For Brad

Desk Copy

ECE 300
Spring Semester, 2006
HW Set #10

Due: March 28, 2006
wlg

Name wlg
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations.** **<u>Be sure to show how you got your answers.</u>** Each problem counts 15 points.

8.24 $i(t) = e^{-5t}[4\cos(19.37t) + 1.03\sin(19.37t)]\ A$

8.27 Do by standard differential equation solution. By this I mean for you to develop the differential equation as a second order differential equation in v(t). Solve the equation.

Ans: $v(t) = [3 - 3e^{-2t}(\cos 2t + \sin 2t)]\ V$

(b) Find the solution using Symbolic MATLAB. Tape-on (attach) your MATLAB program.

(c) Use Simulink of MATLAB for getting a graphical solution. Plot the solution. Use ylabel, xlabel, title in the plot. Show your program and plot-output with your homework.

8.35 $v(t) = [12 - e^{-t}(4\cos 2t + 2\sin 2t)]\ V$

Problem 8.X1

You are given the RLC circuit shown in Figure 8.X1.

(a) Find R so that $\xi = 0.5$.
(b) Develop the differential equation that can be used to solve for v(t). Solve the equation.
(c) Simulate the circuit using Pspice. Obtain a plot for v(t). Attach your Pspice work obtained from the computer.

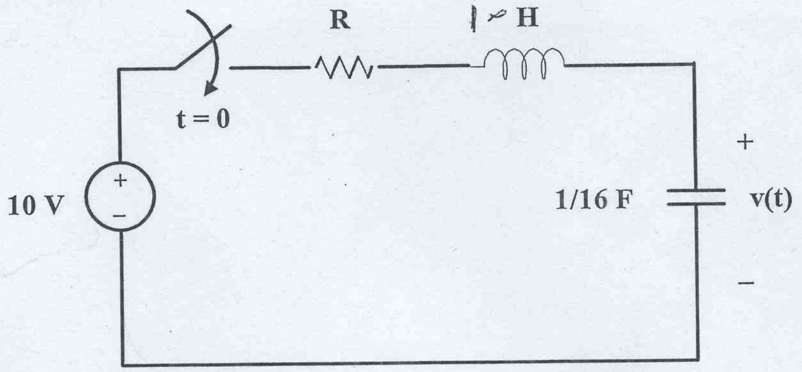
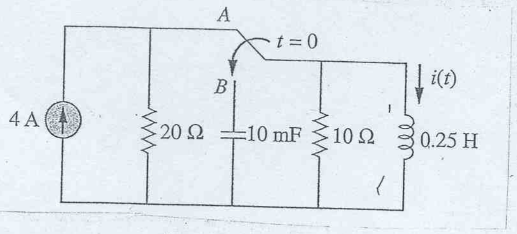


Figure 8.X1: Circuit for problem 8.X1.

8.24

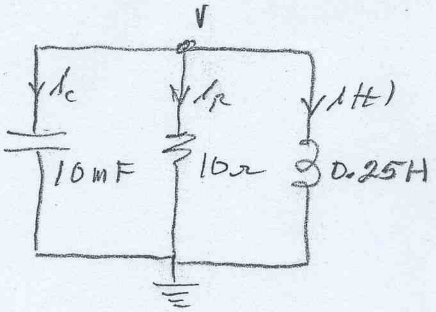


Find $i(t)$ for $t > 0$:

By inspection, for $t < 0$, $i(0^-) = 4\,A$

Since current thru the inductor cannot change instantaneously, $\underline{i(0^+) = i(0^-) = 4\,A}$

The circuit for $t > 0$ is as follows:



$$C\frac{dv}{dt} + \frac{v}{R} + i(t) = 0 \qquad (1)$$

$$\text{but } v(t) = L\frac{di}{dt} \qquad (2)$$

substitute (2) into (1)

$$LC\frac{d^2i}{dt^2} + \frac{L}{R}\frac{di}{dt} + i(t) = 0$$

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i(t)}{LC} = 0 \qquad (3)$$

Put in values

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 400\,i(t) = 0 \qquad (4)$$

8.24 cont.

