ECE 300
Spring '07

Lec 8
Feb 8, '07

Thevenin's Theorem:

Mesh analysis, nodal analysis are key to circuits. Almost as high on the "must" circuit list to these is Thevenin's theorem. The course text justifies Thevenin to a degree. Rather than spend time on the justification let us look at how we use it. Consider Figure 8.1.

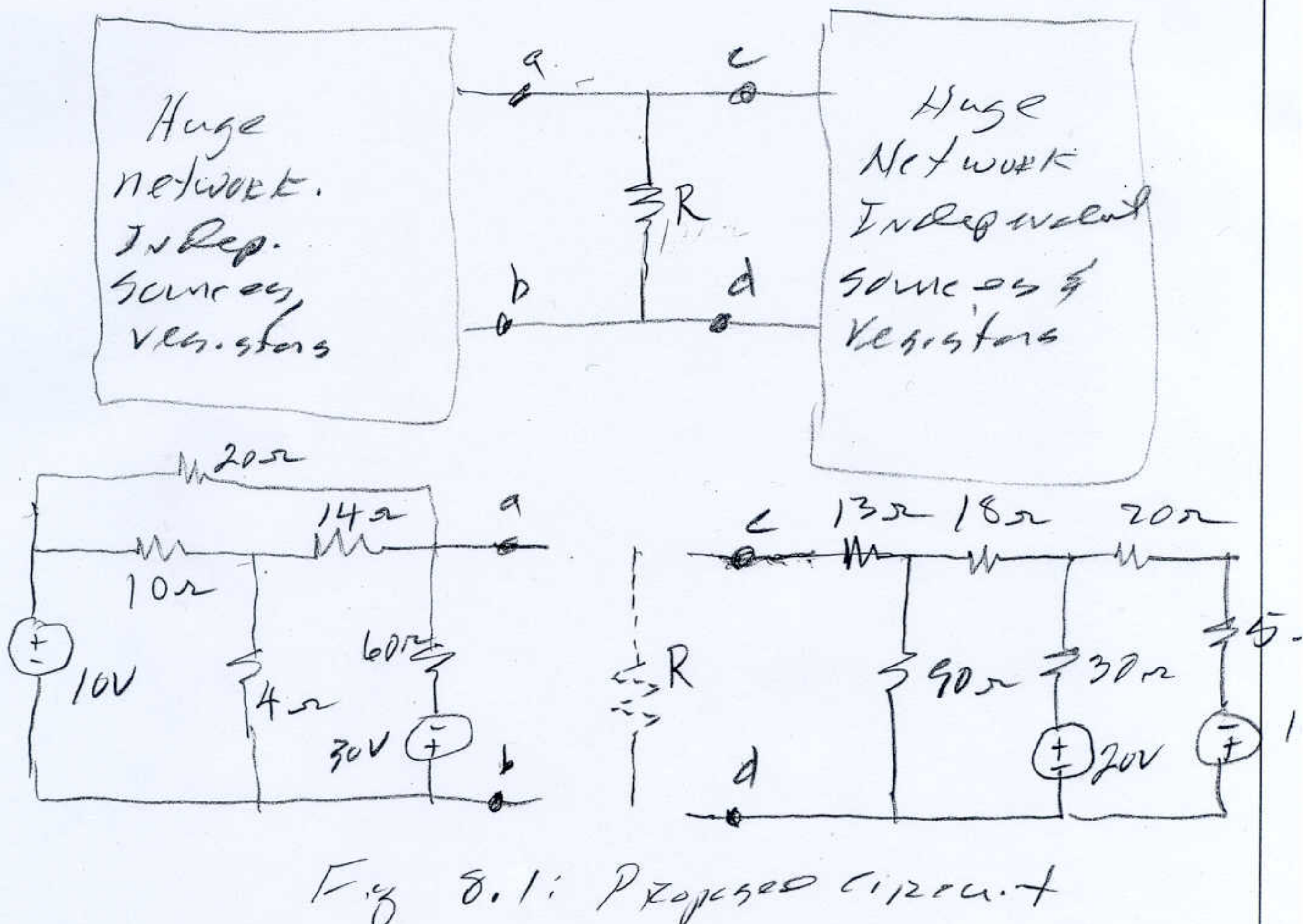


Fig 8.1: Proposed Circuit

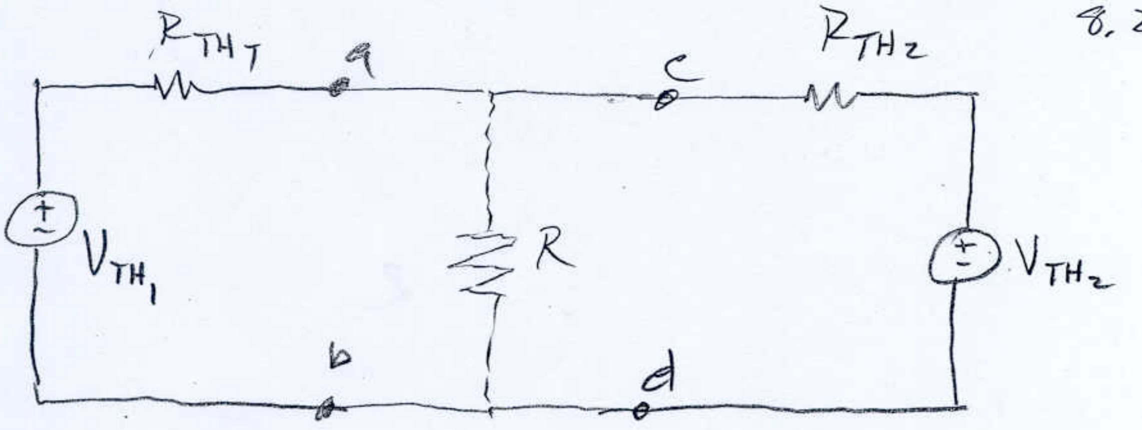


Figure 8.2; Reduced network of Figure 8.1

This illustration (graphical) takes Thevenin to more of a limit than what we normally see; One might more likely see

