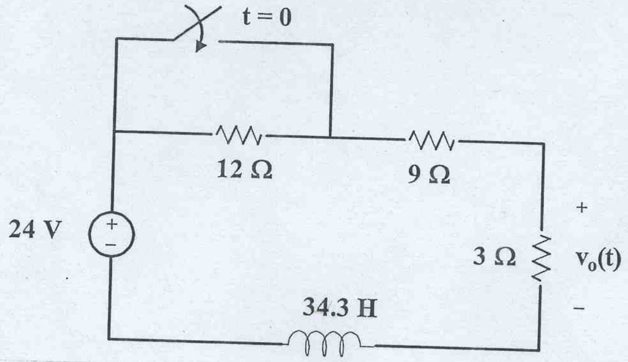
ECE 300
Test #3 A
SPRING 2006

(1) The circuit shown in Figure 1 is at steady state before the switch closes at time t = 0. **Determine** the output voltage, $v_o(t)$, for $t \geq 0$.



Recall:

$$V_o(t) = V_o(\infty) + [V_o(0^+) - V_o(\infty)]\, e^{-\frac{t}{\tau}}$$

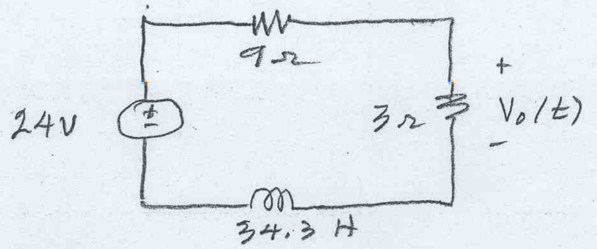
For $t < 0$

$$V_o(0^-) = \frac{24 \times 3}{12 + 9 + 3} = 3\,V$$

Since the in an inductor in this series circuit, $i(0^+) = i(0^-)$. Therefore, $V_o(0^+) = V_o(0^-)$

$$\boxed{V_o(0^+) = 3\,V}$$

For $t > 0$



By inspection, $V_o(\infty) = 6\,V$

$$\tau = \frac{L}{R_{eq}} = \frac{34.3}{12} = 2.86\,s; \quad \frac{1}{\tau} = 0.35\,s^{-1}$$

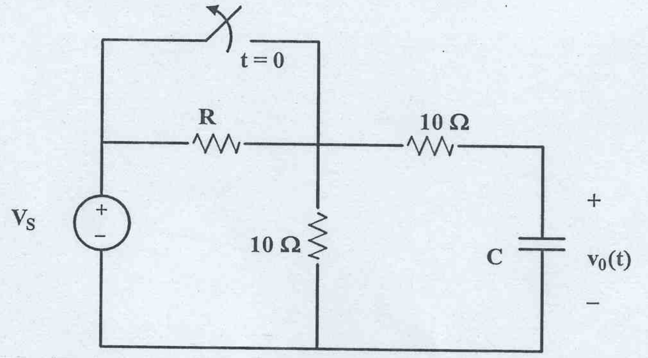
$$\boxed{v_o(t) = [6 - 3e^{-0.35t}]\,V}$$

Test 3A

(2) The circuit shown in Figure 2 in at steady state before the switch opens at t = 0. V_S is a constant dc voltage source. The output of the circuit is the voltage across the capacitor, $v_o(t)$. This output voltage is given by;

$$v_o(t) = 2 + 8e^{-0.5t}\ V;\quad t \geq 0$$

Required: Determine (a) the value of the input voltage, V_S; (b) the value of the capacitor, C; (c) the value of the resistor, R.



FOR $t < 0$

$V_o(0^-) = V_s$

FOR $t > 0$

$V_o(0^+) = V_o(0^-) = V_s = 2 + 8e^{-0.5t}\Big|_{t=0} = 10\,V$

$\boxed{V_s = 10\,V}$

$V_o(\infty) = 2 + 8e^{-.5t}\Big|_{t=\infty} = 2\,V$

$V_o(\infty) = \dfrac{V_s \times 10}{R + 10} = \dfrac{10 \times 10}{R + 10} = 2$

$100 = 2R + 20$

$\boxed{R = 40\,\Omega}$

$\tau = R_{eq}\,C = 2$

(2) Test 3B

$R_{eq} = 10 + 40 \| 10 = 18\,\Omega$

$R_{eq} C = 2$

$\boxed{C = \frac{2}{18} = \frac{1}{9}\ F}$

So;

