ECE 300
Spring Semester, 2006
HW Set #2: version 2

Due: January 26, 2005 **Name**______________________
wlg Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations**. **Be sure to show how you got your answers.** Each problem counts 10 points.

(1) You are given the circuit of Figure 1. The four nodes of the circuit are identified. Using KCL, write the respective equation at each node. This will give you 4 equations. Add the equations you get at nodes 1, 2, and 3. This should give the equation you get at node 4. What this shows is that only 3 of the equations are linearly independent. In general, if a circuit has n nodes, only n-1 of the nodes can be used in applying KCL and getting independent equations. **Rule: If a circuit has n nodes, only n-1 nodes can be used to write independent KCL equations.**

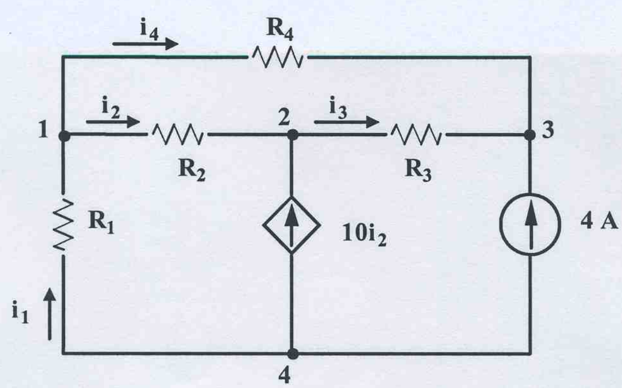

Figure 1: Circuit for problem 1.

(2) You are given the circuit of Figure 2. Find I_1, I_2 and I_3. Ans: I_1 = 5 mA, I_2 = 20 mA, I_3 = -15 mA

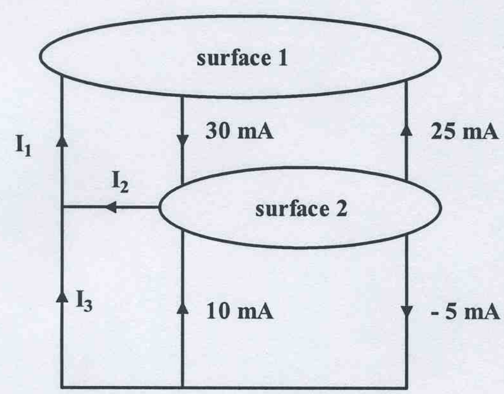

Figure 2: Circuit for problem 2.

(3) You are given the circuit of Figure 3. The loops are identified as loop 1, loop 2, and loop 3. Write KVL around each loop. Add loop 1 and loop 2 together and show that you get the equation for loop 3. This shows that only two of the equations are independent. **<u>Rule: The</u> number of independent KVL equations that may be written for a circuit is equal to the number of mesh (the number of "windows") in the circuit.**

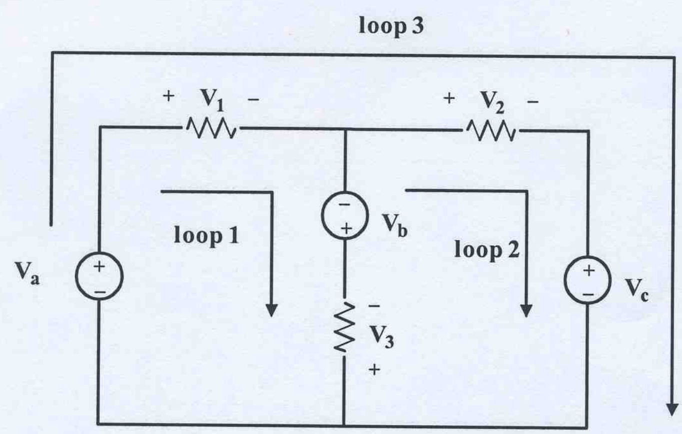

Figure 3: Circuit for problem 3.

(4) You are given the circuit of Figure 4. (a) Find V_{ad} Ans –16 V; (b) Find V_{eb} Ans – 36 V

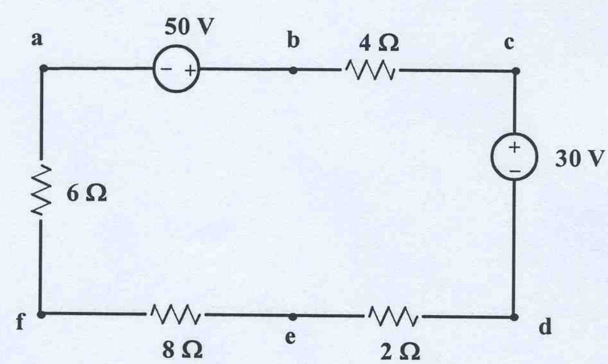

Figure 4: Circuit for problem 4.