The characteristic equation is

$$s^2 + 10s + 400 = 0$$

$$(s + 5 + j19.4)(s + 5 - j19.4) = 0$$

$$i(t) = e^{-5t}\left[A\cos 19.4t + B\sin 19.4t\right] \qquad (5)$$

We know $\boxed{i(0^+) = 4A,}$ and $V(0^-) = 0 = V(0^+) = 0$

$\therefore\ V = L\dfrac{di}{dt}$ so $\boxed{\dfrac{di(0^+)}{dt} = 0}$

From (5)

$$\boxed{4 = A}$$

$$\frac{di}{dt} = e^{-5t}\left[-19.4 \times 4\sin 19.4t + 19.4B\cos 19.4t\right]$$

$$\left[4\cos 19.4t + B\sin 19.4t\right]\left[-5e^{-5t}\right]\Big|_{t=0}$$

$$0 = 19.4B - 20$$

$$B = \frac{20}{19.4} = 1.03$$

$$\therefore\ \boxed{i(t) = e^{-5t}\left[4\cos 19.4t + 1.03\sin 19.4t\right]\ A}$$

8.27

A branch voltage for an RLC circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 24 \qquad (A)$$

Given, $v(0) = \frac{dv(0)}{dt} = 0$, find $v(t)$

The characteristic equation is

$$s^2 + 4s + 8 = 0 \qquad \zeta = 0.707$$

4% O.S.

roots are;

$$(s+2+j2)(s+2-j2) = 0 \qquad \text{underdamped}$$

$\therefore$

$$v(t) = e^{-2t}[A\cos 2t + B\sin 2t] + 3$$

$$v(0) = 0 = A + 3 \longrightarrow A = -3$$

$$v(t) = e^{-2t}[-3\cos 2t + B\sin 2t] + 3$$

$$\frac{dv}{dt} = \Big\{e^{-2t}[6\sin 2t + 2B\cos 2t] + [-3\cos 2t + B\sin 2t][-2e^{-2t}]\Big\}\Big|_{t=0} = 0$$

$$0 = 2B + 6 \longrightarrow B = -3$$

$$v(t) = 3 - e^{-2t}[3\cos 2t + 3\sin 2t] \text{ V}$$

8.27 (b)

solution by symbolic method.

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 24 \qquad v(0) = \dot{v}(0) = 0$$

dsolve('D2v + 4*Dv + 8*v = 24', 'v(0) = 0',
'Dv(0) = 0')

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>>
>>
>> dsolve('D2v + 4*Dv + 8*v = 24',  'v(0) = 0', 'Dv(0) = 0')

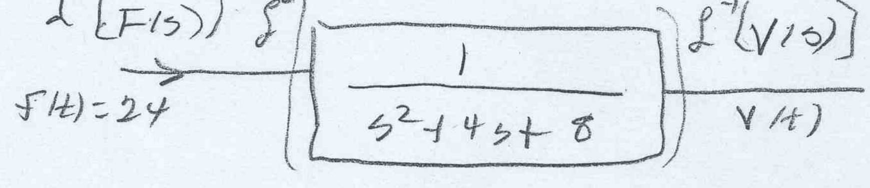
ans =

3-3*exp(-2*t)*sin(2*t)-3*exp(-2*t)*cos(2*t)

>>
```

(c) Do (solve) using simulink. Take the Laplace of Equation A with $24 \to f(t)$. We get

$$(s^2 + 4s + 8)V(s) = F(s)$$



The simulink diagram is shown on the following page

