Desk Copy

ECE 300
Spring Semester, 2006
HW Set #9

Due: March 15, 2006
wlg

Name wlg
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations.** **Be sure to show how you got your answers.** Each problem counts 15 points.

8.4 (a) $i(0^+) = 5$ A, $v(0^+) = 25$ V

(b) $\frac{dv(0^+)}{dt} = 40\ V/s$, $\frac{di(0^+)}{dt} = -160\ A/s$

(c) $i(\infty) = -2.5\,A$, $v(\infty) = 7.5\,V$

√

8.10 $i(t) = [-(10/3)e^{-4t} + (10/3)e^{-t}]\,V$

8.13 R = 120 Ω

8.16 $v(t) = [-9.6te^{-20t}]\,A$

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H.W. #9
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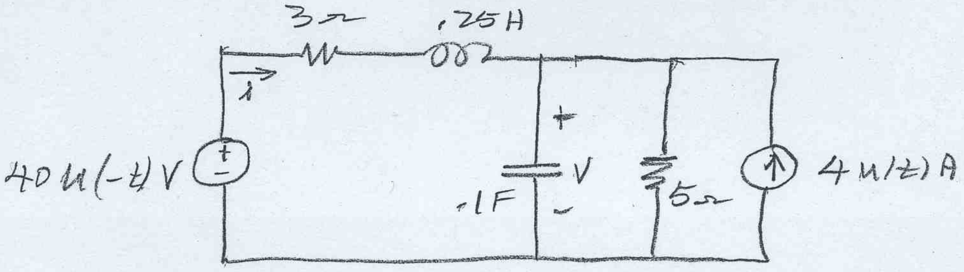
8.4 Consider the circuit below

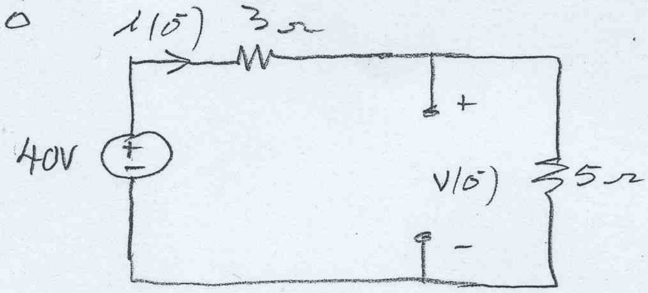


(a) Find $i(0^+)$, $V(0^+)$

(b) $\frac{di(0^+)}{dt}$, $\frac{dV(0^+)}{dt}$

(c) $i(\infty)$, $v(\infty)$

$t<0$



(a)

$i(0^-) = \frac{40}{8} = 5A$ but $i(0^+) = i(0^-)$

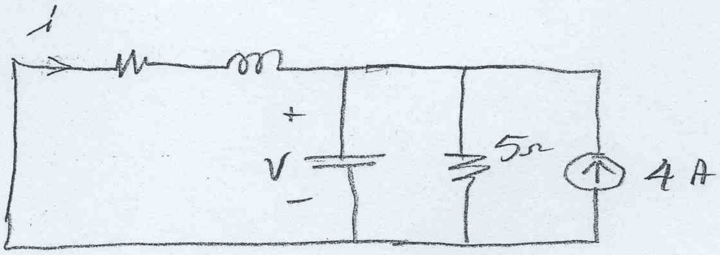
$\boxed{i(0^+) = 5A}$

$V(0^-) = \frac{40 \times 5}{8} = 25 V$ but $V(0^+) = V(0^-)$

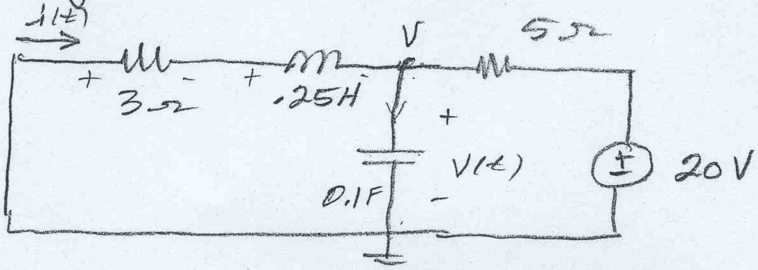
$\boxed{V(0^+) = 25 V}$

8.4 continued

Circuit for $t > 0$



Using a source transformation:



Write a node equation at V:

$$i(t) = .1\frac{dV}{dt} + \frac{V-20}{5}$$

$$\frac{dV(t)}{dt} = 10\,i(t) + 40 - 2V \Big|_{t=0^+}$$

$$\frac{dV(0^+)}{dt} = 10\times 5 + 40 - 2\times 25 = 40$$

$$\boxed{\frac{dV(0^+)}{dt} = 40\ V/s}$$

Also;

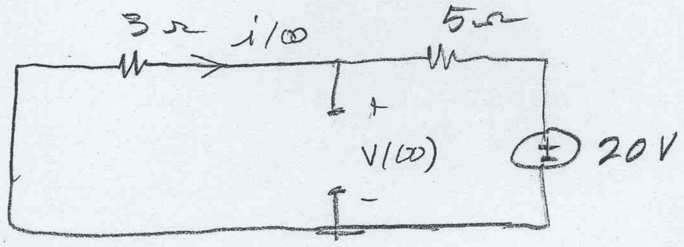
$$3i(t) + .25\frac{di}{dt} + v(t) = 0$$

$$\frac{di(0^+)}{dt} = -4V(0^+) - 12i(0^+) = -100 - 60$$

$$\boxed{\frac{di(0^+)}{dt} = -160\ A/sec}$$

8.4 cont.

In steady state, the circuit is as shown below.



$$v(\infty) = \frac{20 \times 3}{3+5} = 7.5\,V$$

$$i(\infty) = \frac{-20}{3+5} = -2.5\,A$$

$$\boxed{v(\infty) = 7.5\,V}$$

$$\boxed{i(\infty) = -2.5\,A}$$

8.10

The equation describing the voltage in an RLC network is

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 4v = 0$$

$v(0^+) = 10$ V, $\frac{dv(0^+)}{dt} = 0$. Solve for $v(t)$

The characteristic equation is

$$s^2 + 5s + 4 = 0$$

or

$$(s+4)(s+1) = 0$$

Thus

$$v(t) = Ae^{-4t} + Be^{-t}$$

with $v(0^+) = 0$

$$0 = A + B$$

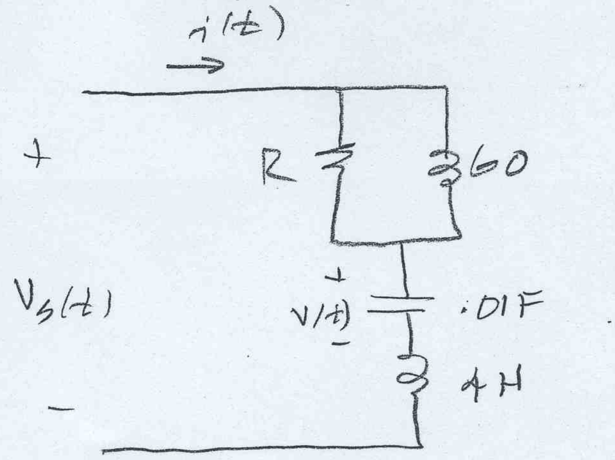
$$\frac{dv}{dt} = -4Ae^{-4t} - Be^{-t}$$

$$\frac{dv(0^+)}{dt} = 10 = -4A - B$$

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \Rightarrow \begin{matrix} A = 3.33 \\ B = -3.33 \end{matrix}$$

$$v(t) = \frac{10}{3}\left[e^{-4t} - e^{-t}\right] \text{ V}$$

8.13 FOR the circuit below, calculate the R that is needed to give critical damping.



$$R_{eq}\, i + 4\frac{di}{dt} + v(t) = V_s(t) \qquad (1)$$

but

$$i(t) = C\frac{dv}{dt} = .01\frac{dv}{dt} \qquad (2)$$

substitute (2) into (1)

$$.01 R_{eq}\frac{dv}{dt} + 4\times.01\times\frac{d^2v}{dt^2} + v(t) = V_s(t)$$

$$\frac{d^2v}{dt^2} + 0.25 R_{eq}\frac{dv}{dt} + 25\, v(t) = \frac{V_s(t)}{.04}$$

Characteristic equation is

$$s^2 + .25 R_{eq}\, s + 25 = 0$$

$$\omega_n^2 = 25 \implies \omega_n = 5$$

$$2\zeta\omega_n = 0.25 R_{eq} \qquad (3)$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

8.13 cont.