(5) You are given the circuit of Figure 5. Find i_1, i_2 and i_3. Ans: $i_1 = 0.654$ A, $i_2 = 0.462$ A, $i_3 = 0.192$

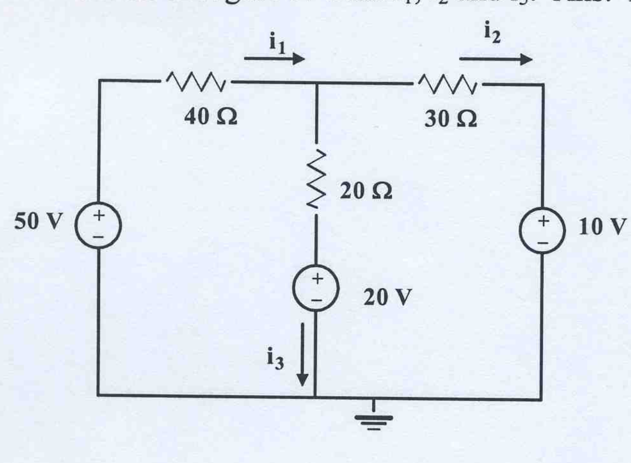

Figure 5: Circuit for problem 5.

(6) For the circuit in Problem 5, show that the power absorbed by the 40 Ω, 30 Ω and 20 Ω resistors is equal to the power supplied by the 50 V, 20 V and 10 V sources.

(7) You are given the circuit of Figure 7. Compute (determine) the values of the currents i_1, i_2, i_3, i_4.
Ans: $i_1 = 2$ A, $i_2 = 0.167$ A, $i_3 = 1.83$ A, $i_4 = -1.17$ A.

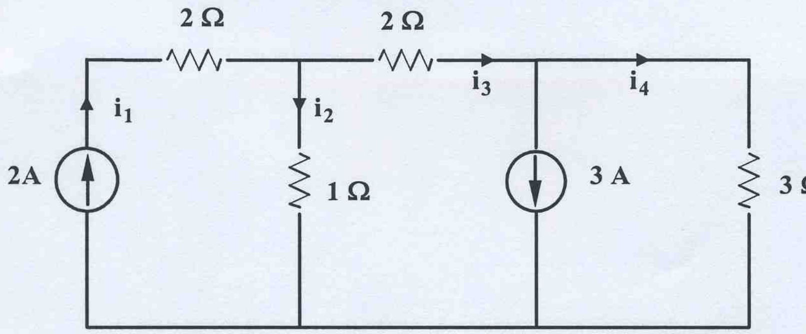

Figure 7: Circuit for problem 7.

(8) A certain circuit having two independent voltage sources and three linear resistors produces the following equations in matrix form.

$$\begin{bmatrix} 25 & -15 \\ -15 & 45 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 30 \end{bmatrix}$$

(a) Draw a circuit using 3 linear resistors and 2 constant independent voltage sources that will produce these equations. Show the locations and values of R_1, R_2, R_3, V_1 and V_2.
Ans: On your own.

(b) Solve for i_1 and i_2. Ans: $i_1 = 3$ A, $i_2 = 1.67$ A

(9) You are given the circuit of Figure 9. Use current division, directly, to find I_{30}. Ans: $I_{30} = 4$ A

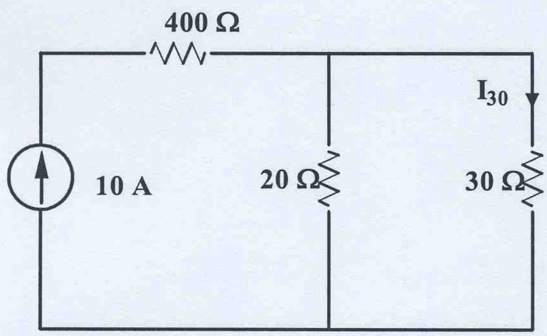

Figure 9: Circuit for problem 9.

(10) (a) Find R_{ab} for the circuit of Figure 10a. Ans: 10 Ω

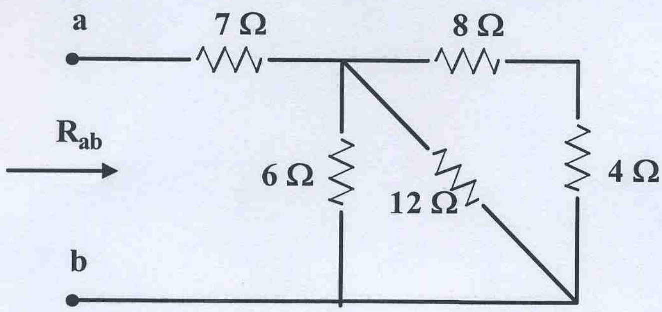

Figure 10a: Circuit for problem 10a.

(b) Find R_{ab} seen looking into terminals a-b for the circuit of Figure 10b. Ans: 20 Ω

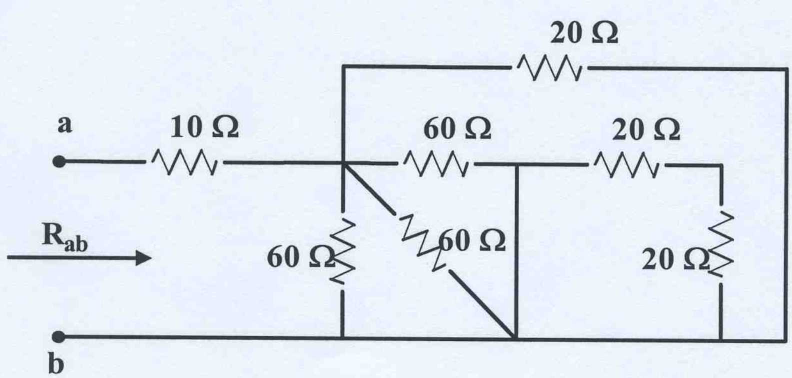

Figure 10b: Circuit for problem 10b

(11) Obtain the equivalent resistance R_{ab} in the circuit of Figure 11. Ans: On your own.

