"Desk Copy"

ECE 300
Spring Semester, 2008
HW Set #2

Due: January 29, 2005 **Name** wlg
wlg Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 10 points.

2.XX You are given the circuits shown in Figures 2.XX(a), 2.XX(b), 2.XX(c). You are to to find node voltages V_1, V_2, V_3 for each diagram. You are to find currents I_A, I_B, I_C and I_D for each diagram. You are to also find V_A, V_B, V_C for each diagram. From this study, state what you have learned about changing reference points in nodal analysis.

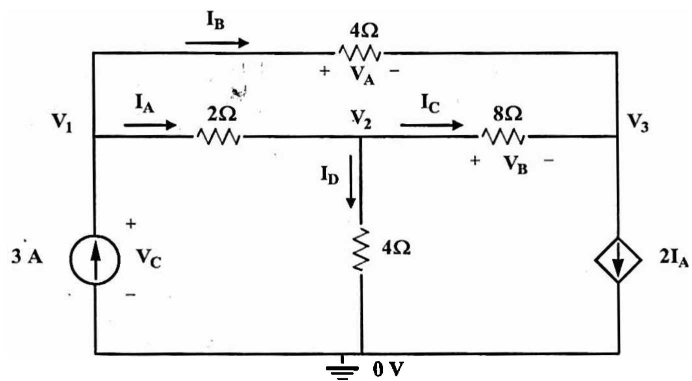

Figure 2.XX(a)

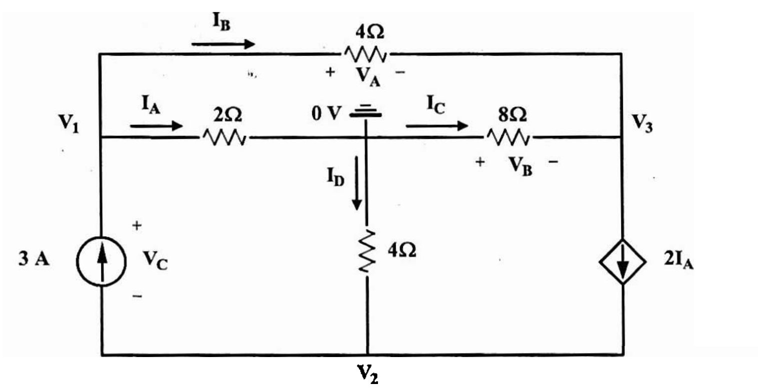

Figure 2.XX(b)

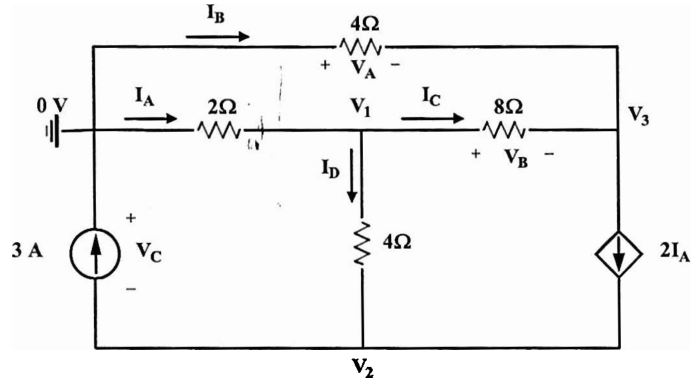

Figure 2.XX(c)

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H.W. #2
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2.XX You are given the circuits shown in Figures 2.XX(a), 2.XX(b), 2.XX(c). You are to to find node voltages V_1, V_2, V_3 for each diagram. You are to find currents I_A, I_B, I_C and I_D for each diagram. You are to also find V_A, V_B, V_C for each diagram. From this study, state what you have learned about changing reference points in nodal analysis.

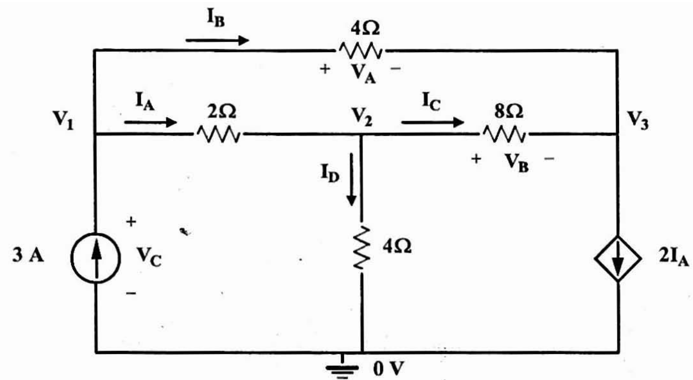


Figure 2.XX(a)

First Determine V_1, V_2, V_3.

