(1) You are given the circuit of Figure 1. The make-before-break switch has been in position 1 for a very long time. At \( t = 0 \), the switch is moved to position 2.

![Circuit Diagram](image)

Figure 1: Circuit for Problem 1.

(a) Determine \( v(0^+) \) and \( \frac{dv(0^+)}{dt} \).

(b) Develop the differential equation that can be used to determine \( v(t) \). The equation is of the form

\[
\frac{d^2v(t)}{dt^2} + K_3 \frac{dv(t)}{dt} + K_2v(t) = K_1
\]

Determine \( K_1, K_2, \) and \( K_3 \) using numerical values.

(c) Give the characteristic equation and the roots of the characteristic equation.

(d) Which of the following apply?

(i) The response for \( v(t) \) is overdamped.
(ii) The response for \( v(t) \) is underdamped.
(iii) The response for \( v(t) \) is critically damped.
(iv) Since this is a linear, time invariant, second order differential equation, none of the above apply.

(e) Determine \( \xi, \omega_n \) and \( \omega_d \).

(f) Determine the solution for \( v(t) \) for \( t \geq 0 \).

(g) Approximately how long will it take (within 20%) before the response decays to 1% of its initial value?

(a) Looking at the circuit,

\[ v(0^-) = v(0^+) = 0 \text{ V} \]
(1) cont

Looking at the circuit

\[ i'(10^2) = \frac{15}{15} = 1 \text{ A} \]

\[ i > 0 \]

\[
\begin{align*}
L \frac{di'}{dt} + V(t) + R i' &= 0 \\
i' &= C \frac{dV}{dt} \quad (A) \\
R C \frac{dV}{dt} + L C \frac{d^2V}{dt^2} + V(t) &= 0 \quad (B) \\
\end{align*}
\]

From (B)

\[ i'(0^-) = -1 \text{ A} \]

\[ i'(0^+) = -1 \text{ A} = -1 \text{ A} \]

From (B)

\[ \frac{dV(0^+)}{dt} = \frac{i'(0^+)}{C} = \frac{-1}{-}\text{A} = -100 \text{ V/s} \]

\[ V(0^-) = 0; \quad \frac{dV(0^-)}{dt} = -100 \text{ V/s} \]
(1) cont.

\[ \frac{\partial^2 v}{\partial t^2} + \frac{\partial v}{\partial t} + \frac{v}{t} = 0 \]

with numbers

\[ \frac{\partial^2 v}{\partial t^2} + 10 \frac{\partial v}{\partial t} + 100 \frac{v}{\partial t} = 0 \]

\[ b^2 + 10b + 100 = 0 \]

\[ (5 + 5j8.66)(5 - 5j8.66) = 0 \]

(b) \[ \frac{\partial^2 v}{\partial t^2} + 10 \frac{\partial v}{\partial t} + 100 \frac{v}{\partial t} = 0 \]

\[ k_1 = 10, \quad k_2 = 100, \quad k_3 = 0 \]

(c) Characteristic Eqn:

\[ b^2 + 10b + 100 = 0 \]

\[ s_1 = -5 + j8.66 \]

\[ s_2 = -5 - j8.66 \]

(d) Underdamped

1e) \[ w_n^2 = 100 \]

\[ w_n = 10 \]

\[ 2\pi w_n = 10 \]

\[ \frac{\pi}{2} = 0.5 \]

\[ \frac{\pi}{2} w_n = 5 \text{ check} \]

\[ \omega_D = 10 \sqrt{1 - 0.5^2} = 8.66 \text{ check} \]
(c) cont

(5)

\[ v(t) = e^{-5t} \left[ B_1 \cos(\omega t) + B_2 \sin(\omega t) \right] \]

\[ v(t) = e^{-5t} \left[ B_1 \cos(\omega t) + B_2 \sin(\omega t) + 1 \right] \]

\[ e^{t=0} \rightarrow v(0^+) = 0 \]

\[ 0 = B_1 \]

\[ v(t) = e^{-5t} B_2 \sin(\omega t) \]

\[ \frac{\Delta v}{\Delta t} = e^{-5t} \omega B_2 \cos(\omega t) - 5e^{-5t} B_2 \sin(\omega t) \]

\[ e^{t=0^+} \]

\[ \frac{\Delta v(0^+)}{\Delta \theta} = -100 = \omega \theta B_2 \]

\[ B_2 = \frac{-100}{\omega \theta} = -11.55 \]

\[ v(t) = -e^{-5t} \left[ 11.55 \sin(8.66 t) \right] \]

(6)

\[ e^{-4} \left( t=\frac{2}{3} \right) = 0.183 = 1.83\% \]

\[ e^{-5} \left( t=1 \right) = 0.0067 = 0.67\% \]

\[ t > 0.8 \text{ will take it to less than } 1\% \]
(2) Use mesh analysis to find the phasor current $I_0$ shown in the circuit of Figure 2. Express your answer in polar form.