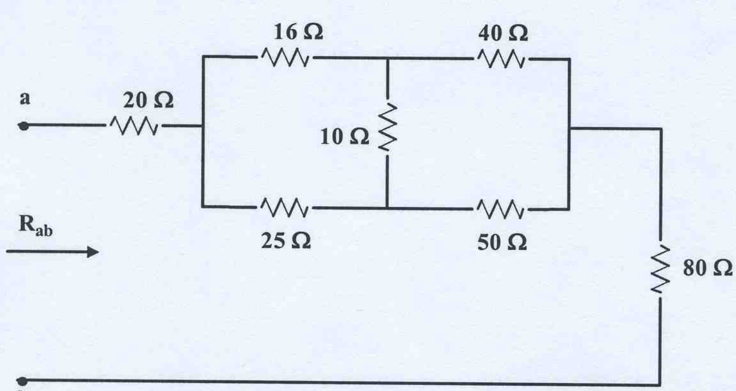

Figure 11: Circuit for problem 11. 132.

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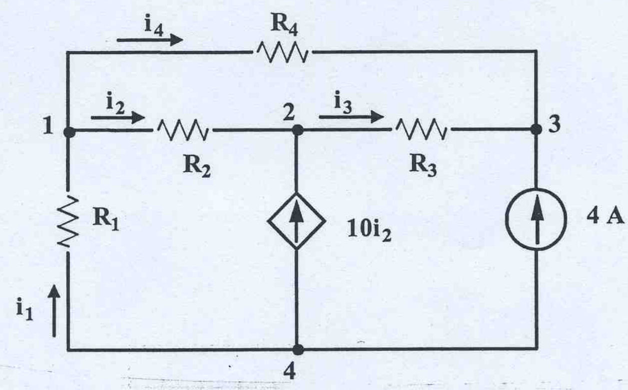
(1) You are given the circuit of Figure 1. The four nodes of the circuit are identified. Using KCL, write the respective equation at each node. This will give you 4 equations. Add the equations you get at nodes 1, 2, and 3. This should give the equation you get at node 4. What this shows is that only 3 of the equations are linearly independent. In general, if a circuit has n nodes, only n-1 of the nodes can be used in applying KCL and getting independent equations. **Rule: If a circuit has n nodes, only n-1 nodes can be used to write independent KCL equations.**



At #1:

$$i_1 - i_2 + 0i_3 - i_4 = 0 \qquad (1)$$

At # 2:

$$0i_1 + 11i_2 - i_3 + 0i_4 = 0 \qquad (2)$$

At # 3:

$$0i_1 + 0i_2 + i_3 + i_4 = -4 \qquad (3)$$

At # 4:

$$-i_1 - 10i_2 + 0i_3 + 0i_4 = 4 \qquad (4)$$

Adding (1), (2), (3):

$$i_1 - i_2 + 11i_2 - i_3 + i_3 - i_4 + i_4 = -4$$

OR

$$i_1 + 10i_2 = -4 \qquad (5)$$

Multiply (5) by (-1) gives

(2) You are given the circuit of Figure 2. Find I_1, I_2 and I_3. Ans: $I_1 = 5$ mA, $I_2 = 20$ mA, $I_3 = -15$ mA

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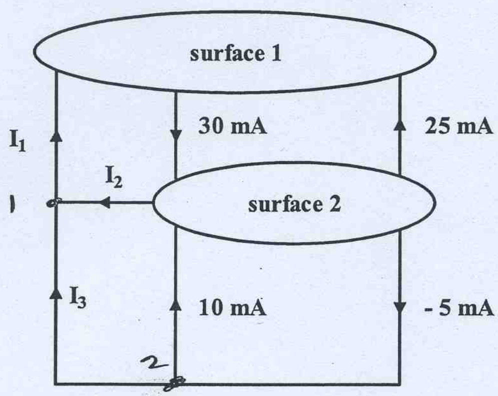

At surface 1:

$$I_1 + 25_{mA} - 30_{mA} = 0 \Rightarrow \underline{I_1 = 5\text{ mA}}$$

At surface 2:

$$10\text{ mA} + 30\text{ mA} - I_2 + 5\text{ mA} - 25\text{ mA} \Rightarrow \underline{I_2 = 20\text{ mA}}$$

At Node #1

$$I_2 + I_3 - I_1 = 0$$

$$I_3 = I_1 - I_2; = 5\text{ mA} - 20\text{ mA} \Rightarrow \underline{I_3 = -15\text{ mA}}$$