```
>> plot(t,v)
>> grid
>> ylabel('Voltage (volts)')
>> xlabel('time (sec)')
>> title('Step Response for problem 8.27 Alexander')
>>
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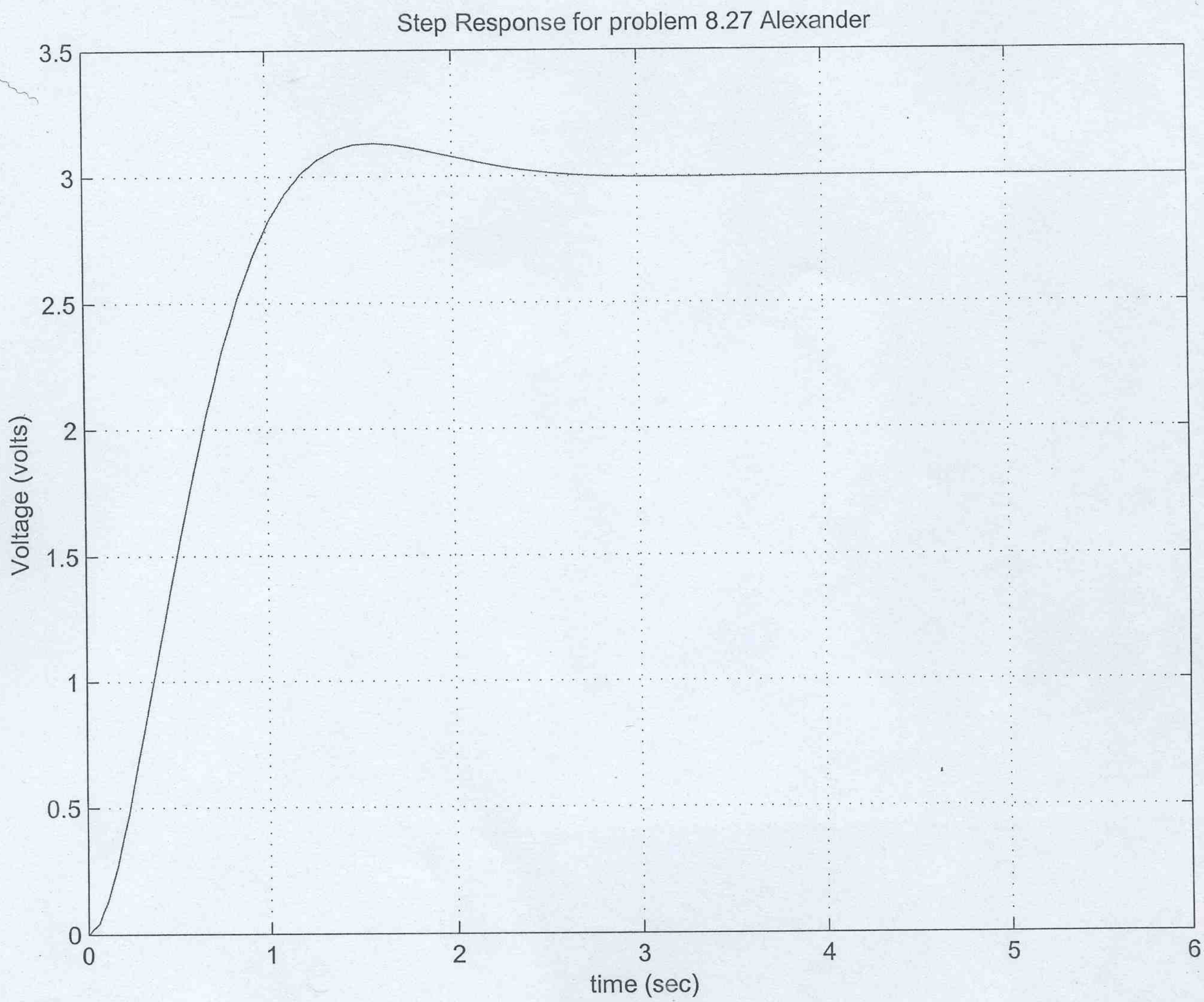
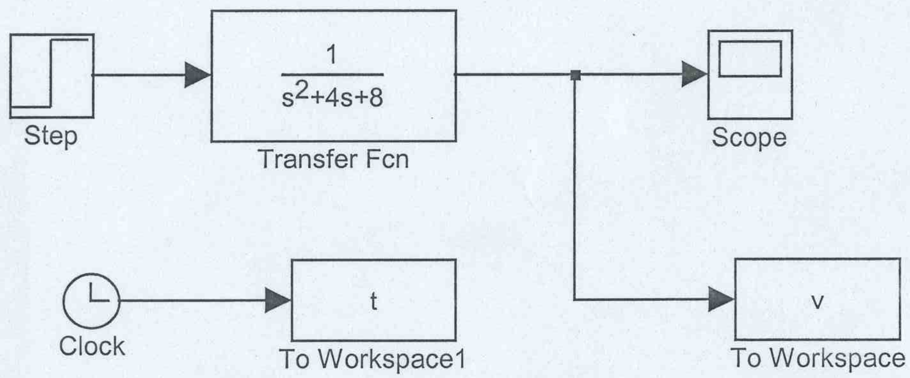
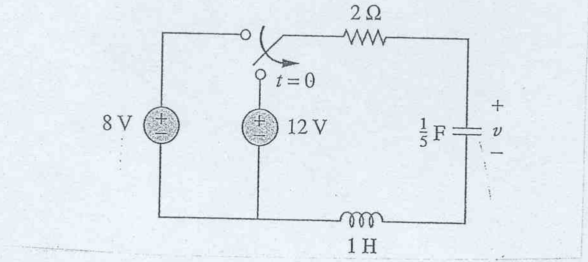