At V_1

$$4\left(\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} - 3 = 0\right.$$

$$2V_1 - 2V_2 + V_1 - V_3 = 12$$

$$\boxed{3V_1 - 2V_2 - V_3 = 12}$$

At V_2

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{8} + \frac{V_2}{4} = 0$$

$$\boxed{-4V_1 + 7V_2 - V_3 = 0}$$

At V_3

$$\frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} + 2 I_A = 0$$

$$I_A = \frac{V_1 - V_2}{2}$$

$$8\left(\frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} + 2\left(\frac{V_1 - V_2}{2}\right) = 0\right)$$

$$V_3 - V_2 + 2V_3 - 2V_1 + 8V_1 - 8V_2 = 0$$

$$6V_1 - 9V_2 + 3V_3 = 0$$

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 6 & -9 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$V_1 = 4.8\,V$; $V_2 = 2.4\,V$; $V_3 = -2.4V$

$$I_A = \frac{V_1 - V_2}{2} = 1.2A$$

$$I_B = \frac{V_1 - V_3}{4} = \frac{7.2}{4} = 1.8A$$

$$I_C = \frac{V_2 - V_3}{8} = \frac{2.4 + 2.4}{8} = 0.6A$$

$$I_D = \frac{V_2}{4} = 0.6A$$

(A) SUMMARY (A) $I_A = 1.2A$; $I_B = 1.8A$; $I_C = 0.6A$; $I_D = 0.6A$

2.XX

FOR V_A

$V_1 - V_A - V_3 = 0$

$V_A = V_1 - V_3 = 4.8 - (-2.4)$

$V_A = 7.2\,V$

FOR V_B

$V_2 - V_B - V_3 = 0$

$V_B = V_2 - V_3 = 2.4 - (-2.4)$

$V_B = 4.8\,V$

FOR V_3

$V_C = V_1$

$V_C = 4.8\,V$

Summary (A)

$V_A = 7.2\,V \quad V_B = 4.8\,V; \quad V_C = 4.8\,V$

2.XX (b)

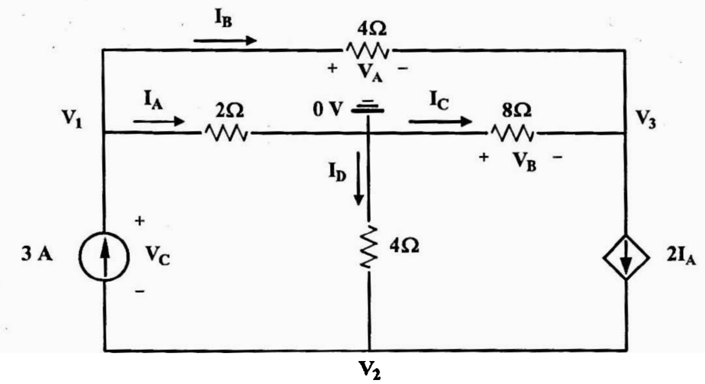


Figure 2.XX(b)

2.xx

(b) At V_1

$$4\left(\frac{V_1}{2} + \frac{V_1 - V_3}{4} - 3 = 0\right)$$

$$2V_1 + V_1 - V_3 = 12$$

$$\boxed{3V_1 + 0V_2 - V_3 = 12}$$

At V_2

$$\frac{V_2}{4} + 3 - 2I_A = 0$$

$$I_A = \frac{V_1}{2}$$

$$4\left(\frac{V_2}{4} + 3 - 2\left(\frac{V_1}{2}\right) = 0\right)$$

$$V_2 + 12 - 4V_1 = 0$$

$$\boxed{-4V_1 + V_2 + 0V_3 = -12}$$

At V_3

$$\frac{V_3}{8} + \frac{V_3 - V_1}{4} + 2I_A = 0$$

$$I_A = \frac{V_1}{2}$$

$$8\left(\frac{V_3}{8} + \frac{V_3 - V_1}{4} + 2\left(\frac{V_1}{2}\right) = 0\right)$$