For Check

At node 2;

$$I_3 + 10\text{ mA} - (-5\text{ mA}) \Rightarrow \boxed{I_3 = -15\text{ mA}}$$

Check

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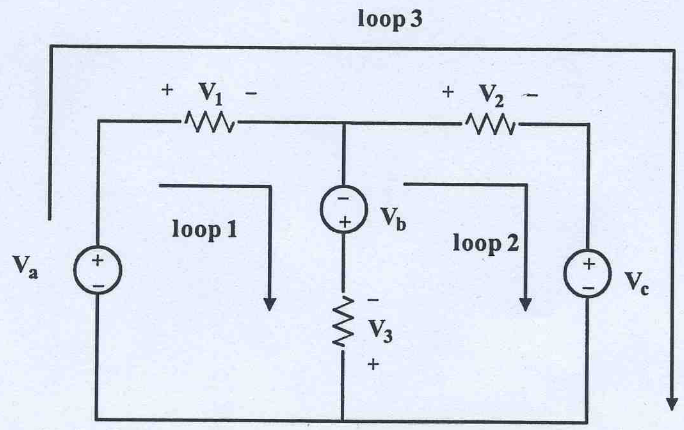
(3) You are given the circuit of Figure 3. The loops are identified as loop 1, loop 2, and loop 3. Write KVL around each loop. Add loop 1 and loop 2 together and show that you get the equation for loop 3. This shows that only two of the equations are independent. **Rule: The number of independent KVL equations that may be written for a circuit is equal to the number of mesh (the number of "windows") in the circuit.**



Loop 1:

$$-V_a + V_1 - V_b - V_3 = 0$$

so

$$-V_a - V_b + 0V_c + V_1 + 0V_2 - V_3 = 0 \qquad (1)$$

Loop 2:

$$+V_3 + V_b + V_2 + V_c = 0$$

so

$$0V_a + V_b + V_c + 0V_1 + V_2 + V_3 = 0 \qquad (2)$$

Loop 3

$$-V_a + V_1 + V_2 + V_c = 0$$

so

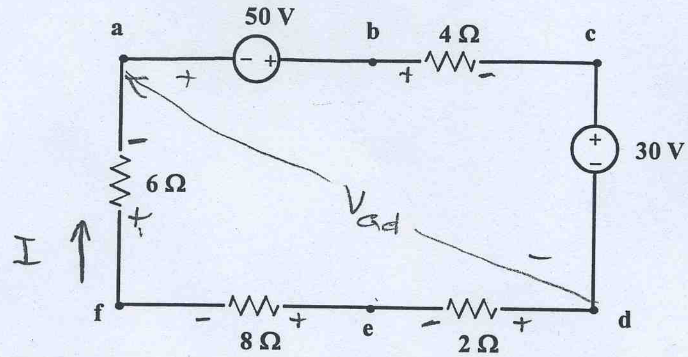
$$-V_a + 0V_b + V_c + V_1 + V_2 + 0V_3 = 0 \qquad (3)$$

Adding (1) and (2) gives

$$-V_a + 0V_b + V_c + V_1 + V_2 + 0V_3 = 0 \qquad (4)$$

Eq (4) is the same as Eq (3) which shows only 2 of the 3 equations are linearly independent.

(4) You are given the circuit of Figure 4. (a) Find V_{ad} Ans –16 V; (b) Find V_{eb} Ans – 36 V



Assume a current I as shown. Use the default sign convention for voltage drops, marking the diagram accordingly.

Start at "b", go clockwise, use Σ drops = 0;

$$4I + 30 + 2I + 8I + 6I - 50 = 0$$

$$20I_1 = 20$$

$$I_1 = 1\ A$$

For V_{ab}

Start at "d", go up through V_{ab}, go left, back to "d".

$$-V_{ab} - 6I - 8I - 2I = 0$$

$$V_{ab} = -16I \quad ; \quad I = 1\ A$$

$$V_{ab} = -16\ V$$

For V_{eb};

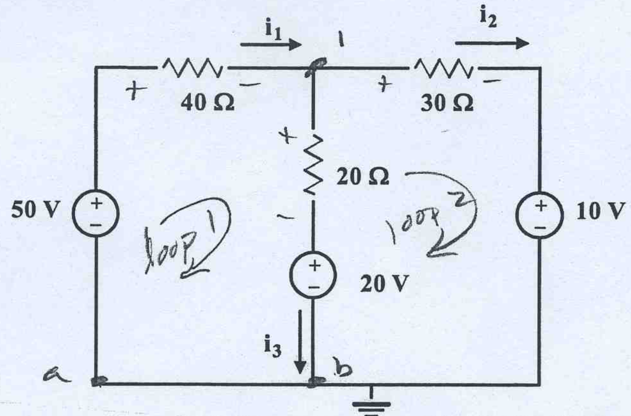
Start at b, go through V_{eb}, go left, back to b,

$$-V_{eb} - 2I - 30 - 4I = 0$$

$$V_{eb} = -30 - 6I = -36V$$

$$V_{eb} = -36\ V$$

(5) You are given the circuit of Figure 5. Find i_1, i_2 and i_3. Ans: $i_1 = 0.654$ A, $i_2 = 0.462$ A, $i_3 = 0.192$