Configuration for Problem 8.27 Alexander text: Spring, 2006: wlg

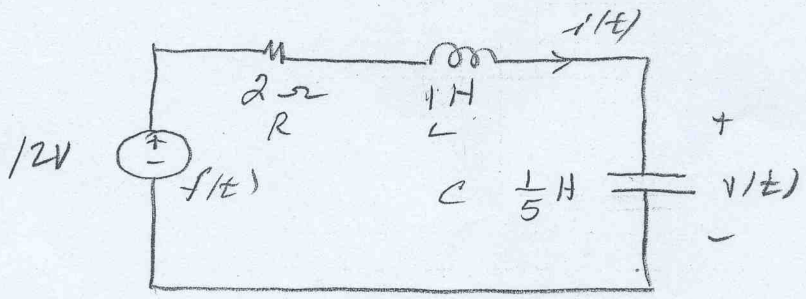
8.35



Find $v(t)$ for $t > 0$.

For $t < 0$, $v(0^-) = \boxed{8\text{ V} = v(0^+)}$

For $t > 0$, analyze the following ckt.



$$Ri + L\frac{di}{dt} + v(t) = f(t)$$

OR

$$2i(t) + L\frac{di}{dt} + v(t) = 12 \qquad (1)$$

$i(0^-) = 0$: current cannot change inst. because of the inductor.

$i(0^+) = i(0^-) = 0$

$$i(t) = C\frac{dv}{dt}$$

$$\boxed{\frac{dv(0^+)}{dt} = \frac{i(0^+)}{C} = 0} \qquad \boxed{v(0^+) = 8\text{ V}} \qquad (2)$$

The above are the initial conditions we will need.

8.35

substitute $i = C\frac{dv}{dt}$ into (1)

$$2 \times C\frac{dv}{dt} + C\frac{d^2v}{dt} + v(t) = 6$$

$$\frac{d^2v}{dt^2} + \frac{2dv}{dt} + 5v(t) = 60 \qquad (3)$$

Char. Eq. is:

$$s^2 + 2s + 5 = 0$$

$$(s+1+j2)(s+1-j2) = 0$$

$$v(t) = 12 + e^{-t}[A\cos 2t + B\sin 2t] \text{ V} \qquad (4)$$

From (2),

$$8 = 12 + A \longrightarrow A = -4.$$

$$v(t) = 12 + e^{-t}[-4\cos 2t + B\sin 2t] \qquad (5)$$

$$\frac{dv}{dt} = 0 = \left(e^{-t}[+8\sin 2t + 2B\cos 2t] + [-4\cos 2t + B\sin 2t][-1e^{-4t}]\right)\Big|_{t=0} = 0$$