\[ 80 \angle 60 \text{V} \left( I_1 - I_2 \right) = 100 \angle 120^\circ \]

\[ (80 - j60)I_1 + j60I_2 + 0I_3 = 100 \angle 120^\circ \]

\[ -j60(I_2 - I_1) + j30I_2 - j60(I_2 - I_3) = 0 \]

\[ 60I_1 - j60I_2 + j60I_3 = 0 \]

\[ -j60(I_3 - I_2) + 80I_3 = -60 \angle -30^\circ \]

\[ 0I_1 + j60I_2 + (80 - j60)I_3 = -60 \angle -30^\circ \]

\[
\begin{bmatrix}
80 - j60 & j60 & 0 \\
-j60 & -j90 & j60 \\
0 & j60 & 80 - j60
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
=
\begin{bmatrix}
100 \angle 120^\circ \\
0 \\
-60 \angle -30^\circ
\end{bmatrix}
\]

\[ I_1 = -0.356 + j1.464 \]

\[ I_2 = -0.864 + j1.823 \]

\[ I_3 = -0.9407 + j1.27 = 1.58 \angle 126.5^\circ \text{A} = I_0 \]
(3) You are given the circuit of Figure 3. Use Nodal Analysis to find the phasor voltages $V_1$ and $V_2$. Express your answers in polar form.

\[ \frac{V_1}{j3} + \frac{V_2}{j6} + \frac{V_2}{12} = 3 \]
\[ j0.333V_1 - j0.1667V_2 + 0.08333V_2 = 3 \]
\[ j0.333V_1 + (0.08333 - j0.1667)V_2 = 3 \]

**Constraint**
\[ V_1 - 10\angle45 - V_2 = 0 \]

\[
\begin{bmatrix}
30.333, (0.08333 - j0.124) \\
1, -1
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = 
\begin{bmatrix}
3 \\
10\angle45
\end{bmatrix}
\]

\[ V_1 = 25.8 \angle -70.43^\circ \text{ check} \]
\[ V_2 = 31.45 \angle -87.13^\circ \text{ check} \]
(4) You are given the AC circuit shown in Figure 4. The following phasor voltages are known.

\[ V_s = 100 \angle 0^\circ V; \quad V_R = 44.8 \angle -63.43^\circ V; \quad V_L = 156.8 \angle 26.57^\circ V \]

The source frequency is 300 rad/sec.

\[ \omega = 300 \text{ rad/sec} \]

![AC circuit diagram](image)

Figure 4: Circuit for Problem 4.

(a) Solve for the phasor voltage \( V_C \). Express your answer in polar form.

(b) Draw the phasor diagram (close to scale) showing \( V_s, V_R, V_L, \) and \( V_C \).

(c) Determine the phasor current \( I \) shown in the diagram. Express your answer in polar form.

(d) Determine the value of the capacitor \( C \), expressed in \( \mu F \).

\[
(A) \quad V_C = V_s - V_R - V_L = 100 - 44.8 \angle -63.43^\circ - 156.8 \angle 26.57^\circ
\]

\[
V_C = 67.36 \angle -153.5^\circ V
\]
\( I = \frac{V_R}{R} = \frac{44.8}{10} \quad \text{A} \)

\( I = 4.48 \angle -63.43^\circ \quad \text{A} \)

(\( d) \)

\[ V_c = I \cdot \left( \frac{1}{\text{j}wc} \right) \]

\[ C = \frac{I}{V_c (\omega)} = \frac{4.48 \angle -63.43^\circ}{6.736 \angle -153.5^\circ (0.300)} \]

\[ C = 22.17 \mu \text{F} \]
(5) You are given the circuit shown in Figure 5. The switch has been closed for a very long time and is opened at t=0. Use the step-by-step method to find \( v_0(t) \).

\[
\begin{align*}
R_{TH} &= 2 + 2/2 = 3 \, \Omega \\
v_{TH} &= \frac{8 \times 2}{4} = 4 \, V \\
\end{align*}
\]

\[t < 0\]

\[
\frac{V_{c(0^+)} - 12}{1} + \frac{V_{c(0^+)} - 4}{3} = 0
\]

\[4 \times 10^3 = 40\]

\[V_{c(0^+)} = 10 \, V\]

\[t > 0\]

\[
\begin{align*}
12V & \quad + \\
\end{align*}
\]

\( V_0(10^2) = \frac{2 \times V(10^2)}{2 + 2} = 0.5 \ V(10^2) \)
\[ V_0(10^2) = 0.5 \times 10 = 5 \ V \]
\[ V_0(100) = \frac{10 \times 2}{2 + 2 + 1} = \frac{20}{5} = 4.8 \ V \]
\[ V_0(100) = 4.8 \ V \]
\[ R_C = \frac{1}{4} = \frac{4}{5} = 0.8 \]
\[ \tau = R_C C = 0.8 \times 2 = 1.6 \]
\[ V(t) = V(\infty) + [V(10^2) - V(\infty)] e^{-\frac{t}{\tau}} \]
\[ V(4) = 4.8 + [5 - 4.8] e^{-0.625 \times 4} \]
\[ V(4) = 4.8 + 0.2 e^{-0.625 \times 4} \ V \]