Solutions to Exam #1

Test A

ECE 300

Spring Semester, 2006

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ECE Department
(1) You are given the circuit of Figure 1.
(a) Find $V_{eb}$.
(b) Find the power supplied by the 15 V source.

Assume $I$ as shown. Use default sign convention. Start from "a".

(a) $10I - 10 + 20I - 20 + 15I - 18 + 18I + 32I + 15 + 12I = 0$

$107I = 33$

$I = \frac{33}{107} = 0.308 \text{ A}$

$-V_{eb} - 20I + 10 - 10I - 12I = 0$

$V_{eb} = -42I + 10 = -2.94 \text{ V}$

(b) $P_{up} = \frac{-15 \times I}{15} = -4.62 \text{ W}$

(being changed)
(2) You are given the circuit shown in Figure 2. Find the voltage $V_o$.

$15\%$

From the diagram using Ohm's Law:

$I_1 = \frac{10}{20} = 0.5 A$

$V_{ab} = 20 I_1 = 20 \times 0.5 = 10 V$

$I_2 = \frac{20}{20} = 1 A$

$I_3 = I_1 + I_2 = 0.5 + 1 = 1.5 A$

$V_o = 10 I_3 + 20$

$V_o = 35 V$
(3) You are given the circuit of Figure 3. Find the resistance $R_{eq}$.

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At the right side we have

$30 \times 20 = 12$ Ω

Then we have $12$ Ω in parallel with $12$ Ω, giving

$\frac{12 \times 12}{12 + 12} = 6$ Ω

Then we have $6$ Ω in series with $12$ Ω, giving $6 + 12 = 10$ Ω

Then we have $10$ Ω in parallel with $10$ Ω, giving

$\frac{10 \times 10}{10 + 10} = 5$ Ω

Then we have $5$ Ω in series with $5$ Ω, giving $5 + 5 = 10$ Ω

Then we have $10$ Ω in parallel with $10$ Ω, giving $\boxed{R_{eq} = 5}$ Ω
(4) You are given the circuit of Figure 4. Find the gain $v_o/v_s$.

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\[ V_o = (20k)(5I_o) \] (#1) (at the right side)

On the left:

\[ -V_o - 4kI_o + 0.2V_o = 0 \]

so

\[ 4kI_o = 0.2V_o - V_s \]

\[ I_o = \frac{0.2V_o - V_s}{4k} \] (#2)

Substitute (#2) into (#1)

\[ V_o = 20k \times 5 \left[ \frac{0.2V_o - V_s}{4k} \right] = 25 \left[ \frac{0.2V_o - V_s}{4k} \right] \]

\[ V_o = 15V_o - 25V_s \]

\[ -4V_o = -25V_s \]

\[ \frac{V_o}{V_s} = 6.25 \]
(5) You are given the circuit of Figure 5. Use mesh analysis to find the current $I_0$.

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**Assign mesh currents as shown above. Redraw the circuit with the current inactive, as such.**

**Write the two mesh equations:**

\[-15 + 4(i_1 - i_2) + 8(i_3 - i_2) + 5i_3 = 0\]

or

\[4i_1 = 12i_2 + 13i_3 = 15\]  \hspace{1cm} (\#1)

\[-4(i_1 - i_2) + 10 + 10i_2 - 8(i_3 - i_2) = 0\]

\[-4i_1 + 22i_2 - 8i_3 = -10\]  \hspace{1cm} (\#2)

**Constraint Equation:**

\[i_3 - i_1 = 2\]

\[-i_1 + 0i_2 + i_3 = 2\]  \hspace{1cm} (\#3)
From (3), (4), (2)

\[
\begin{bmatrix}
4 & -12 & 13 \\
-4 & 22 & -8 \\
-1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{bmatrix}
= \begin{bmatrix}
15 \\
-10 \\
2
\end{bmatrix}
\]

\[\lambda_1 = -0.739\text{ A}, \lambda_2 = -0.130\text{ A}, \lambda_3 = 1.26\text{ A}\]

\[I_0 = \lambda_2 - \lambda_3\]

\[I_0 = -0.13 - 1.26\]

\[I_0 = -1.39\text{ A}\]
**Node Analysis**

![Circuit Diagram]

\[ V_1 - 15 \frac{4}{4} + \frac{V_1 - V_2}{8} - 2 = 0 \]

\[ 2V_1 - 30 + V_1 - V_2 = 120 \]

\[ 3V_1 - V_2 = 46 \]

\[ V_2 = V_1 + \frac{V_2 + 10 - 15}{10} + \frac{V_2}{5} = 0 \]

\[ 5V_2 - 5V_1 + 4V_2 - 20 + 8V_2 = 0 \]

\[ -5V_1 + 17V_2 = 20 \]

\[ V_1 = 17.43 \text{ V}, \quad V_2 = 6.3 \text{ V} \]

\[ I_0 = \frac{V_2 - V_1}{8} = \frac{6.3 - 17.43}{8} = -1.299 \text{ A} \]

*Check* \[ \frac{3}{3} \]
(6) You are given the circuit of Figure 6. Use nodal analysis to find $v_1$ and $v_2$.

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A supernode is present as shown:
We have:
\[
\frac{v_1 - 10}{20} + \frac{v_1}{5} + \frac{v_2 - 10}{20} + \frac{v_2}{25} = 0
\]

\[
0.05v_1 - 0.5 + 0.2v_1 + 0.05v_2 - 0.5 + 0.04v_2 = 0
\]

\[
0.25v_1 + 0.09v_2 = 1
\]

Constraint:
\[
v_1 - 2i_1 - v_2 = 0 \quad i_1 = \frac{v_2}{25} = 0.04v_2
\]

\[
v_1 - 0.08v_2 - v_2 = 0
\]

\[
v_1 - 1.08v_2 = 0
\]

\[
v_1 = 3v_1, \quad v_2 = 2.78V
\]
Mesh Analysis

\[ -10 + 20(i_2 - i_3) + 5(i_2 - i_1) = 0 \]
\[ -5i_1 + 25i_2 - 20i_3 = 10 \]
\[ -5(i_2 - i_1) + 2i_1 + 25i_1 = 0 \]
\[ 32i_1 - 5i_2 + 20i_3 = 0 \]
\[ -20(i_2 - i_3) + 20i_3 - 2i_1 = 0 \]
\[ -2i_1 - 20i_2 + 40i_3 = 0 \]

\[
\begin{bmatrix}
-5 & 25 & -20 \\
32 & -5 & 0 \\
-2 & -20 & 40 \\
\end{bmatrix}
\begin{bmatrix}
\dot{i}_1 \\
\dot{i}_2 \\
\dot{i}_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
10 \\
0 \\
0 \\
\end{bmatrix}
\]

\[ \dot{i}_1 = 0.111 \quad \dot{i}_2 = 0.711 \quad \dot{i}_3 = 0.761 \]

\[ V_2 = 25\dot{i}_1 = 2.78 \text{ V} \quad \text{check} \]

\[ V_1 = (i_2 - i_1) 5 = 3 \text{ V} \quad \text{check} \]