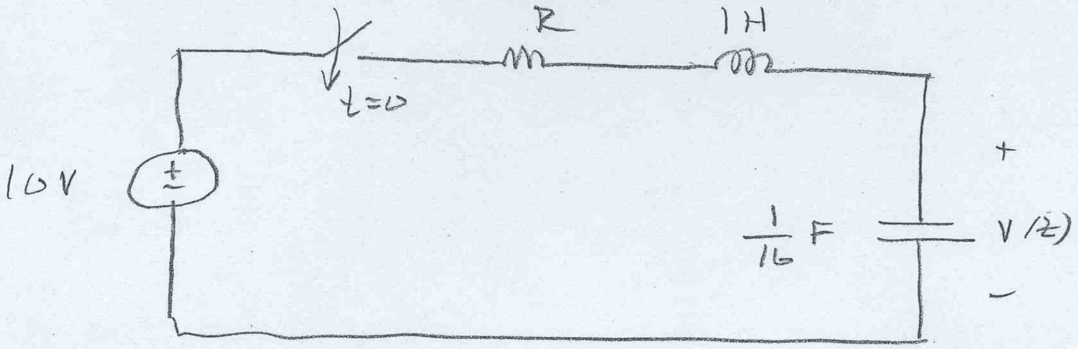
$$0 = 2B + 4$$

$$B = -2$$

$$\boxed{v(t) = 12 + e^{-t}[4\cos 2t + 2\sin 2t] \text{ V}}$$

Problem 8.X1

Given the following circuit.



1a) Find R so that $\xi = 0.5$.

$$Ri + L\frac{di}{dt} + v(t) = 10 \qquad i = C\frac{dv}{dt} \qquad (1)$$

$$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v(t) = 10 \qquad (2)$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v(t)}{LC} = \frac{10}{LC} \qquad (3)$$

with numbers;

$$\frac{d^2v}{dt^2} + R\frac{dv}{dt} + 16v = 160 \qquad (4)$$

$$s^2 + Rs + 16 = 0 \qquad (5)$$

compare with

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \qquad (6)$$

$$\omega_n^2 = 16 \longrightarrow \omega_n = 4 \qquad (7)$$

8.X1) with $\xi = 0.5$, $w_n = 4$

$$s^2 + 4s + 16 = 0 \qquad (8)$$

compare with

$$s^2 + Rs + 16 = 0 \qquad (9)$$

$$\therefore \boxed{R = 4} \qquad (10)$$

(b) The differential equation is given in (4)

$$\boxed{\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 16v(t) = 160}$$

$$s^2 + 4s + 16 = 0$$

$$(s+2+j3.46)(s+2-j3.46) = 0$$

$$v(t) = 10 + e^{-2t}[A\cos 3.46t + B\sin 3.46t)$$

$v(0) = 0$, $i = C\frac{dv(0)}{dt}$;

but $i(0^+) = 0$ so $\frac{dv(0^+)}{dt} = 0$

$$0 = 10 + A \longrightarrow A = -10$$

$$v(t) = 10 + e^{-2t}[-10\cos 3.46t + B\sin 3.46t]$$

$$\frac{dv}{dt} = e^{-2t}[+10\times 3.46\sin 3.46t + 3.46B\cos 3.46t] + [-10\cos 3.46t + B\sin 3.46t][-2e^{-2t}\Big|_{t=0}$$

$$0 = 3.46B + 20$$

$$B = -5.78$$

$$v(t) = 10 - e^{-2t}\left[10\cos 3.46t + 5.78\sin 3.46t\right] \quad (8)$$

Check by Symbolic

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Using Toolbox Path Cache.  Type "help toolbox_path_cache" for more info.

