8.24 \[ i(t) = e^{-st}[4\cos(19.37t) + 1.03\sin(19.37t)] \, A \]

8.27 Do by standard differential equation solution. By this I mean for you to develop the differential equation as a second order differential equation in \( v(t) \). Solve the equation.

Ans: \[ v(t) = [3 - 3e^{-2t}(\cos 2t + \sin 2t)] \, V \]

(b) Find the solution using Symbolic MATLAB. Tape-on (attach) your MATLAB program.

(c) Use Simulink of MATLAB for getting a graphical solution. Plot the solution. Use ylabel, xlabel, title in the plot. Show your program and plot-output with your homework.

8.35 \[ v(t) = [12 - e^{-t}(4\cos 2t + 2\sin 2t)] \, V \]

Problem 8.X1

You are given the RLC circuit shown in Figure 8.X1.

(a) Find \( R \) so that \( \xi = 0.5 \).
(b) Develop the differential equation that can be used to solve for \( v(t) \). Solve the equation.
(c) Simulate the circuit using Pspice. Obtain a plot for \( v(t) \). Attach your Pspice work obtained from the computer.

![Figure 8.X1: Circuit for problem 8.X1.](image)
Find \( i(t) \) for \( t > 0 \):

By inspection, for \( t < 0 \), \( i(0^-) = 4 \, \text{A} \)

Since current thru the inductor cannot change instantaneously, \( i'(0^+) = i'(0^-) = 4 \, \text{A} \)

The circuit for \( t > 0 \) is as follows:

\[
\frac{C}{R} \frac{dv}{dt} + \frac{v}{R} + i'(t) = 0 \tag{1}
\]

but \( v(t) = L \frac{di}{dt} \) \tag{2}

Substitute (2) into (1)

\[
\frac{L}{C} \frac{d^2i}{dt^2} + \frac{1}{R} \frac{di}{dt} + i'(t) = 0
\]

\[
\frac{d^2i}{dt^2} + \frac{1}{RC} \frac{di}{dt} + \frac{1}{LC} i(t) = 0 \tag{3}
\]

Put in \( v(t) = 0 \)

\[
\frac{d^2i}{dt^2} + 10 \frac{di}{dt} + 400 i(t) = 0 \tag{4}
\]
The characteristic equation is

\[ s^2 + 10s + 400 = 0 \]

\[((s+5+j19.4)(s+5-j19.4)) = 0\]

\[ i(t) = e^{-5t} \begin{bmatrix} A \cos 19.4t + B \sin 19.4t \end{bmatrix} \]  \hspace{1cm} (5)

We know \( v(100) = 4A \), and \( v(0) = 0 = v(0^+) = 0 \)

\[ v = \frac{dv}{dt} \hspace{1cm} \Rightarrow \frac{dv}{dt} = 0 \]

From (5)

\[ H = A \]

\[ e_{i(t)} = e^{-5t} \begin{bmatrix} -19.4x4.4\sin19.4t + 19.4B \cos 19.4t \\
4 \cos 19.4t + B \sin 19.4t \end{bmatrix} \begin{bmatrix} -5e^{-5t} \\ 0 \end{bmatrix} \]

\[ B = \frac{20}{19.4} = 1.03 \]

\[ i(t) = e^{-5t} \begin{bmatrix} 4 \cos 19.4t + 1.03 \sin 19.4t \end{bmatrix} A \]
A branch voltage for an RLC circuit is described by
\[
\frac{d^2 v}{dt^2} + 4 \frac{dv}{dt} + 6v = 24
\]  
(1)

Given \( v(0) = 0 \), \( \frac{dv(0)}{dt} = 0 \), find \( v(t) \)

The characteristic equation is
\[
5^2 + 4 \cdot 3 + 8 = 0 \quad \gamma = 0.707
\]
roots are:
\[
(5 + 2\sqrt{2})(5 - 2\sqrt{2}) = 0 \quad \text{underdamped}
\]

\[
v(t) = e^{-2t} \left[ A \cos 2t + B \sin 2t \right] + 3
\]
\[
v(0) = 0 = A + 3 \quad \rightarrow A = -3
\]
\[
v(t) = e^{-2t} \left[ -3 \cos 2t + B \sin 2t \right] + 3
\]
\[
\frac{dv}{dt} = \left. e^{-2t} \left[ -6 \sin 2t + 2B \cos 2t \right] + 3 \right|_{t=0} = 0
\]
\[
0 = 2B + 6 \quad \rightarrow B = -3
\]
\[
v(t) = 3 - e^{-2t} \left[ 3 \cos 2t + 3 \sin 2t \right] \quad V
\]
(b) Solution by symbolic method.

\[
\frac{d^2 v}{dt^2} + 4 \frac{dv}{dt} + 8v = 24 \quad v(0) = i(0) = 0
\]

\[
dsolve('D2v + 4*Dv + 8*v = 24', 'v(0)=0', 'Dv(0)=0')
\]

\[
ans =
3 - 3*exp(-2*t)*sin(2*t) - 3*exp(-2*t)*cos(2*t)
\]

(c) Do (solve) using Simulink. Take the Laplace of Equation A with \( s^2 \to sH \). We get

\[
(s^2 + 4s + 8)v(s) = F(s)
\]

The Simulink diagram is shown on the following page.
>> plot(t,v)
>> grid
>> ylabel('Voltage (volts)')
>> xlabel('time (sec)')
>> title('Step Response for problem 8.27 Alexander')
>>
Configuration for Problem 8.27 Alexander text: Spring, 2006; wlg
Find $V(t)$ for $t > 0$.