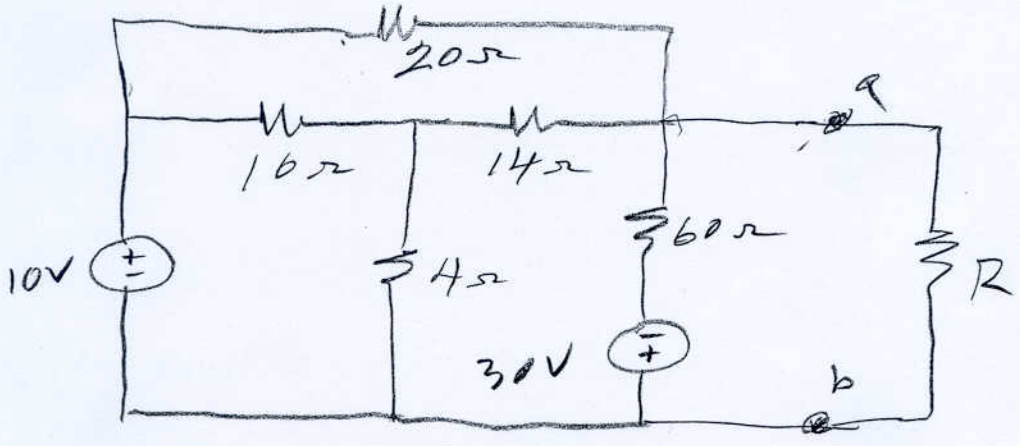


Figure 8.3: A reduced circuit for applying Thevenin.

Thevenin's Theorem tells us how to reduce the network to the left of a-b to a single source in series with a single resistor.

How do we do this? Think of it this way: Disconnect "R" from the load of Figure 8.3, that is, let terminals a-b be hanging loose. Now connect a voltmeter across a-b. We read a single voltage. This is V_{TH}. Now, make all the independent sources in-operative. Connect an ohmmeter across a-b. The meter reads R_{TH}. We have two things going here:

- We can determine the Thevenin equivalent by making direct measurements of the circuit.
- We can determine the Thevenin equivalent circuit by analysis; that is, we determine the open-circuit voltage, and we find, analytically, the equivalent resistance of the circuit

Let's take a simple case and see how this works.

Example 8.1

Consider the following circuit. Find the Thevenin circuit to the left of a-b. Connect this circuit to the 100 Ω resistor and find the current I in the 100 Ω resistor

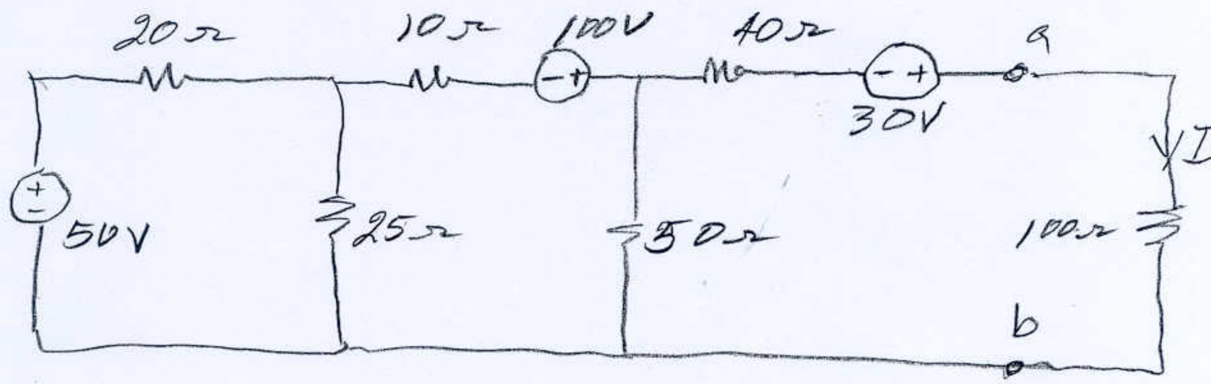


Figure 8.4: Circuit to illustrate Thevenin's theorem, Example 8.1.

Find the open circuit voltage. We use the circuit of Figure 8.5 to do this

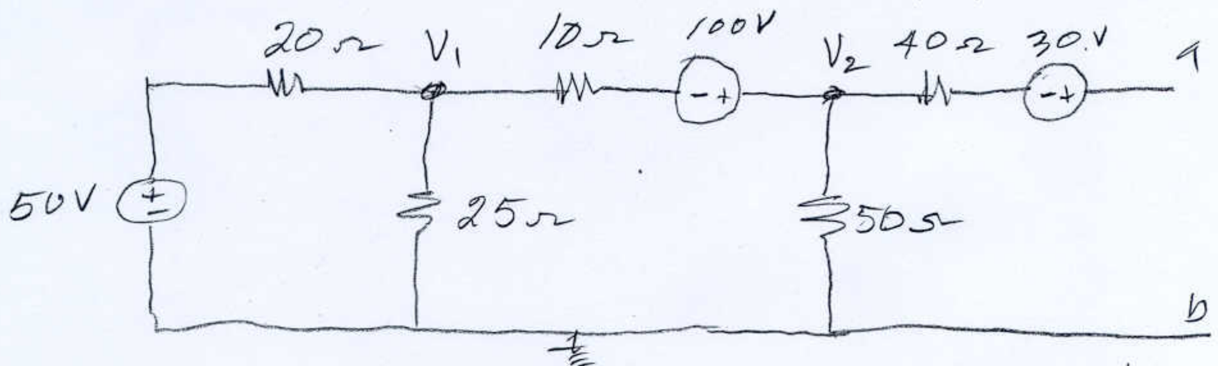


Figure 8.5: Circuit for finding V_{TH} for Example 8.1.