To get started, select "MATLAB Help" from the Help menu.

>> dsolve('D2v + 4*Dv + 16*v = 160', 'v(0) = 0', 'Dv(0) = 0')

ans =

10-10/3*3^(1/2)*exp(-2*t)*sin(2*3^(1/2)*t)-10*exp(-2*t)*cos(2*3^(1/2)*t)

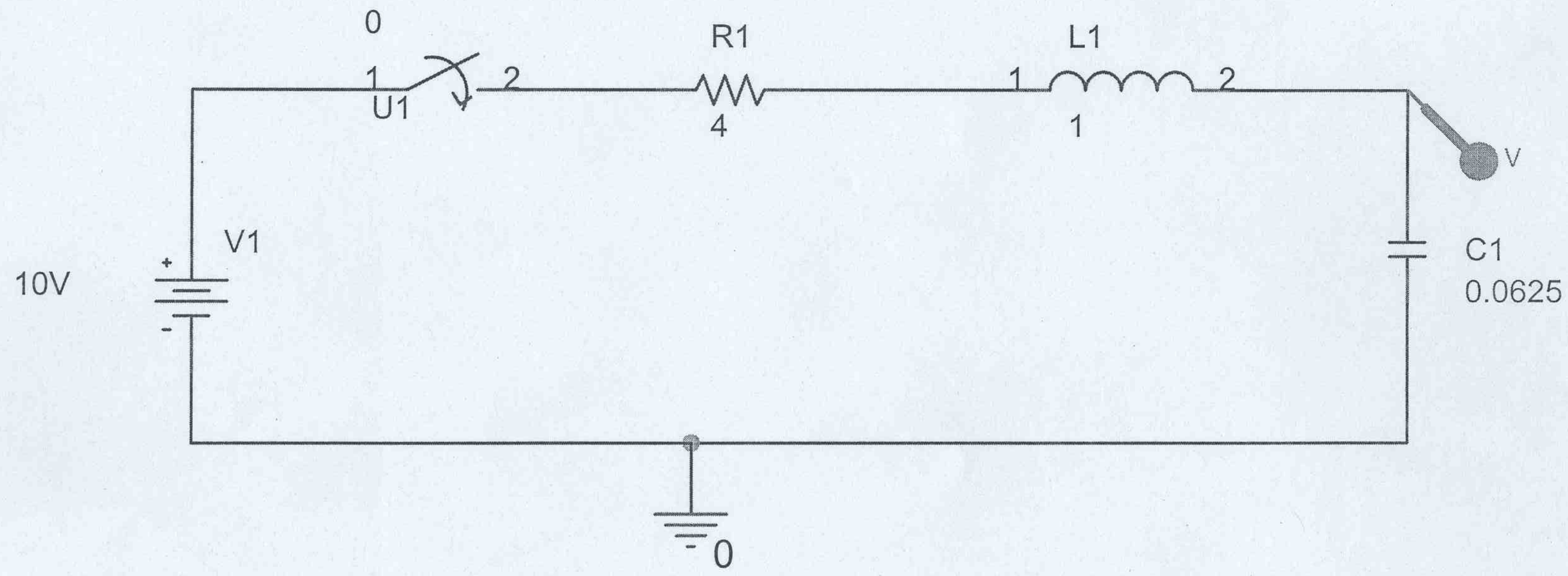
>>
```

This reduces to

$$v(t) = \left[10 - 5.78e^{-2t}\sin 3.46t - 10e^{-2t}\cos 3.46t\right] V$$

which checks with (8) above.

(c) The P-Spice solution is given on the following pages.



0
1
U1
2
R1
4
L1
1
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C1
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** Profile: "SCHEMATIC1-case2" [C:\MY DOCUMENTS\RLC_transient-PSpiceFiles\SCHEMATIC1\case2.sim]
Date/Time run: 03/27/06 Temperature: 27.0