For $t < 0$, $V(0^-) = 8 \text{ V} = V(0^+)$

For $t > 0$, analyze the following circuit:

\[ L i + R i + \frac{1}{C} \int i(t) \, dt + V(t) = f(t) \]
\[ 2i(t) + L \frac{di}{dt} + \frac{1}{C} \int i(t) \, dt + V(t) = 12 \]  \hspace{1cm} (1)

\[ i(10^-) = 0 \text{; current cannot change indefinitely because of the inductor} \]

\[ i(10^+) = i(10^-) = 0 \]

\[ i(t) = C \frac{dV}{dt} \]

\[ \frac{dV(10^+)}{dt} = \frac{i(10^+)}{C} = 0 \]

\[ V(10^+) = 8 \text{ V} \]  \hspace{1cm} (2)

The above are the initial conditions we will need.
$S_{35}$  

Substitute $t=1/r$ into (1) 

$$2x\frac{CV}{\partial t} + C\frac{C^2V}{\partial t} + 4C(t) = 0$$

$$\frac{\partial^2 V}{\partial t^2} + \frac{2\partial V}{\partial t} + 5V(t) = 0$$  \hspace{1cm} (3)

Chap, Eq. 153

$$5^2 + 25 + 5 = 0$$

$$L(5t^2 + 12)(5t - 5) = 0$$

$$V(t) = 12 + e^{-\frac{t}{5}}\left[A \cos 2t + B \sin 2t\right]$$  \hspace{1cm} (4)

From (3),

$$8 = 12 + A \rightarrow A = -4$$

$$V(0) = 12 + e^{-\frac{0}{5}}\left[-4 \cos 2t + B \sin 2t\right]$$  \hspace{1cm} (5)

$$\frac{\partial V}{\partial t} = 0 = \left[e^{-\frac{t}{5}}\left[4 \sin 2t + 2B \cos 2t\right] + L\left[-4 \cos 2t + B \sin 2t\right]\left[-1 e^{-\frac{0}{5}}\right]\right] = 0$$

$$t = 0$$

$$0 = 2B + 4$$

$$B = -2$$

$$V(t) = 12 + e^{-\frac{t}{5}}\left[4 \cos 2t + 2 \sin 2t\right]$$
Problem 8.1

Given the following circuit.

\[ V(t) = 160 \]

1a) Find \( R \) so that \( \phi = 0.5 \).

\[
R + L \frac{dI}{dt} + V(t) = 100
\]

\[
I = \frac{dV}{dt}
\]  

(1)

\[
R \frac{dV}{dt} + L \frac{d^2V}{dt^2} + V(t) = 10
\]

(2)

\[
\frac{d^2V}{dt^2} + \frac{R}{L} \frac{dV}{dt} + \frac{V(t)}{L} = \frac{10}{L}
\]

with numbers:

\[
\frac{d^2V}{dt^2} + \frac{R}{L} \frac{dV}{dt} + 16V = 160
\]  

(4)

\[
s^2 + Rs + 16 = 0
\]  

(5)

compare with

\[
s^2 + 2\bar{\omega}_m s + \omega_m^2 = 0
\]  

(6)

\[
\omega_m^2 = 16 \quad \Rightarrow \quad \omega_m = 4
\]  

(7)
(3.1)

with \( s = 0.5 \), \( w_n = 4 \)

\[ s^2 + 4s + 16 = 0 \]  \hspace{1cm} (10)

compare with

\[ s^2 + rs + 16 = 0 \]  \hspace{1cm} (9)

\[ r = 4 \]  \hspace{1cm} (12)

(b) The differential equation is given

\[ \frac{d^2 v}{dt^2} + 4 \frac{dv}{dt} + 16v = 160 \]

\[ s^2 + 4s + 16 = 0 \]

\[ (s + 2 + j3.46)(s + 2 - j3.46) = 0 \]

\[ v(t) = 10 + e^{-2t} [A \cos 3.46t + B \sin 3.46t] \]

\[ v(0) = 0 \quad \Rightarrow \quad C \frac{dv}{dt}(0) = 0 \]

\[ 0 = 10 + A \quad \Rightarrow \quad A = -10 \]

\[ v(t) = 10 + e^{-2t} \left[ -10 \cos 3.46t + B \sin 3.46t \right] \]

\[ \frac{dv}{dt} = e^{-2t} \left[ +10 \times 3.46 \sin 3.46t + 3.46B \cos 3.46t \right] \]

\[ + \left[ -10 \cos 3.46t + B \sin 3.46t \right] \left[ -2e^{-2t} \right] \]

\[ 0 = 3.46B + 20 \]

\[ B = -5.78 \]
\[ V(t) = 10 - e^{-2t} \left[ 10 \cos 3.46t + 5.78 \sin 3.46t \right] \]

Check by Symbolic (6)

3/27/06 12:54 PM
MATLAB Command Window

Using Toolbox Path Cache. Type "help toolbox_path_cache" for more info.

To get started, select "MATLAB Help" from the Help menu.

>> dsolve('D2v + 4*Dv + 16*v = 160', 'v(0) = 0', 'Dv(0) = 0')
ans =
10 -10/3*3^1/2*exp(-2*t)*sin(2*3^1/2*t) -10*exp(-2*t)*cos(2*3^1/2*t)

>>

This reduces to
\[ V(t) = \left[ 10 - 5.78 e^{-2t} \sin 3.46t - 10 e^{-2t} \cos 3.46t \right] V \]

Which checks with (6) above.

(c) The PSpice solution is given on the following pages.