At V_1

$$\frac{V_1 - 50}{20} + \frac{V_1}{25} + \frac{V_1 + 100 - V_2}{10} = 0$$

this gives;

$$\boxed{19V_1 - 10V_2 = -750} \quad \text{Eq } 8.1$$

At V_2

$$\frac{V_2 - 100 - V_1}{10} + \frac{V_2}{50} = 0$$

(Note; no current goes through the 40 Ω resistor because terminals a-b are open)

this gives;

$$\boxed{-5V_1 + 6V_2 = 500} \quad \text{Eq } 8.2$$

Solving equations 8.1 & 8.2 gives

$V_1 = 7.813\,V$ $\quad V_2 = 89.644\,V$

Now, $V_{TH} = V_{ab}$ of the circuit in Figure 8.5. We can see no current flows through the 40Ω resistor so

$$V_{TH} = V_{ab} = V_2 + 30$$

$$\boxed{V_{TH} = 119.64\,V}$$

To find R_{TH}: We de-energize all the sources of Figure 8.5. This gives the circuit of Figure 8.6

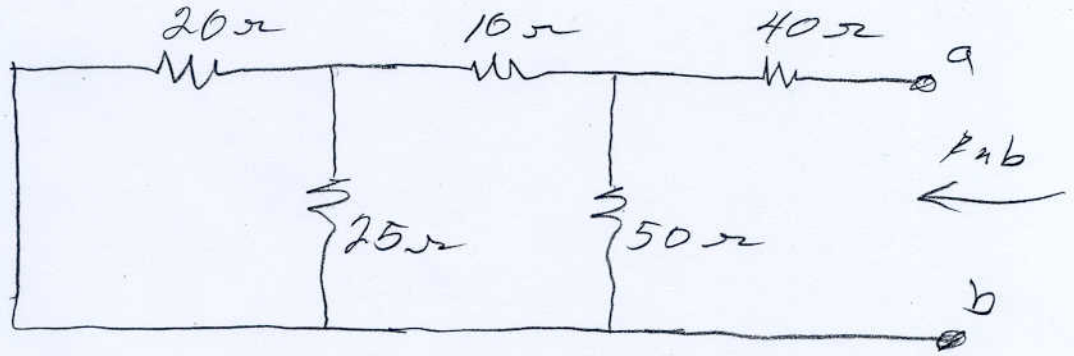


Figure 8.6: Circuit for finding R_{TH} for Example 8.1

Without too much difficulty we find

$$R_{ab} = R_{TH} = (20 \| 25 + 10) \| 50 + 40$$

$$\boxed{R_{ab} = 54.84\ \Omega}$$

The Thevenin is as shown in Fig 8.7

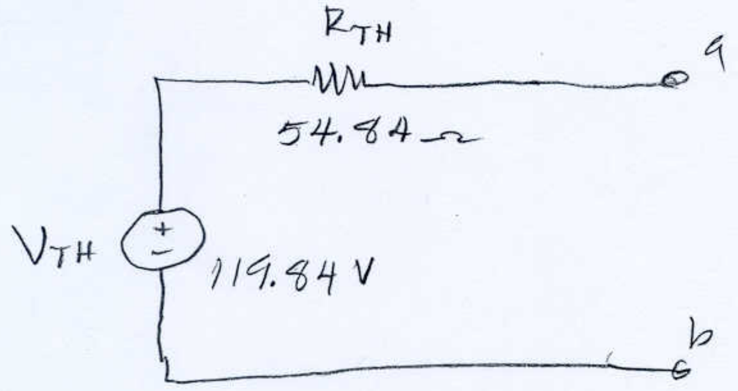


Figure 8.7: Thevenin equivalent circuit for example 8.1.

We can now tie this Thevenin circuit to any linear DC circuit; in particularly to the 100 Ω as shown in Figure 8.8

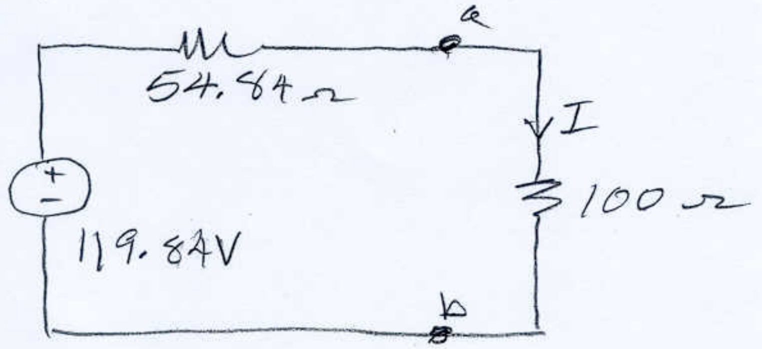


Figure 8.8: Thevenin equivalent circuit tied to the 100 Ω load resistor.

We easily determine that $I = 0.774A$. RED.

An Alternate Way of Finding R_{TH}

Consider the Thevenin Equivalent circuit when we tie a short-circuit across a-b.

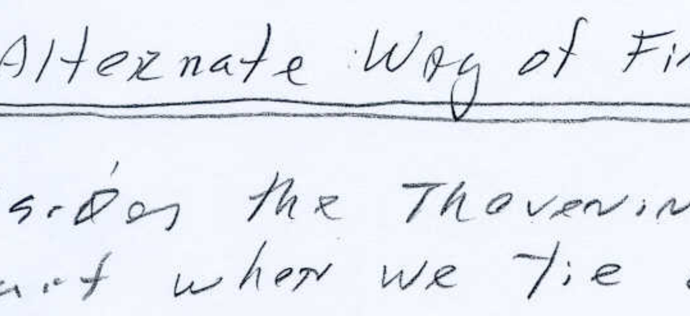


Figure 8.9: Theven circuit with a short-circuit load.