Around Loop 1: KVL

Mark the diagram with default voltage sign convention. Start at "a", go cw, Σ drop = 0

$$-50 + 40 I_1 + 20 I_3 + 20 = 0$$

or

$$\boxed{40 I_1 + 0 I_2 + 20 I_3 = 30} \quad (1)$$

Around Loop 2: KVL

Start at "b", go cw around loop 2, Σ drops = 0

$$-20 - 20 I_3 + 30 I_2 + 10 = 0$$

$$\boxed{0 I_1 + 30 I_2 - 20 I_3 = 10} \quad (2)$$

Apply KCL at Node 1

$$\boxed{I_1 - I_2 - I_3 = 0} \quad (3)$$

Solve (1), (2), (3)

$$\begin{bmatrix} 40 & 0 & 20 \\ 0 & 30 & -20 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 10 \\ 0 \end{bmatrix}$$

$\boxed{I_1 = 0.654 \text{ A}}$, $\boxed{I_2 = 0.462 \text{ A}}$, $\boxed{I_3 = 0.192 \text{ A}}$

(6) For the circuit in Problem 5, show that the power absorbed by the 40 Ω, 30 Ω and 20 Ω resistors is equal to the power supplied by the 50 V, 20 V and 10 V sources.

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Absorbed Power

$P_{40} = I_1^2 40 = (0.654)^2 \times 40$

$P_{40} = 17.12 \text{ W}$

$P_{30} = I_2^2 \times 30 = (0.462)^2 30$

$P_{30} = 6.4 \text{ W}$

$P_{20} = I_3^2 20 = (0.192)^2 20$

$P_{20} = 0.737 \text{ W}$

$\Sigma P_{ABSORBED} = P_{40} + P_{30} + P_{20} = (17.12 + 6.4 + 0.737)\text{W}$

$\boxed{P_{ABSORBED} = 24.26 \text{ W}}$

Supplied Power

$P_{50\,supplied} = 50 \times I_1 = 50 \times 0.654 = 32.7 \text{ W}$

$P_{50\,supplied} = 32.7 \text{W}$

$P_{20\,supplied} = -20 I_3 = -20 \times 0.192 = -3.84 \text{ W}$

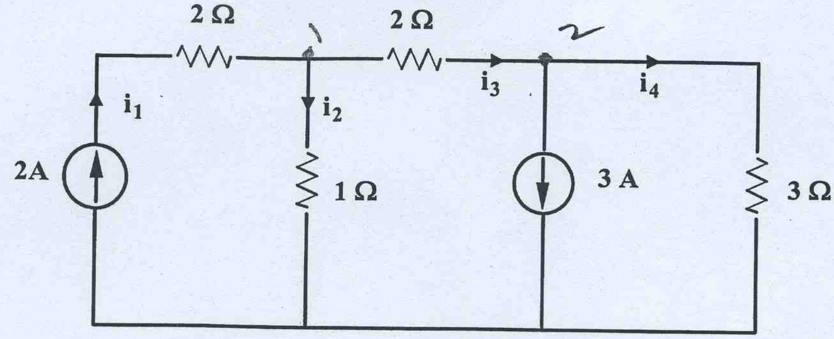
$P_{20\,supplied} = -3.84 \text{W}$

$P_{10\,supplied} = -10\,I_2 = -10 \times .462 = -4.62 \text{W}$

$\Sigma P_{supplied} = (32.7 - 3.84 - 4.62)\text{W} = 24.24 \text{W}$

$\Sigma P_{ABS} = 25.26 \text{ W}$ (very close)

(7) You are given the circuit of Figure 7. Compute (determine) the values of the currents i_1, i_2, i_3, i_4.
Ans: $i_1 = 2$ A, $i_2 = 0.167$ A, $i_3 = 1.83$ A, $i_4 = -1.17$ A.



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By inspection, (KCL)

$$\boxed{i_1 = 2A}$$

At Node 1 (KCL)

$$i_1 = i_2 + i_3$$

OR

$$\boxed{i_2 + i_3 = 2} \qquad (1)$$

At Node 2 (KCL)

$$i_3 = i_4 + 3$$

$$\boxed{i_3 - i_4 = 3} \qquad (2)$$

Around the loop that does not have a current source,

$$\boxed{-1 \times i_2 + 2 i_3 + 3 i_4 = 0} \qquad (3)$$

so we have

$$\begin{cases} i_2 + i_3 + 0\, i_4 = 2 \\ 0\, i_2 + i_3 - i_4 = 3 \\ -i_2 + 2 i_3 + 3 i_4 = 0 \end{cases}$$

Gives

$$\boxed{i_2 = 0.167A} \quad \boxed{i_3 = 1.83 A,} \quad \boxed{i_4 = -1.167A}$$

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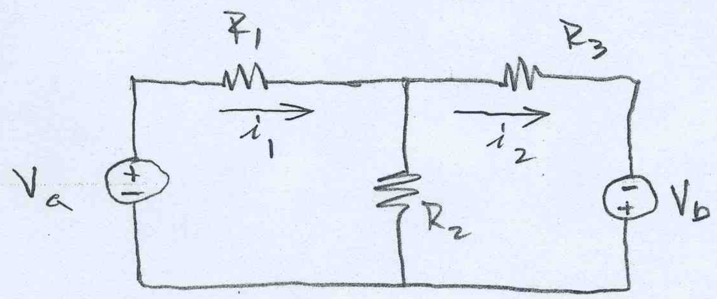
(8) A certain circuit having two independent voltage sources and three linear resistors produces the following equations in matrix form.

$$\begin{bmatrix} 25 & -15 \\ -15 & 45 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 30 \end{bmatrix}$$

(a) Draw a circuit using 3 linear resistors and 2 constant independent voltage sources that will produce these equations. Show the locations and values of R_1, R_2, R_3, V_1 and V_2. Ans: On your own.

