Thévenin's Theorem:

Mesh analysis, nodal analysis are key to circuits. Almost as high on the "must" circuit list to  
these is Thévenin's theorem. The course text justifies Thévenin  
to a degree. Rather than spend  
time on the justification let us look  
at how we use it. Consider Figure  
6.1.

![Diagram of the circuit showing Thévenin's Theorem application.]

Fig 6.1: Resistor Circuit
Figure 8.2: Reduced network of Figure 8.1

This illustration (graphical) takes Thévenin to more of a limit than what we normally see, one might more likely see

Figure 8.3: A reduced circuit for applying Thévenin.

Thévenin's Theorem tells us how to reduce the network to the left of a-b to a single source in series with a single resistor.
How do we do this? Think of it this way: Disconnect \( R \) from the load of Figure 8.3, that is, let terminals \( a-b \) be hanging loose. Now connect a voltmeter across \( a-b \). We read a single voltage. This is \( V_{th} \).

Now, make all the independent sources in-operative. Connect an ammeter across \( a-b \). The meter reads \( R_{th} \). We have two things going here:

- We can determine the Thévenin equivalent by making direct measurements of the circuit.

- We can determine the Thévenin equivalent circuit by analysis; that is, we determine the open-circuit voltage, and we find, analytically, the equivalent resistance of the circuit.
Let's take a simple case and see how this works.

**Example 8.1**

Consider the following circuit. Find the Thévenin circuit to the left of a-b. Connect this circuit to the 100 Ω resistor and find the current I in the 100 Ω resistor.

![Circuit Diagram](image)

**Figure 8.4:** Circuit to illustrate Thévenin's theorem, Example 8.1.

Find the open circuit voltage. We use the circuit of Figure 8.5 to do this.

![Circuit Diagram](image)

**Figure 8.5:** Circuit for finding Vout for Example 8.1.
\( V_1 - 50 \frac{V_1}{20} + \frac{V_1 + 100}{10} - V_2 = 0 \)

This gives:
\[ 19V_1 - 10V_2 = -750 \quad \text{Eq} \ 8.1 \]

\( V_2 - 100 - V_1 \frac{V_2}{10} + \frac{V_2}{50} = 0 \)

Note: No current goes through the 40 \( \text{m} \) resistor because terminals \( a-b \) are open

This gives:
\[ -5V_1 + 6V_2 = 600 \quad \text{Eq} \ 8.2 \]

Solving equations 8.1 and 8.2 gives:
\[ V_1 = 7.873 \text{V} \quad V_2 = 89.844 \text{V} \]

Now, \( V_{TH} = V_{ab} \) of the circuit in Figure 8.5. We can see no current flows through the 40 \( \text{m} \) resistor so:
\[ V_{TH} = V_{ab} = V_2 + 30 \]

\[ V_{TH} = 119.84 \text{V} \]
To find $R_{TH}$, we de-energize all the sources of Figure 8.5. This gives the circuit of Figure 8.6.

![Circuit diagram](image)

**Figure 8.6:** Circuit for finding $R_{TH}$ for Example 8.1.

Without too much difficulty, we find:

$$R_{ab} = R_{TH} = \frac{(20 + 25 + 10)}{1150} + 40$$

$$R_{ab} = 54.84 \, \Omega$$

The Thevenin is as shown in Fig. 8.7.

![Thevenin circuit diagram](image)

**Figure 8.7:** Thevenin equivalent circuit for example 8.1.
We can now tie this Thévenin circuit to any linear DC circuit, in particular to the 100Ω as shown in Figure 8.8.

![Figure 8.8: Thévenin equivalent circuit tied to the 100Ω load resistor.](image)

We easily determine that $I = 0.774A$.

**An Alternate Way of Finding $R_{TH}$**

Consider the Thévenin equivalent circuit when we tie a short-circuit across a-b.

![Figure 8.9: Thévenin circuit with a short-circuit load.](image)
We see from Figure 8.9:

\[ V_{TH} = I_{SC} R_{TH} \]

We solve for \( R_{TH} \):

\[ R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{V_{OC}}{I_{SC}} \quad \text{Eq. 8.3} \]

This is slick: it gives us another way to find \( R_{TH} \). Let us apply this to Example 8.1. If we short the load we have the following circuit of Figure 8.10.