We see from Figure 8.9;

$$V_{TH} = I_{SC} R_{TH}$$

we solve for R_{TH}:

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{V_{OC}}{I_{SC}} \quad \text{Eq } 8.3$$

This is slick: it gives us another way to find R_{TH}. Let us apply this to Example 8.1. If we short the load we have the following circuit of Figure 8.10

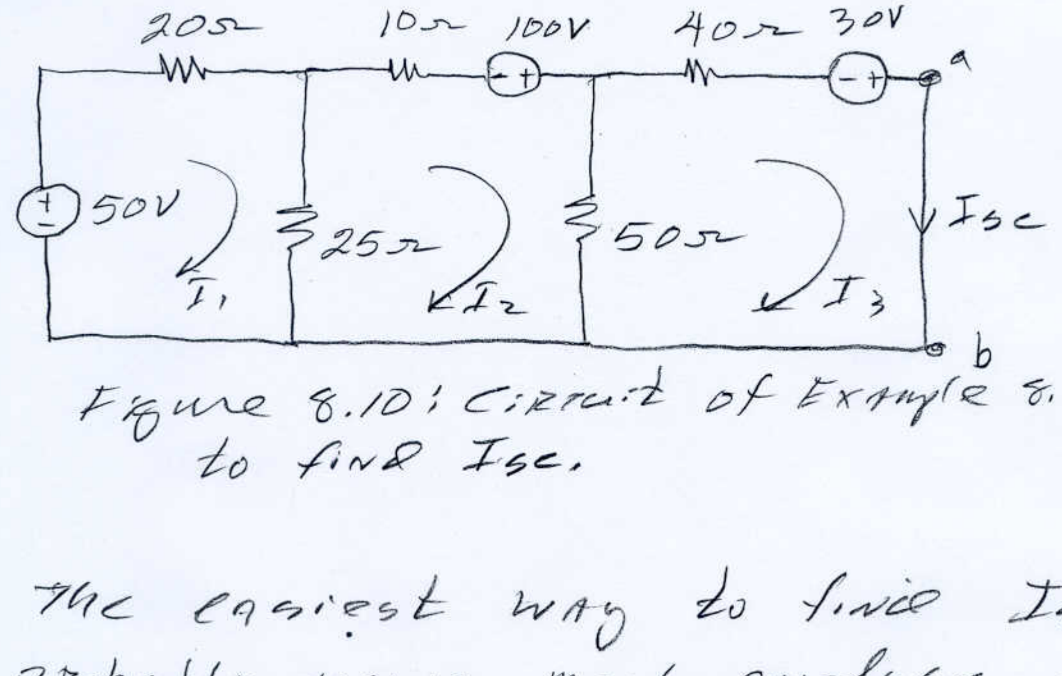


Figure 8.10: Circuit of Example 8.1, set to find I_{SC}.

The easiest way to find I_{SC} is probably using mesh analysis and I_1, I_2, I_3 as shown in Figure 8.10

$$I_{SC} = I_3$$

The equation for the circuit of Figure 8.10 are given in matrix form (by inspection) as

$$\begin{bmatrix} 45 & -25 & 0 \\ -25 & 85 & -50 \\ 0 & -50 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 100 \\ 30 \end{bmatrix}$$

This gives $I_3 = 2.185\ A$

Therefore

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{V_{OC}}{I_{SC}}$$

$$\boxed{R_{TH} = \frac{119.84}{2.185} = 54.85\ \Omega}$$

This agrees with our previous work.

If we want to find the Norton circuit we can again

- make direct measurements of the circuit
- Use analytical methods to find I_{SC} and R_{TH}.
- Use source transformation

The Norton circuit looks like that of Figure 8-11

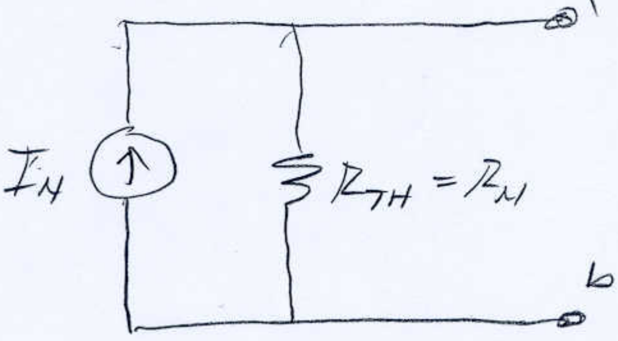


Figure 8.11: General configuration of the Norton equivalent ckt.

We know that $I_N = I_{sc}$. We also know that R_{TH} is the same resistance we found for the Thevenin equivalent circuit.

- Laboratory method for finding I_N: We simple put a short across a-b and place an Amp meter there to find I_{sc}.

  We find $R_N = R_{TH}$ the same way we did earlier by De-energizing the independent sources.

- Analytical method: Find I_{sc} by placing a short across a-b and using the most expedient method to find I_{sc}.