(b) Solve for i_1 and i_2. Ans: $i_1 = 3$ A, $i_2 = 1.67$ A

After some thought, we realize that the desired equations can be derived from the following circuit.



Other structures are possible

We have

$$(R_1 + R_2)i_1 - R_2 i_2 = V_a \qquad (1)$$

and

$$-R_2 i_1 + (R_2 + R_3) i_2 = V_b \qquad (2)$$

Comparing coefficients of Eq (1) and (2) with the given equations, we have;

$$R_1 + R_2 = 25\Omega, \quad R_2 = 15\Omega \quad V_a = 50V$$

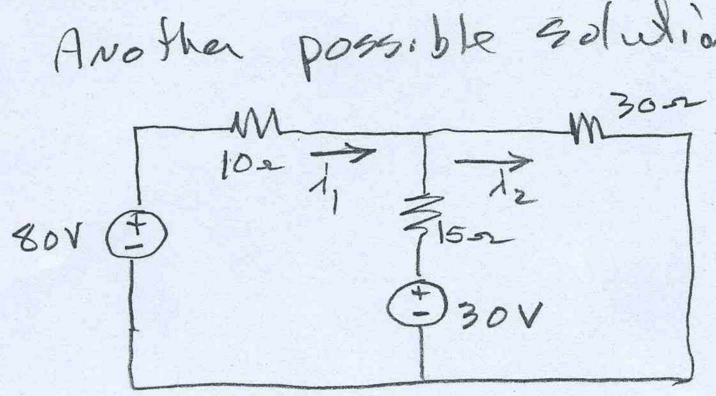
$$R_2 + R_3 = 45, \quad V_b = 30$$

so $\boxed{V_a = 50V}$, $\boxed{V_b = 30V}$, $\boxed{R_1 = 10\Omega}$, $\boxed{R_2 = 15\Omega}$

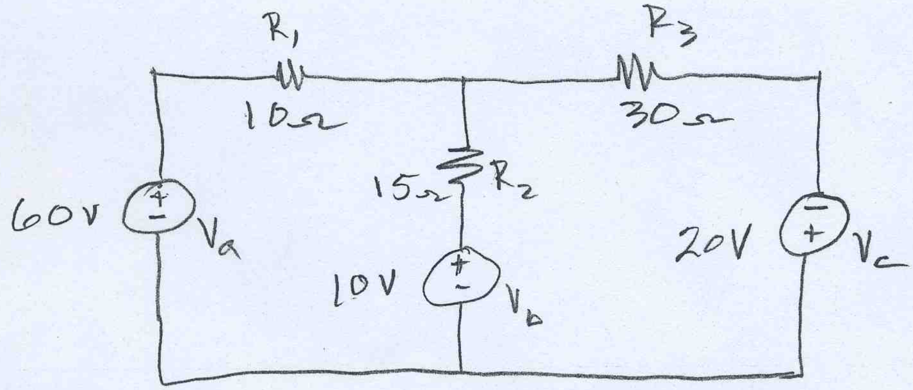
$\boxed{R_3 = 30\Omega}$ Other answers possible.

(8) continued

Another possible solution for (8)



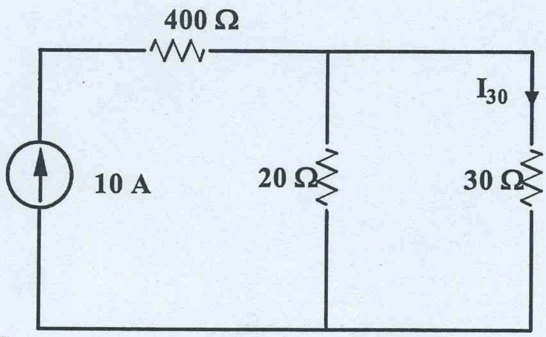
Another Form



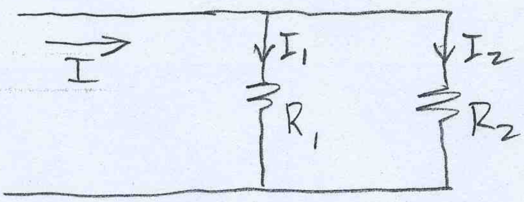
Lots of combinations of V_a, V_b, V_c will work.

I believe R_1, R_2, R_3 are unique.