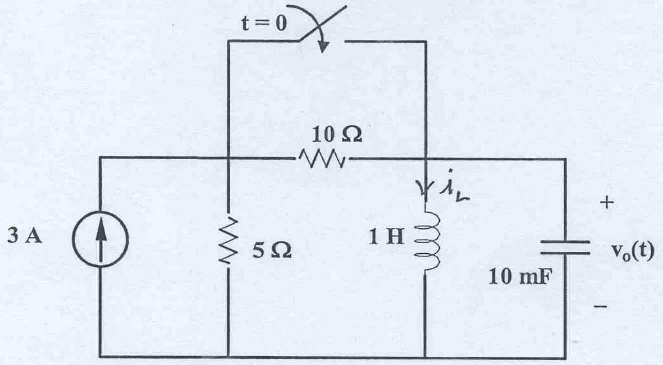
$V_s = 10\ V$

$R = 40\,\Omega$

$C = \frac{1}{9}\ F$

(3) T3A & T3B

(3). You are given the circuit of Figure 3. **Determine** the expression for the voltage $v_o(t)$.

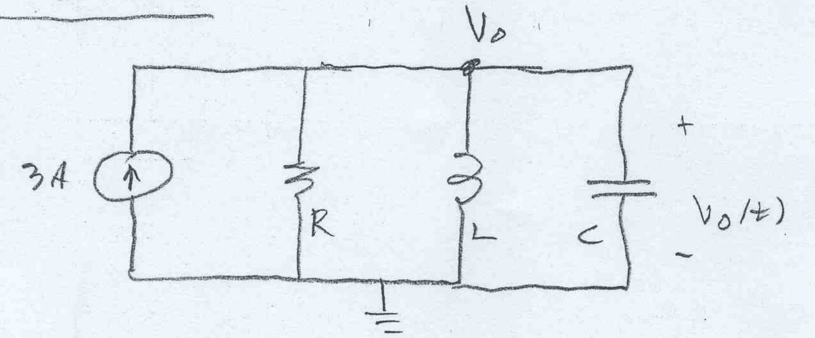


FOR $t < 0$

$$i_L(0^+) = i_L(0^-) = \frac{3 \times 5}{10+5} = 1A$$

$$V_o(0^+) = V_o(0^-) = 0$$

FOR $t > 0$



Parallel RLC ckt

Using nodal:

$I.C. = i_L(0^+)$

$$\frac{V_o}{R} + C\frac{dV_o}{dt} + \frac{1}{L}\int_0^t V_o\,dt + 1 = 3 \qquad (1)$$

Will need $\frac{dV_o(0^+)}{dt}$ in solving the D.E.

Go ahead and get it now from (1)

(3)

$$C\frac{dV(0^+)}{dt} = 3 - 1 - \frac{\cancelto{0}{V_o(0^+)}}{R}$$

$$\frac{dV(0^+)}{dt} = \frac{2}{C} = \frac{2}{10\times10^{-3}} = 200\ V/s$$

so;

$$V_c(0^+) = 0\ , \quad \frac{dV(0^+)}{dt} = 200\ V/s \qquad (2)$$

Go to (1) and take $\frac{d(\cdot)}{dt}$, this gives

$$\frac{1}{R}\frac{dV_o}{dt} + C\frac{d^2V_o}{dt^2} + \frac{V_o}{L} = 0$$

$$\frac{d^2V_o}{dt^2} + \frac{1}{RC}\frac{dV_o}{dt} + \frac{V_o}{LC} = 0$$

Put in values;

$$\frac{d^2V_o}{dt^2} + 20\frac{dV_o}{dt} + 100V_o = 0$$

$$s^2 + 20s + 100 = 0$$

$$(s+10)(s+10) = 0$$

$$V_o = (A + Bt)e^{-10t} \qquad (3)$$

$$V_o(0) = A = 0$$

$$V_o(t) = Bte^{-10t}$$

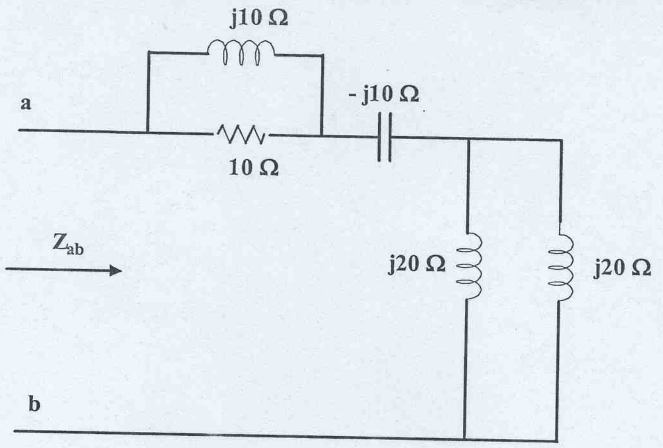
$$\left.\frac{dV_o}{dt}\right|_{t=0} = 200 = \left[-10Bte^{-10t} + Be^{-10t}\right]\Big|_{t=0}$$

$$B = 200$$

$$\boxed{V_o(t) = 200te^{-10t}\ V}$$ Ans

Test 3A

(4) You are given the circuit shown in Figure 4. **Determine** the input impedance, Z_{ab},



