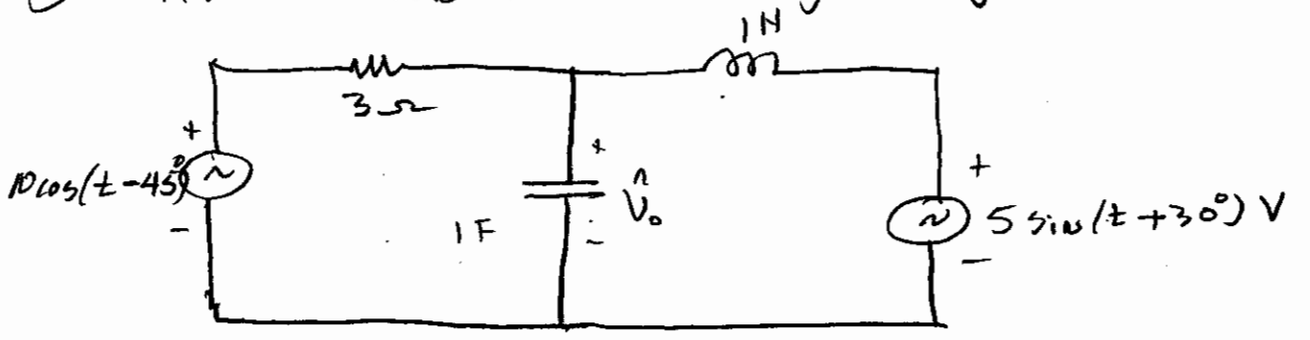
$0 = -2K_1 - 8K_2$

$\boxed{K_1 + 4K_2 = 0}$

$\begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} \begin{Bmatrix} K_1 \\ K_2 \end{Bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$ $K_1 = -13.33$, $K_2 = 3.33$

$V_c(t) = \left(10 - 13.33 e^{-2t} + 3.33 e^{-8t}\right)$ V

(5) Find $\hat{V}_o$ for the following using Nodal



Make the phasor ckt

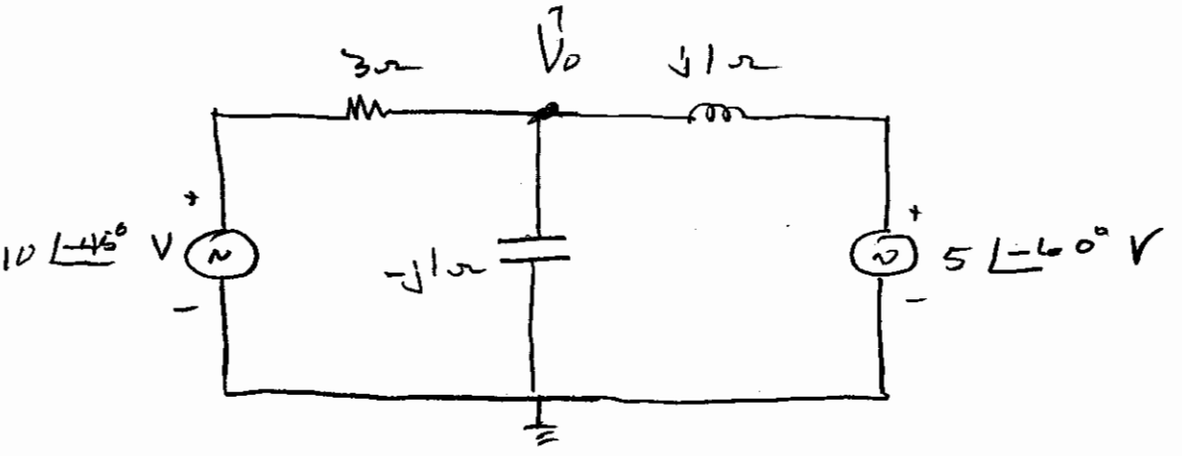
$1\,H \rightarrow j\omega L = j1\,\Omega$

$1\,F \rightarrow \frac{-j}{\omega C} = -j1\,\Omega$

$10\cos(t-45^\circ) \rightarrow 10\angle -45$

$5\sin(t+30) = 5\cos(t-60) \rightarrow 5\angle -60$

Phasor ckt



$$\frac{\hat{V}_o - 10\angle -45^\circ}{3} - \frac{\hat{V}_o}{j1} + \frac{\hat{V}_o - 5\angle -60}{j1} = 0$$

$$.333\hat{V}_o + j\hat{V}_o - j\hat{V}_o = 3.331 - j5\angle -60$$

$$\frac{\hat{V}_o}{3} = \frac{10\angle -45 + 5\angle -150}{3}$$

$$\hat{V}_o = 10\angle -45 + 15\angle -150 = -5.12 - j14.57\ V$$

$$\hat{V}_1 \approx 15.73\angle -112.1\ V = 15.73\angle 247.9^\circ\ V$$

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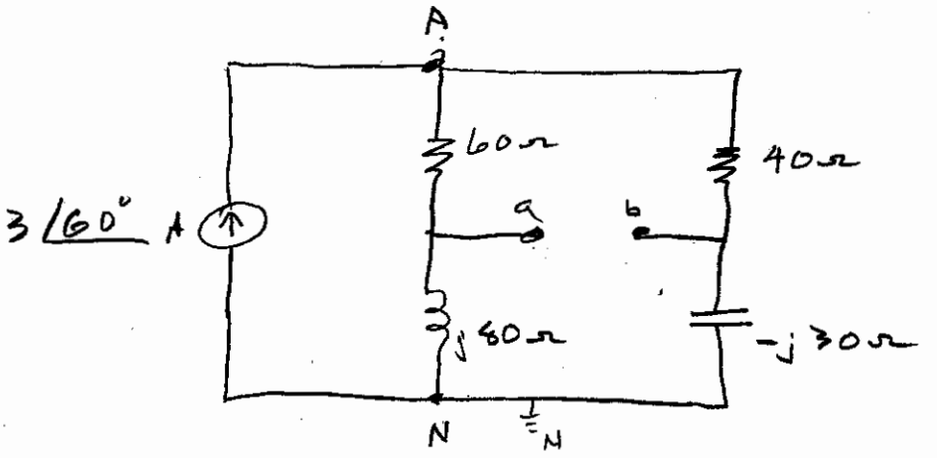
(6) FIND V_{TH} & Z_{TH} for the following circuit and draw the thevenin circuit.



$$Z_{TH} = \frac{100(j50)}{100+j50}$$

$$\boxed{Z_{TH} = 44.72 \angle 63.4}$$

$$Z_{AN} = \frac{(60+j80)(40-j30)}{100+j50} = 44.72\angle -10.3^\circ \Omega$$

$$V_{AN} = (3\angle 60)(44.72\angle -10.3 = 134.16\angle 49.3$$

$$V_{aN} = \frac{(V_{AN})\times(j80)}{60+j80} = 107.33\angle 86.17$$

$$V_{bN} = \frac{V_{AN}(30\angle -90)}{40-j30} = 80.49\angle -3.83$$

$$V_{ab} = V_{aN} - V_{bN} = 134.16\angle 123^\circ$$