From Eq (3), for critical damping, $\zeta = 1$

$$2 \times 1 \times 5 = .25 R_{eq}$$

$$R_{eq} = \frac{10}{.25} = 40\ \Omega$$

but $R_{eq} = \dfrac{60 \times R}{60 + R} = 40$

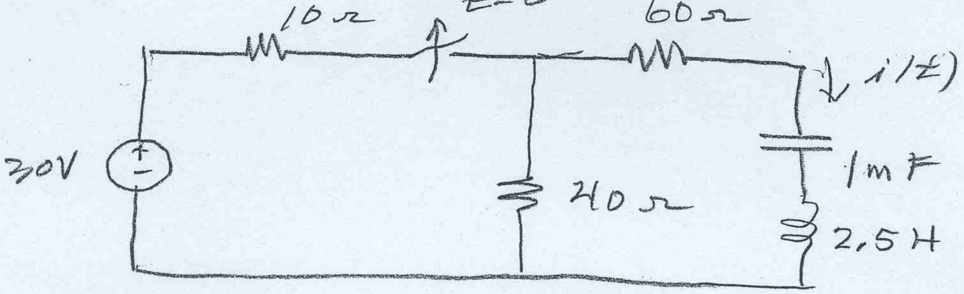
$$60R = 40 \times 60 + 40R$$

$$20R = 2400$$

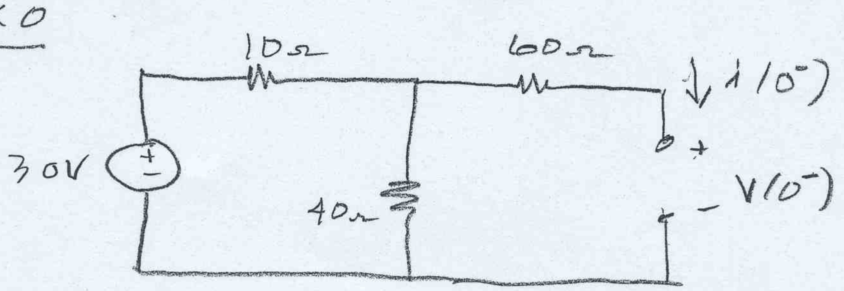
$$\boxed{R = 120\ \Omega}$$

8.16

Find $i(t)$ for $t>0$ in the following circuit.



$t<0$

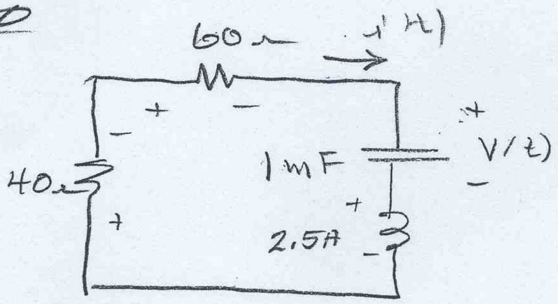


$i(0^-) = 0$ but $i(0^+) = i(0^-) = 0$

$V(0^-) = \frac{30 \times 40}{10 + 40} = 24V$ but $V(0^+) = V(0^-)$

$\boxed{i(0^+) = 0}$; $\boxed{V(0^+) = 24V}$ (1)

$t>0$



$100\, i(t) + V(t) + 2.5 \frac{di}{dt} = 0$ (2)

$\frac{di}{dt} = -0.4V(t) - 25\, i(t)$

$\boxed{\frac{di(0^+)}{dt} = 0.4 \times 24 - 40 \times 0 = -9.6 \text{ A/sec}}$

8.16

From equation (2)

$$100i(t) + \frac{1}{1\times10^{-3}}\int_0^t i(t)dt + 24 + 2.5\frac{di}{dt} = 0$$

Take the derivative

$$100\frac{di}{dt} + 1000i(t) + 2.5\frac{d^2i}{dt^2} = 0$$

20 x 20

400

$$\frac{d^2i}{dt^2} + 40\frac{di}{dt} + 400i(t) = 0$$

$$s^2 + 40s + 400 = 0$$

$$(s+20)(s+20) = 0$$ critically damped

$$i(t) = (A + Bt)e^{-20t}$$

$$i(0) = 0 = A$$

$$\frac{di}{dt} = \left[-20Bte^{-20t} + Be^{-20t}\right]\Big|_{t=0} = -9.6$$

$$B = -9.6$$

$$i(t) = -9.6te^{-20t} \text{ A}$$