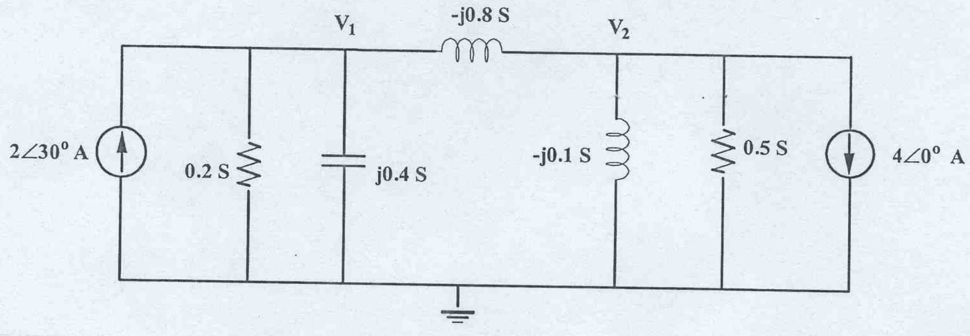
$$j20 \parallel j20 = \frac{(j20)(j20)}{j20 + j20} = j10 \,\Omega$$

$j10\,\Omega$ in series with $-j10\,\Omega = 0$

so $$Z_{ab} = \frac{10\,(j10)}{10 + j10}$$

$$\boxed{Z_{ab} = 5 + j5 = 7.07\angle 45^\circ \,\Omega}$$

(5) You are given the circuit of Figure 5. **Solve** for the phasor voltages V_1 and V_2. Express your answers in polar form. **Note:** The unit for the circuit parameters is given in Seiman (S).



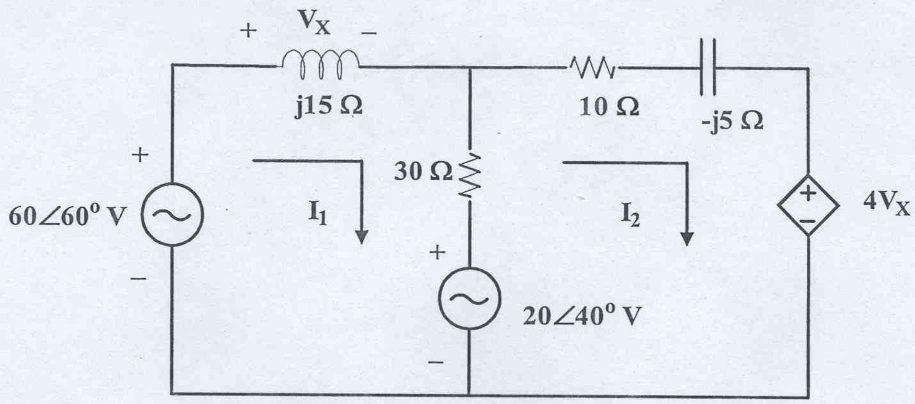
By inspection

$$\begin{bmatrix} (0.2-j0.4), & j0.8 \\ j0.8, & (0.5-j0.9) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2\angle 30 \\ -4\angle 0 \end{bmatrix}$$

$V_1 = 5.16 \angle 95.45^\circ$ V

$V_2 = 0.4 \angle 135^\circ$ V

(6) **Solve** for the phasor currents I_1 and I_1 indicated in the circuit of Figure 6. Express your answers in polar form.



$$(30+j15)\hat{I}_1 - 30\hat{I}_2 = 60\angle 60 - 20\angle 40$$

$$-30\hat{I}_1 + (40-j5)\hat{I}_2 = 20\angle 40 - 4V_x$$

$$= 20\angle 40 - 4(j15)I_1$$

$$(-30+j60)I_1 + (40-j5)I_2 = 20\angle 40$$

$$\begin{bmatrix} (30+j15, & -30 \\ (-30+j60), & (40-j5) \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 60\angle 60 - 20\angle 40 \\ 20\angle 40 \end{bmatrix}$$

$I_1 = 0.99 \angle -24.04° \; A$

$I_2 = 1.40 \angle -63.98° \; A$