2.XX

$$V_3 + 2V_3 - 2V_1 + 8V_1 = 0$$

$$\boxed{6V_1 + 0V_2 + 3V_3 = 0}$$

(b)

$$\begin{bmatrix} 3 & 0 & -1 \\ -4 & 1 & 0 \\ 6 & 0 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 0 \end{bmatrix}$$

$V_1 = 2.4V$; $V_2 = -2.4V$; $V_3 = -4.8V$

$$I_A = \frac{V_1}{2} = 1.2A$$

$$I_B = \frac{V_1 - V_3}{4} = \frac{2.4 + 4.8}{4} = 1.8A$$

$$I_C = \frac{-V_3}{8} = \frac{4.8}{8} = 0.6A$$

$$I_D = \frac{-V_2}{4} = \frac{2.4}{4} = 0.6A$$

Summary (b)

$I_A = 1.2A$; $I_B = 1.8A$; $I_C = 0.6A$; $I_D = 0.6A$

FOR VA

$$V_A = I_B \times 4 = 1.8 \times 4 = 7.2V$$

FOR VB

$$V_B = 8I_C = 4.8V$$

FOR V_3: $V_1 - V_3 - V_2 = 0$

$$V_3 = V_1 - V_2 = 2.4 - (-2.4) = \underline{4.8V}$$

2.XX (c)

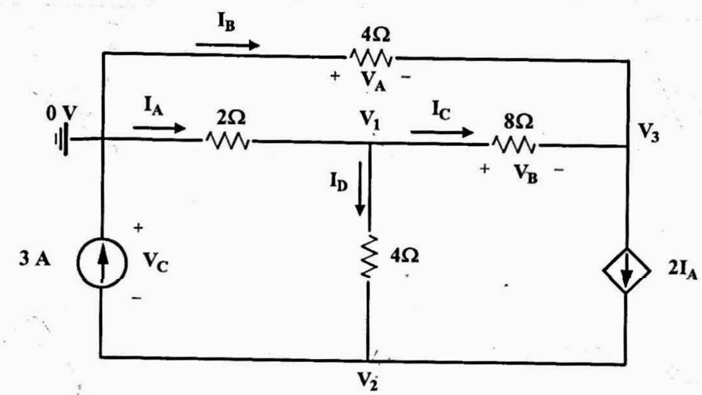


Figure 2.XX(c)

Part (c)

At V_1

$$8\left(\frac{V_1 - V_2}{4} + \frac{V_1 - V_3}{8} + \frac{V_1}{2} = 0\right)$$

$$2V_1 - 2V_2 + V_1 - V_3 + 4V_1 = 0$$

$$\boxed{7V_1 - 2V_2 - V_3 = 0}$$

At V_2

$$\frac{V_2 - V_1}{4} + 3 - 2I_A = 0$$

$$4\left(\frac{V_2 - V_1}{4} + 3 - 2\frac{(-V_1)}{2} = 0\right)$$

$$V_2 - V_1 + 12 + 4V_1 = 0$$

$$\boxed{3V_1 + V_2 + 0V_3 = -12}$$

2.XY

At V_3

$$8\left(\frac{V_3 - V_1}{8} + \frac{V_3}{4} + 2I_A = 0\right)$$

$$I_A = -\frac{V_1}{2}$$

$$8\left(\left(\frac{V_3 - V_1}{8}\right) + \frac{V_3}{4} + 2\left(-\frac{V_1}{2}\right) = 0\right)$$

$$V_3 - V_1 + 2V_3 - 8V_1 = 0$$

$$-9V_1 + 0V_2 + 3V_3 = 0$$

$$\begin{bmatrix} 7 & -2 & -1 \\ 3 & 1 & 0 \\ -9 & 0 & 3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \\ 0 \end{bmatrix}$$