(9) You are given the circuit of Figure 9. Use current division, directly, to find I_{30}. Ans: $I_{30} = 4$ A



Generally, if we are given,



we can show;

$$I_1 = \frac{I \times R_2}{R_1 + R_2} \; ; \quad I_2 = \frac{I \times R_1}{R_1 + R_2}$$

words

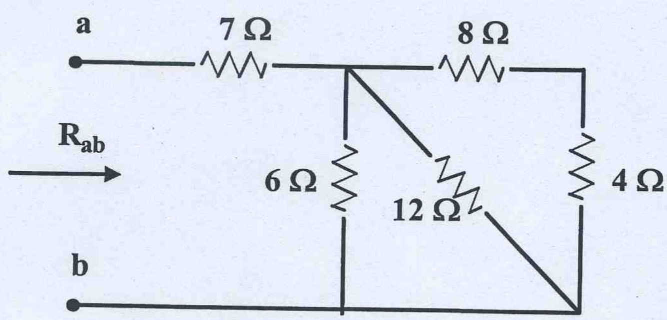
$$I_1 = \frac{(\text{incoming current}) \times (\text{opposite resistor})}{(\text{sum of the two resistors})}$$

For this case;

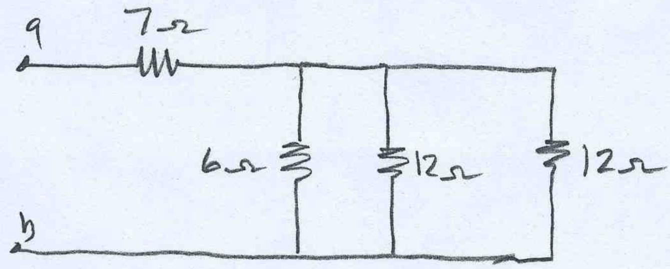
$$I_{30} = \frac{10 \times 20}{20 + 30}$$

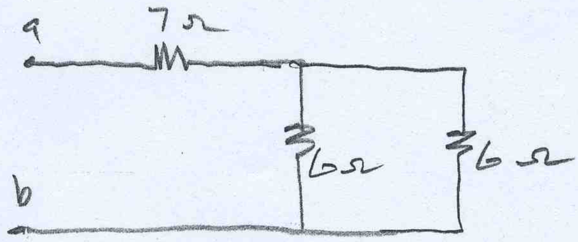
$$\boxed{I_{30} = 4\,A}$$

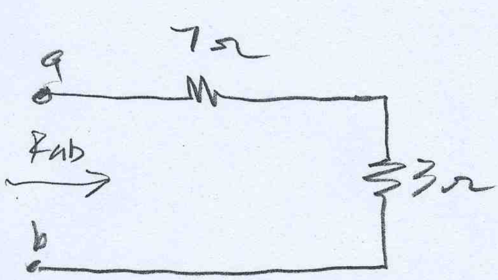
(10) (a) Find R_{ab} for the circuit of Figure 10a. Ans: 10 Ω



we can redraw as follows:

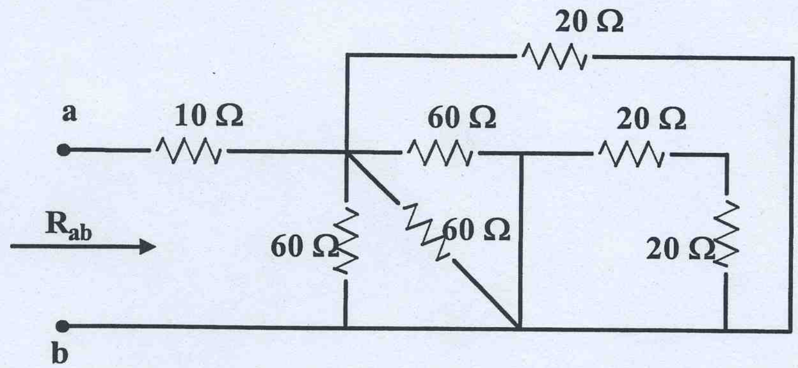




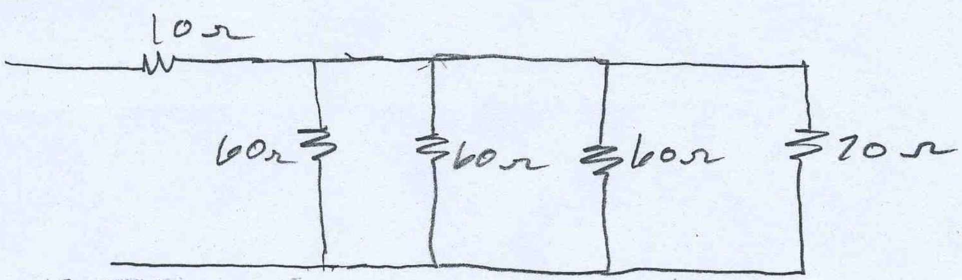


$$\boxed{R_{ab} = 10\,\Omega}$$

10 (b) Find R_{ab} seen looking into terminals a-b for the circuit of Figure 10b. Ans: 20 Ω