![Figure 8.10: Circuit of Example 8.1 set to find \( I_{SC} \).](image)

The easiest way to find \( I_{SC} \) is probably using mesh analysis and \( I_1, I_2, I_3 \) as shown in Figure 8.10.

\[ I_{SC} = I_3 \]
The equation for the circuit of Figure 8.10 are given in matrix form (by inspection) as:

\[
\begin{bmatrix} 45 & -25 & 0 \\ -25 & 85 & -50 \\ 0 & -50 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 100 \\ 30 \end{bmatrix}
\]

This gives \( I_3 = 2.185 \text{ A} \)

Therefore

\[
R_{TH} = \frac{V_{TH}}{I_{sc}} = \frac{V_{oc}}{I_{sc}}
\]

\[
R_{TH} = \frac{119.84}{2.185} = 54.85 \text{ } \Omega
\]

This agrees with our previous work.

If we want to find the Norton circuit, we can again:
- Make direct measurements of the circuit
- Use analytical methods to find \( I_{sc} \) and \( V_{TH} \)
- Use source transformation
The Norton circuit looks like that of Figure 8.11:

\[ \frac{I_N}{3R_{TH}} = \frac{I}{R_H} \]

Figure 8.11: General configuration of the Norton equivalent circuit.

We know that \( I_N = I_{SE} \). We also know that \( R_{TH} \) is the same resistance we found for the Thévenin equivalent circuit.

A laboratory method for finding \( I_N \) is:

- Short across \( a-b \) and place an ammeter there to find \( I_{SE} \).
- We find \( I_N = R_{TH} \) the same way we did earlier by de-energizing the independent source.
Analytical methods: Find $I_{oc}$ by placing a short across a-b and using the most expedient method to find $I_{oc}$.

Place an open between a-b (just open a-b) and use the most expedient circuit solution method to determine $V_{ab} = V_{oc}$.

We know:

$$I_N = I_{oc}$$

$$R_H = R_{TH} = \frac{V_{oc}}{I_N}$$

Source transformation.

We can go Norton to Thevenin or Thevenin to Norton as shown in Figure 8.12.

Figure 8.12: Switching from Thevenin to Norton; Norton to Thevenin.
Essentially, if we understand the essence of Thévenin and Norton, the application of these two theorems boil down to our ability to work circuit using (a) mesh, (b) nodal, (c) source transformation, perhaps.

**Example 8.2**

Find the Thévenin equivalent circuit for the following:

![Circuit Diagram](image)

*Have the class to first write RTM.*

*Next have class solve for VTB.*
For \( R_{TH} \)

\[
\begin{align*}
\text{This gives} \\
R_{TH} &= 80 \Omega
\end{align*}
\]

Different with the circuit of Figure 3.

\[
\begin{align*}
20 \times 60 &= 1200 \\
\frac{20 \times 60}{60 + 30} &= 13.33 \text{V}
\end{align*}
\]

Use source transformation on the current source and 20 \( \Omega \) resistor.
We redraw the circuit.

\[ V_{ab} = V_{ac} = V_{TH} = 13.33 + 40 = 53.33 \text{V} \]

\[ R_{TH} = 20 + 20 + 40 = 80 \Omega \]

RED. Of course there are other ways to find \( V_{ab} \). It was desired to show some of the utility of source transformation.

\[ 60 \Omega \]

\[ 53.33 \text{V} \]

Figure 8.14: Thévenin equivalent circuit for Example 8.2.
Problem 4.48

Determine the Norton equivalent circuit between terminals a-b for the following circuit.

\[ R_{TH} = \frac{-4}{3} \Omega \]
\[ V_{TH} = -12 \text{V} \]
\[ J_N = 2 \text{A} \]

To find \( V_{oc} \):

\[ V_x = 2 \times 4 = 8 \text{V} \]

Since \( i_0 = 2 \text{A} \), the voltage across the 10\( \Omega \) dep. source is 20\text{V}.