  Place an open between a-b (just open a-b) and use the most expedient circuit solution method to determine $V_{ab_{open}} = V_{oc}$

We know

$$I_N = I_{sc}$$

$$R_N = R_{TH} = \frac{V_{oc}}{I_N}$$

- Source transformation. We can go Norton to Thevenin or Thevenin to Norton as shown in Figure 8.12

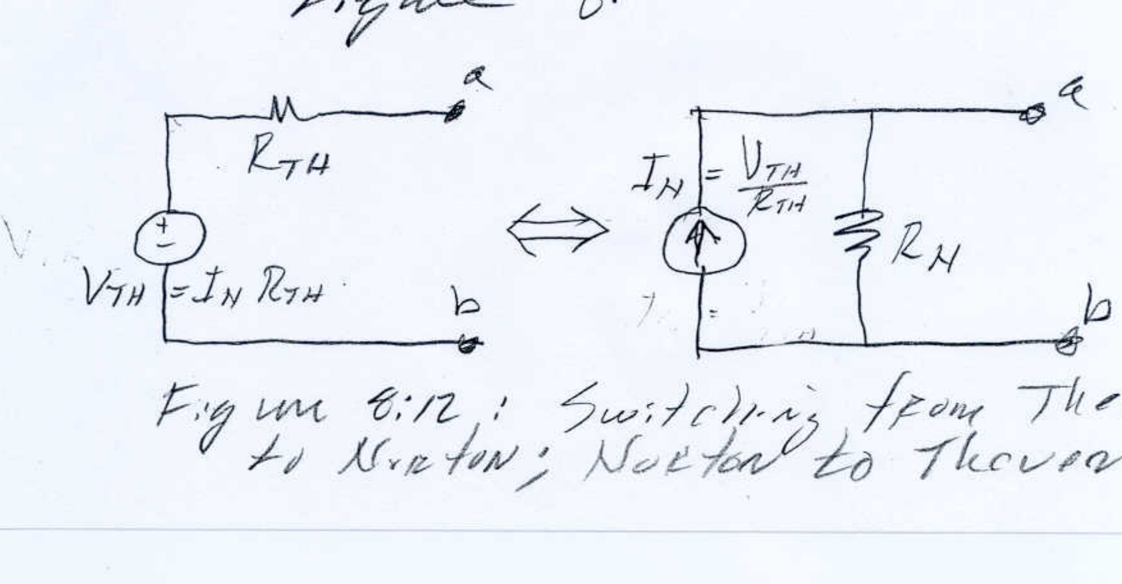


Figure 8.12: Switching from Thevenin to Norton; Norton to Thevenin.

Essentially, if we understand the essence of Thevenin and Norton the application of these two theorems boil down to our ability to work circuit using (a) mesh, (b) nodal, (c) source transformation, perhaps.

Example 8.2

Find the Thevenin equivalent circuit for the following:

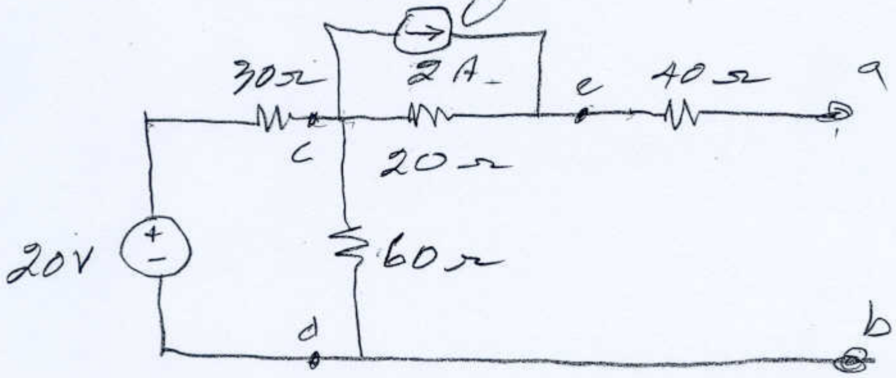
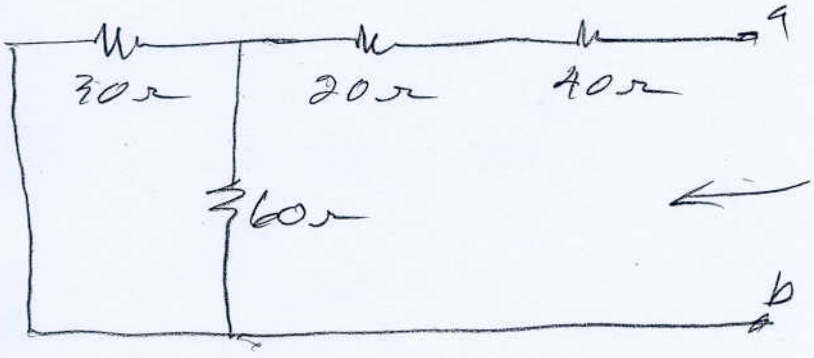


Figure 8.13: Circuit for Example 8.2

- Have the class to first find R_{TH}.

- Next have class solve for V_{TH}.

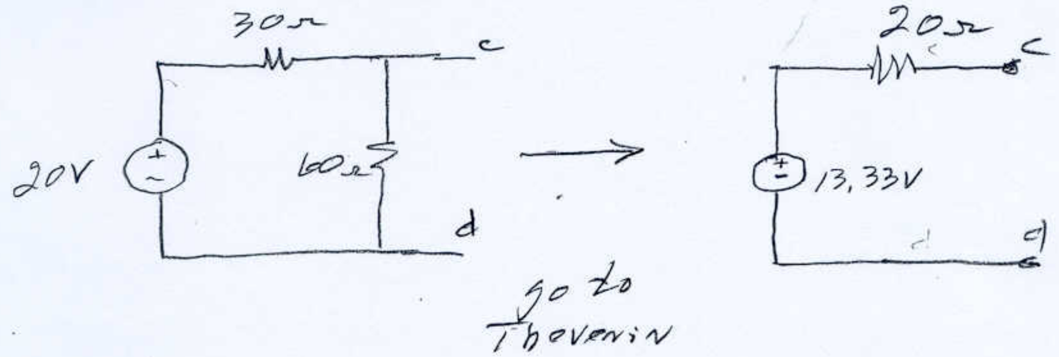
FOR R_{TH}



This give

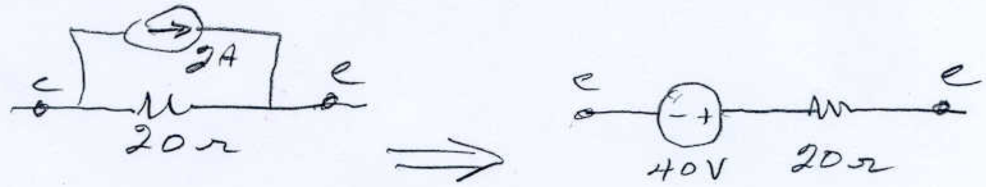
$$R_{TH} = 80\,\Omega$$

Deal with the circuit of Figure 8.

