(1) The circuit shown in Figure 1 is in steady state before the switch opens at \( t = 0 \). The input is 10 V, constant. The output is the voltage across the capacitor, \( v_o(t) \).

**Required:** Determine the expression for the output voltage \( v_o(t) \) for \( t \geq 0 \).

\[ v_o(0^-) = V_s = 10 \text{ V} \]

For \( t \geq 0 \)

Since voltage across the capacitor cannot change instantaneously, \( v_o(t) \) at \( t = 0^+ \) is the same as at \( t = 0^- \):

\[ v_o(0^+) = v_b(0^-) = 10 \text{ V} \]

\[ v_o(\infty) = \frac{V_s \times 10}{40 + 10} = \frac{10 \times 10}{50} = 2 \text{ V} \]

\[ R_{eq} = 10 + \frac{40 \times 10}{40 + 10} = 18 \Omega \]

\[ T = \frac{R_{eq}C}{9} = 2 \]

\[ v_o(t) = v_o(\infty) + \left[ v_b(0^+) - v_o(\infty) \right] e^{-\frac{t}{T}} \]

\[ v_b(t) = 2 + 8 e^{-0.5t} \text{ V} \]
(2) The circuit shown in Figure 2 is in steady state before the switch closes at \( t = 0 \). The input to the circuit is a constant dc voltage of 24 V. The output, \( v_o(t) \), is the voltage across the 3 \( \Omega \) resistor and is given by

\[
v_o(t) = 6 - 3e^{-0.35t} \ V; \quad t \geq 0
\]

**Determine** the (a) value of the inductor \( L \), (b) the values of the resistor \( R_1 \) and \( R_2 \).

Look at \( v_o(0^+) \)

\[
v_o(\infty) = 6 - 3e^{-0.35 \times \infty} = 6 \ V
\]

From the circuit,

\[
v_o(\infty) = \frac{24 \times 3}{R_2 + 3} = 6
\]

\[
R_2 = 6R_2 + 18
\]

\[
R_2 = 9 \ \Omega
\]

Look at \( v_o(0^-) = v_o(0^+) \)

From the equation:

\[
v_o(0^-) = 6 - 3e^{-0.35 \times 0} = 3 \ V
\]

but

\[
v_o(0^-) = \left( \frac{24}{R_1 + 9 + 3} \right) \times 3 = 3
\]
\[ \frac{24}{R_1 + 9 + 3} = 1 \]
\[ 24 = R_1 + 12 \]
\[ \boxed{R_1 = 12} \]
\[ \gamma = \frac{1}{35} = \frac{L}{R_0} \]
\[ L = 2.86 = \frac{L}{12} \]
\[ 2L = 12 \times 2.86 = 34.32 \text{ H} \]

1. \[ R_1 = 12 \text{ \Omega} \]
2. \[ R_2 = 9 \text{ \Omega} \]
3. \[ L = 34.32 \text{ H} \]
(3) You are given the circuit of Figure 3. **Determine** the expression for the voltage $v_o(t)$.

For $t < 0$

\[ i_L(0^+) = i_L(0^-) = \frac{2 \times 5}{10 + 5} = 1 \text{A} \]

\[ v_o(0^+) = v_o(0^-) = 0 \]

For $t > 0$

Using nodal:

\[ \frac{v_o}{R} + C \frac{dv_o}{dt} + \frac{1}{L} \int v_o \, dt + 1 = 3 \quad (1) \]

Will need $\frac{dv_o(t)}{dt}$ in solving the O.D.E.

Go ahead and get it now from (1)
\[
\frac{dV(t)}{dt} = 3 - 1 - \frac{V_0(t)}{R} \\
\frac{d^2V(t)}{dt^2} = \frac{2}{C} = \frac{2}{10 \times 10^{-3}} = 200 \ \text{V/s}
\]

so:
\[V_0(t) = 0, \quad \frac{dV_0(t)}{dt} = 200 \ \text{V/s} \ \ (2)\]

Go to (1) and take \(\frac{d}{df}\), this gives:

\[
\frac{1}{R} \frac{dV_0}{dt} + \frac{C}{L} \frac{d^2V_0}{dt^2} + \frac{V_0}{L} = 0
\]

\[
\frac{d^2V_0}{dt^2} + \frac{1}{RC} \frac{dV_0}{dt} + \frac{V_0}{LC} = 0
\]

Put in values:

\[
\frac{d^2V_0}{dt^2} + 200 \frac{dV_0}{dt} + 100V_0 = 0
\]

\[s^2 + 20s + 100 = 0\]

\[(s + 10)(s + 10) = 0\]

\[V_0 = (A + Bt)e^{-10t} \ \ (3)\]

\[V_0(0) = A = 0\]

\[V_0(0) = B = e^{-10t}\]

\[\frac{dV_0}{dt} = 200 = \left[ -10Be^{-10t} + Be^{-10t} \right]_{t=0}\]

\[B = 200\]

\[V_0(t) = 200e^{-10t} \]

\[\text{Ans}\]
(4) You are given the circuit of Figure 4. **Determine** the impedance, \( Z_{ab}. \)

\[
\begin{align*}
(\frac{-j \cdot 20}{1}) \parallel (\frac{-j \cdot 20}{1}) &= \frac{(-j \cdot 20)(-j \cdot 20)}{-j \cdot 20 - j \cdot 20} = -j \cdot 10 \\
-j \cdot 10 \text{ in series with } +j \cdot 10 &= 0 \\
&= 0
\end{align*}
\]

\[
Z_{ab} = \frac{40 \parallel j \cdot 20}{40 + j \cdot 20}
\]

\[
Z_{ab} = 8 + j \cdot 16 = 17.81 \angle 3.43^\circ \Omega
\]
(5) You are given the circuit of Figure 5. Solve for the phasor voltages $V_1$ and $V_2$. Express your answers in polar form. **Note:** The unit for the circuit parameters is given in Seinan (S).

By inspection

\[
\begin{bmatrix}
0.2 - j0.4 & j0.8 \\
j0.8 & (0.5 - j0.1)
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
2130 \\
-412
\end{bmatrix}
\]

$V_1 = 5.16 \angle 96.45^\circ \ \checkmark$

$V_2 = 0.4 \angle 135^\circ \ \checkmark$
(6) **Solve** for the phasor currents $I_1$ and $I_2$ indicated in the circuit of Figure 6. Express your answers in polar form.

\[
(30 + j15)I_1 - 30I_2 = 60\angle 60° - 20\angle 40°
\]

\[-30I_1 + (40 - j5)I_2 = 20\angle 40° - 4V_x\]

\[= 20\angle 40° - 4(j15)I_1\]

\[-(30 + j60)I_1 + (40 - j5)I_2 = 20\angle 40°\]

\[
\begin{bmatrix}
30 + j15, & -30 \\
-30 + j60, & (40 - j5)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
= \begin{bmatrix} 60\angle 60° - 20\angle 40° \\
20\angle 40° \end{bmatrix}
\]

\[I_1 = 0.94\angle -24.04° \text{ A}\]

\[I_2 = 1.40\angle -63.98° \text{ A}\]