Therefore:

\[ V_{ab} = V_{oc} = 8 - 20 = -12 \text{V} \]
\[ V_{TH} = -12 \text{V} \]

There are two ways to find \( R_{TH} \) when we have a dependent source and at least one independent source.
Method 1

De-energize all independent sources. Apply either a known voltage (say 1 V or 1 mA) or a known current (say 1 A). \( R_{TH} = \frac{V}{I} \), as shown below.

\[
\begin{align*}
\text{Network} & \quad I_x \\
\text{Network} & \quad V_x \\
R_{TH} &= \frac{1V}{I_x} \\
R_{TH} &= \frac{V_x}{1A}
\end{align*}
\]

Method 2

Leave all sources active; find \( J_{sc} \) (I short circuit).

\[
\begin{align*}
\text{Network} & \quad J_{sc} \\
R_{TH} &= \frac{V_{oc}}{J_{sc}}
\end{align*}
\]
There is no rule that says that

\[ R_{TH} = \frac{1V}{I_x} \quad \text{or} \quad R_{TH} = \frac{V_x}{1A} \quad \text{or} \quad R_{TH} = \frac{V_{ac}}{I_{ac}} \]

is easier. In other words, I can't tell you that one is easier than the other. It sort of depends on the problem.

Let us apply each method to problem 4.48.

**Method 1**

Apply 1V, find resulting I:

First, de-energize the 2 A independent source. We have

![Diagram]

Obviously, 10 = I_x
Writing KVL we have

\[-1 + 2I_x - 10I_x + 4I_x = 0\]
\[-4I_x = -1\]
\[I_x = -\frac{1}{4}\]

\[R_{TH} = \frac{1}{\frac{1}{14}} = -4\Omega\]

Now apply a current source say 1A

For this case \( I_0 = 1A \). Writing KVL

\[-V_x + 2 - 10 + 4 = 0\]
\[V_x = -4\]

\[\therefore R_{TH} = \frac{V_x}{I} = -4\Omega\]

Now find the rest for the circuit shown on the following page.
Use mesh as above

4(Isc - 2) + 10I_0 + 2Isc = 0

but I_0 = 2 - Isc

4Isc - 8 + 20 - 10Isc + 2Isc = 0

4Isc = 12

Isc = 3 A

\[ R_{TH} = \frac{V_{oc}}{Isc} \]

\[ R_{TH} = \frac{-12}{3} = -4 \Omega \]

Some it's the case
Work another problem that has a

equivalent source.

Note:

The open-circuit voltage (Thévenin's voltage) for any circuit that contains only

equivalent sources is zero.

Problems 4.62, 4.63, 4.64 in the
text are like this. Why is the open-
circuit voltage equal to zero?

There is no "juice" to make the

circuit work, so the voltage, everywhe-

and the current everywhere in the
circuit is zero. It is a dead circuit.

Ever thought \( I_n = V_{Th} = 0 \), \( R_{Th} \) generally

is not zero. Consider Problem 4.63, p. 148

\[ 4.63 \]

Find the Norton equivalent for the

following circuit.

\[ V_o = 20 \text{ V}, R = 10 \text{ }\]
Normally we would place a short circuit across the output terminals and find $I_{sc}$, as below.

Since there is no "juice" in the circuit, $V_0 = 0$, $0.5V_0 = 0$; the circuit is dead. $I_{sc} = I_{in} = 0$.

We apply a 1 A source to determine $V_x$.

We have:

\[-V_x + 10(1 + 0.5V_0) + V_0 = 0\]

\[-V_x + 10 + 6V_0 = 0\]

But $V_0 = \frac{V_x \times 20}{10+20} = \frac{2V_x}{3}$

\[-V_x + 10 + 6 \left( \frac{2V_x}{3} \right) = 0\]
\[ -V_x + 10 + 4V_x = 0 \]

\[ 3V_x = -10 \]

\[ V_x = -\frac{10}{3} \]

\[ Z_m = \frac{V_x}{1} = -\frac{10}{3} \]

The Norton circuit is

\[ \begin{array}{c}
\begin{array}{c}
\text{a}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\frac{10}{3} \text{ ohm}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{b}
\end{array}
\end{array} \]