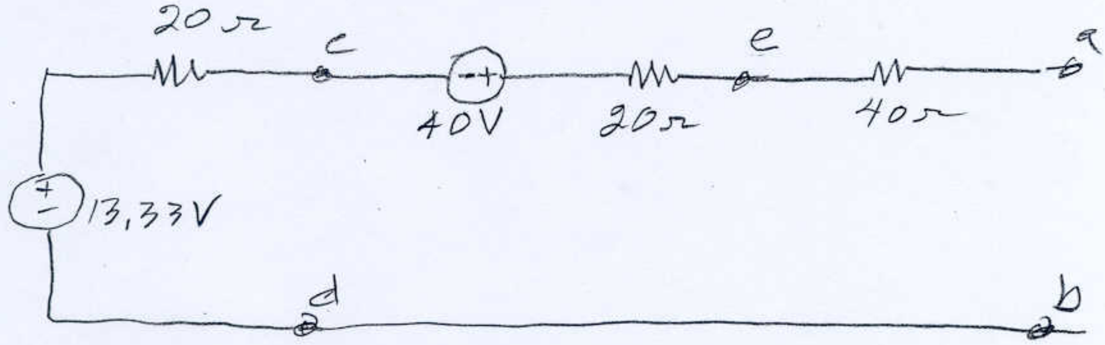


$$\frac{20 \times 60}{60 + 30} = 13.33\text{V}$$

Use source transformation on the current source and 20 resistor.



We redraw the circuit



$\underline{V_{ab}} = \underline{V_{oc}} = \underline{V_{TH}} = 13.33 + 40 = \underline{53.33V}$

$R_{TH} = 20 + 20 + 40 = 80\,\Omega$

QED. Of course there are other ways to find V_{ab}. It was desired to show some of the utility of source transformation.

So;

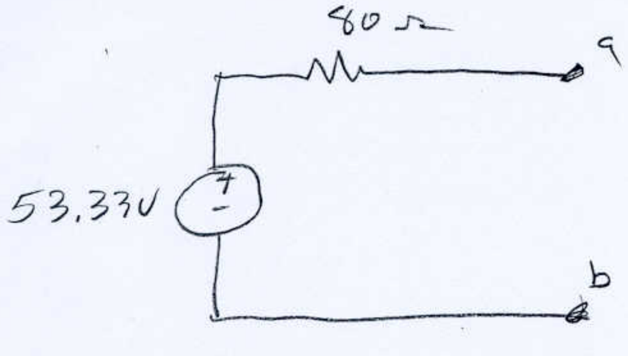
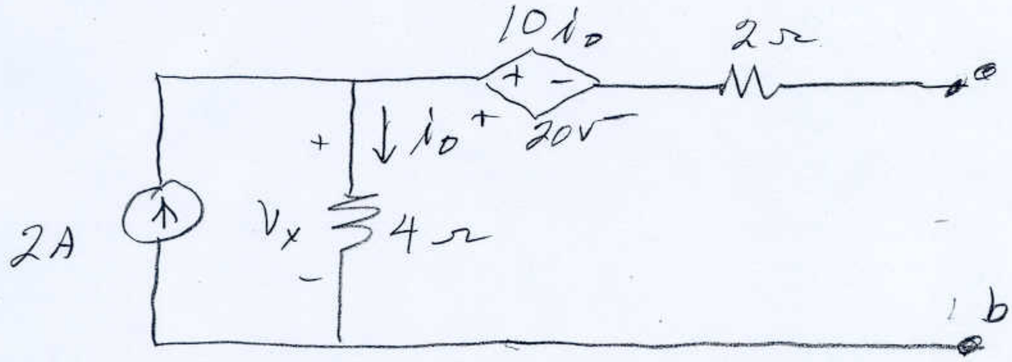


Figure 8.14: Thevenin equivalent circuit for Example 8.2

Problem 4.48

Determine the Norton equivalent circuit between terminal a-b for the following circuit.



$R_{TH} = -4\,\Omega$
$V_{TH} = -12V$
$I_N = 3A$

To find Voc:

$V_x = 2 \times 4 = 8V$

Since $i_o = 2A$, the voltage across the $10 i_o$ dep. source is 20V.