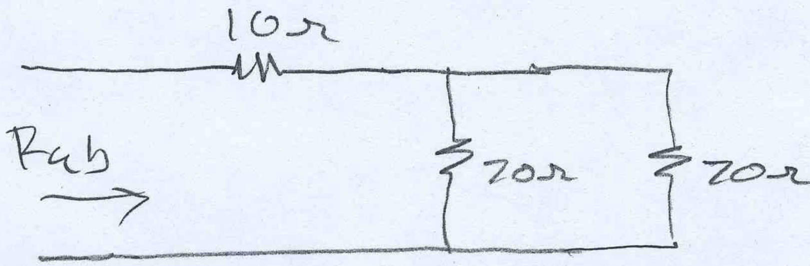


Redraw



(the 2, 20Ω resistors are shorted out)

Redraw

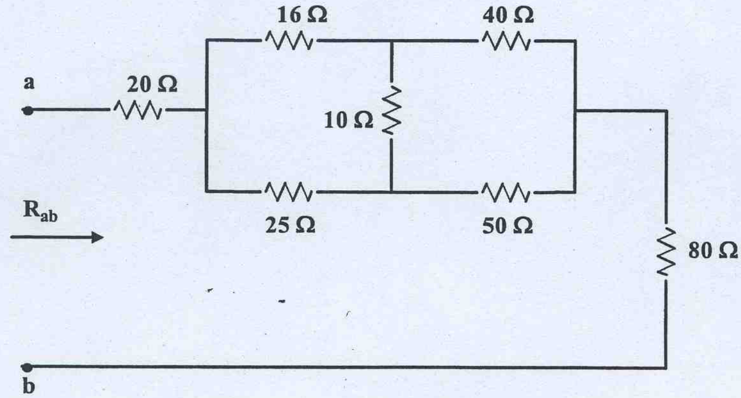


$R_{ab} = 10\,\Omega + 10\,\Omega$

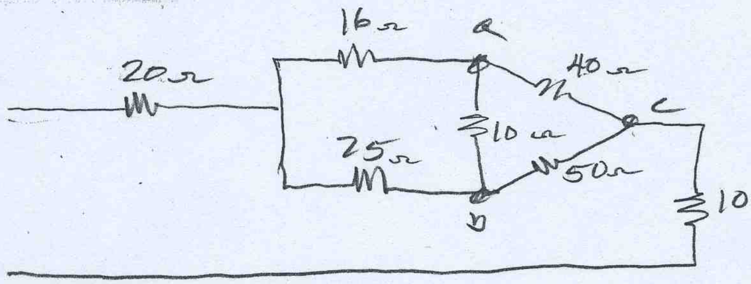
$\boxed{R_{ab} = 20\,\Omega}$

(11) Obtain the equivalent resistance R_{ab} in the circuit of Figure 11. Ans: On your own.

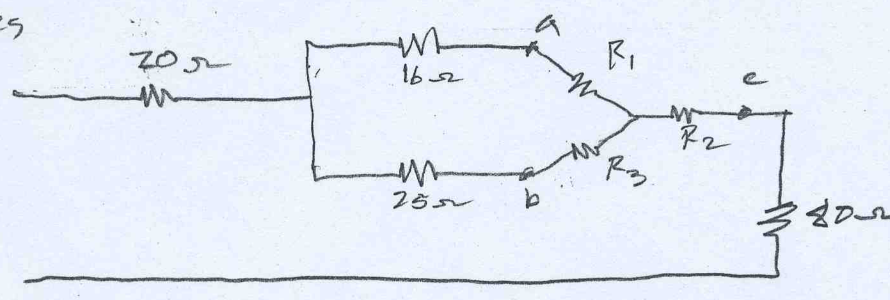
why



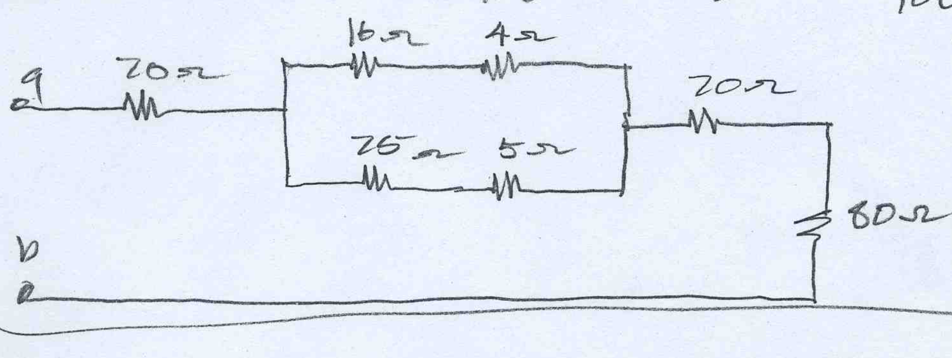
There is a Δ on the front side and a Δ on the back side of the upper resistors. Using the backside (numbers come out even) we have



becomes



$R_1 = \dfrac{40 \times 10}{40+50+10} = 4\,\Omega \quad R_2 = \dfrac{40 \times 50}{100} = 20\,\Omega, \quad R_3 = \dfrac{50 \times 10}{100} = 5\,\Omega$



$$\boxed{R_{ab} = 20 + 12 + 20 + 80 = 132\,\Omega}$$