Therefore

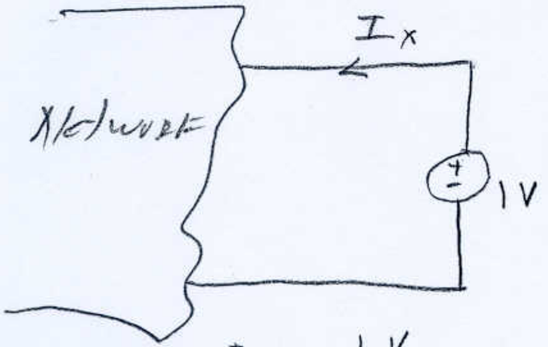
$$V_{ab} = V_{oc} = 8 - 20 = -12V$$

$$V_{TH} = -12V$$

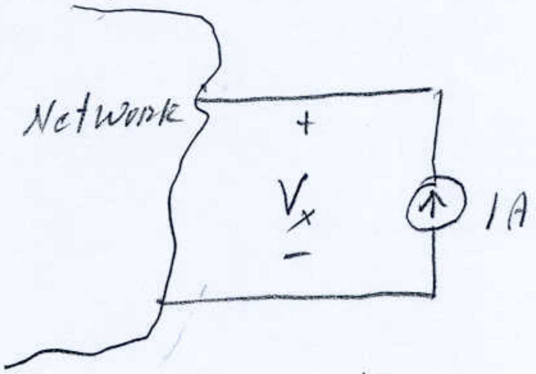
There are two ways to find R_{TH} when we have a dependent source and at least one independent source.

method 1

De-energize all independent sources. Apply either a known voltage (say 1 volt) or a known current (say 1 amp). $R_{TH} = \frac{V}{I}$, as shown below



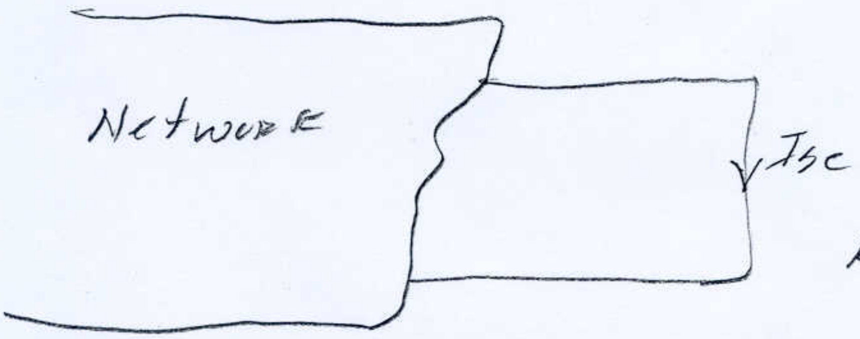
$$R_{TH} = \frac{1\,V}{I_x}$$



$$R_{TH} = \frac{V_x}{1\,A}$$

method 2

Leave all sources active; Find I_{sc} (I short circuit)



$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$

There is no rule that says that

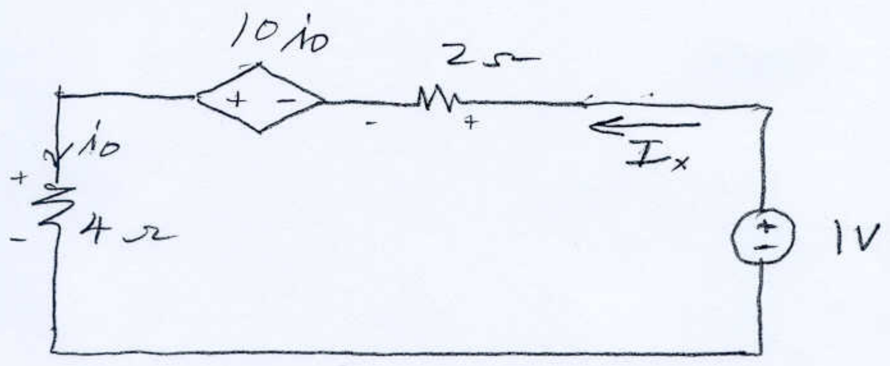
$$R_{TH} = \frac{1V}{I_x} \quad \text{OR} \quad R_{TH} = \frac{V_x}{1A} \quad \text{OR} \quad R_{TH} = \frac{V_{oc}}{I_{sc}}$$

is easier. In other words, I can't tell you that one is easier than the other. It sort of depends on the problem.

Let us apply each method to problem 4.48.

<u>Method 1</u>

Apply 1V, find resulting I:
First, de-energize the 2A independent source. We have



obviously, $i_o = I_x$

Writing KVL we have

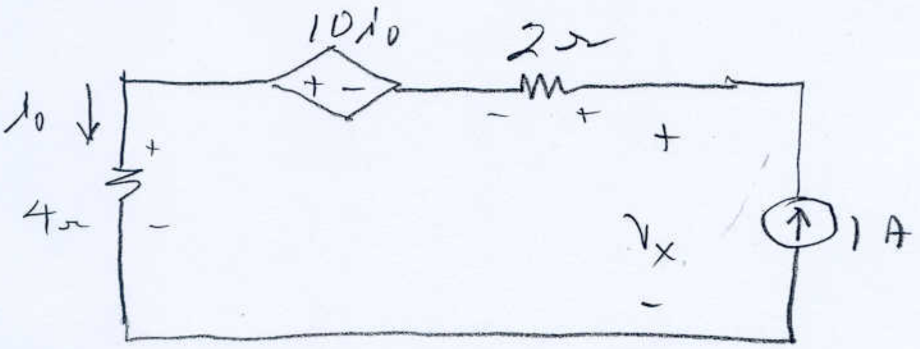
$$-1 + 2I_x - 10I_x + 4I_x = 0$$

$$4I_x = -1$$

$$I_x = -\frac{1}{4}$$

$$\boxed{R_{TH} = \frac{1}{-1/4} = -4\,\Omega}$$

Now apply a current source, say 1A



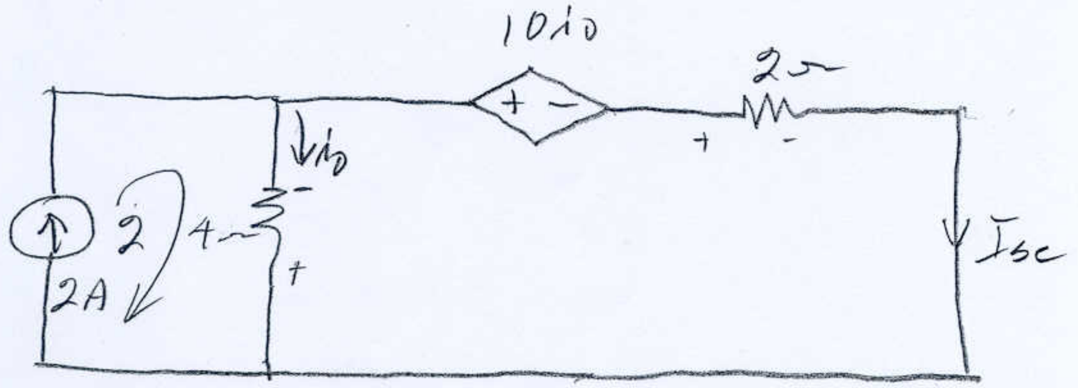
For this case $i_0 = 1A$. Writing KVL

$$-V_x + 2 - 10 + 4 = 0$$

$$V_x = -4$$

$$\therefore \boxed{R_{TH} = \frac{V_x}{1} = -4\,\Omega}$$

Now find i_{sc} for the circuit shown on the following page.



Use mesh as above

$$4(I_{sc}-2) + 10 i_0 + 2 I_{sc} = 0$$

but $i_0 = 2 - I_{sc}$

$$4 I_{sc} - 8 + 20 - 10 I_{sc} + 2 I_{sc} = 0$$

$$4 I_{sc} = 12$$

$$I_{sc} = 3 \text{ A}$$

$$\therefore \quad R_{TH} = \frac{V_{oc}}{I_{sc}}$$

$$R_{TH} = \frac{-12}{3} = -4\Omega$$

$$\boxed{R_{TH} = -4\Omega}$$

Same as before

Work another problem that has a dependent source.

Note:

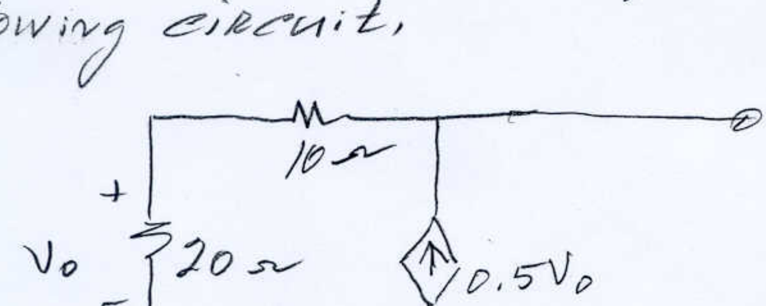
The open-circuit voltage (Thevenin's voltage) for any circuit that contains only dependent sources is zero.

Problems 4.62, 4.63, 4.64 in the text are like this. Why is the open-circuit voltage equal to zero?
There is no "juice" to make the circuit work, so the voltage everywhere and the current everywhere in the circuit is zero. It is a dead circuit.

Even though $I_N = V_{TH} = 0$, R_{TH} generally is not zero. Consider Problem 4.63, p165

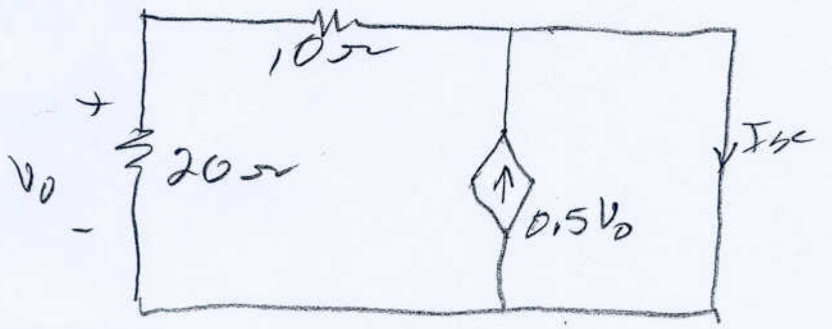
4.63

Find the Norton equivalent for the following circuit.



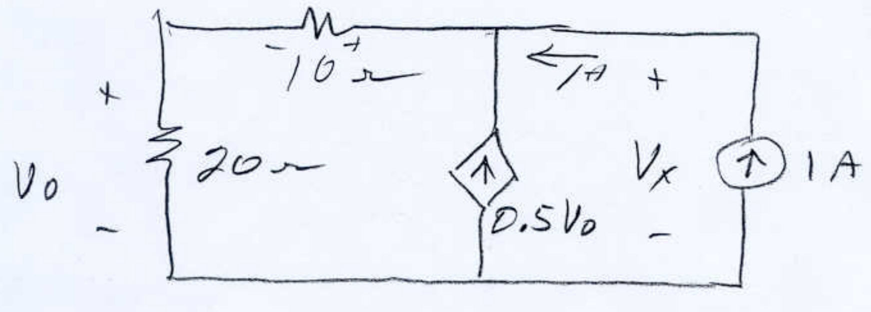
8.21

Normally we would place a short circuit across the output terminals and find I_{SC}, as below.



Since there is no "juice" in the circuit, $V_0 = 0$, $0.5V_0 = 0$; the circuit is dead. $I_{SC} = I_N = 0$.

We apply a 1 A source; determine V_x



We have $-V_x + 10(1 + 0.5V_0) + V_0 = 0$

$$-V_x + 10 + 6V_0 = 0$$

but $V_0 = \dfrac{V_x \times 20}{10 + 20} = \dfrac{2V_x}{3}$

$$-V_x + 10 + 6\left(\frac{2V_x}{3}\right) = 0$$

Sol

8.22

$$-V_x + 10 + 4V_x = 0$$

$$3V_x = -10$$

$$V_x = -\frac{10}{3}$$

$$\boxed{R_{TH} = \frac{V_x}{1} = -\frac{10}{3}\,\Omega}$$

The Norton circuit is