$V_1 = -2.4V$ $\quad V_2 = -4.8V$ $\quad V_3 = -7.2V$

$$I_A = \frac{-V_1}{2} = \frac{2.4}{2} = 1.2A$$

$$I_B = \frac{-V_3}{4} = \frac{-(-7.2)}{4} = 1.8A$$

$$I_C = \frac{V_1 - V_3}{8} = \frac{-2.4 - (-7.2)}{8} = 0.6A$$

$$I_D = \frac{V_1 - V_2}{4} = \frac{-2.4 + 4.8}{4} = 0.6A$$

2.XX

For V_A

$V_A = I_B \times 4 = 1.8 \times 4 = 7.2\,V$

For V_B

$V_B = I_C \times 8 = 0.6 \times 8 = 4.8\,V$

For V_C

$V_C = -V_2 = -(-4.8) = 4.8V$

Overall Summary

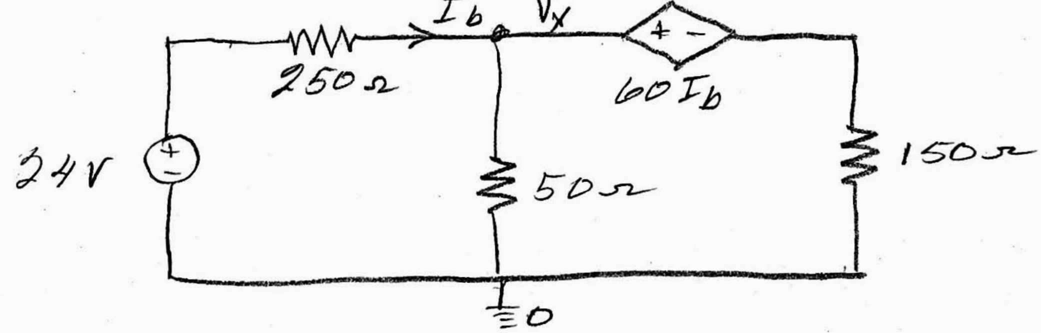
	Part A	Part B	Part C
V_1	4.8V	2.4V	−2.4V
V_2	2.4V	−2.4V	−4.8V
V_3	−2.4V	−4.8V	−7.2V
I_A	1.2A	1.2A	1.2A
I_B	1.8A	1.8A	1.8A
I_C	0.6A	0.6A	0.6A
I_D	0.6A	0.6A	0.6A
V_A	7.2V	7.2V	7.2V
V_B	4.8V	4.8V	4.8V
V_C	4.8V	4.8V	4.8V

Conclusion

Changing reference nodes, changes the reference node voltages, However, All currents and voltages across elements remain the same.

3.9

Determine I_b for the following circuit. Use nodal analysis.



At V_x: Σi leaving = 0; using rises.

$$75\left(\frac{V_x - 24}{250} + \frac{V_x}{50} + \frac{V_x - 60I_b}{150} = 0\right)$$

$$3V_x - 72 + 15V_x + 5V_x - 300I_b = 0$$

$$I_b = \frac{24 - V_x}{250}$$

so

$$23V_x - 300\left(\frac{24 - V_x}{250}\right) = 72$$

$$23V_x - 28.8 + 1.2V_x = 72$$

$$24.2V_x = 100.8$$

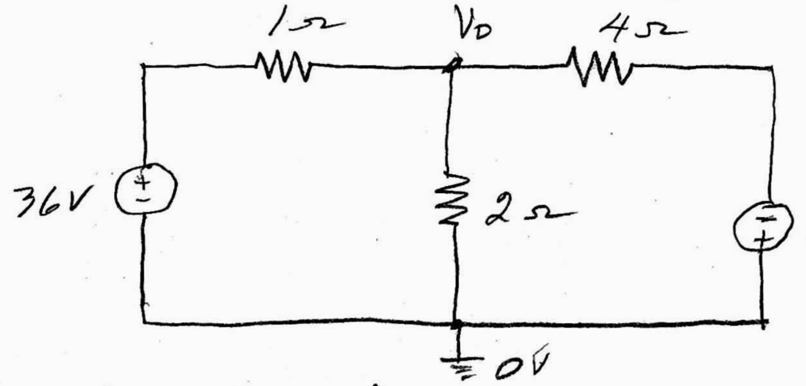
$$V_x = 4.165\text{ V}$$

$$I_b = \frac{24 - V_x}{250} = \frac{24 - 4.165}{250} = 79.3\text{ mA}$$

$$\boxed{I_b = 79.3\text{ mA}}$$

3.11

Find V_o and the power dissipated in all the resistors in the following circuit.



At V_o: Σi leaving = 0; use rises.

$$4\left(\frac{V_o-36}{1} + \frac{V_o}{2} + \frac{V_o+12}{4} = 0\right)$$

$$4V_o - 144 + 2V_o + V_o + 12 = 0$$

$$7V_o = 132$$

$$V_o = 18.86V = 18.9\,V$$

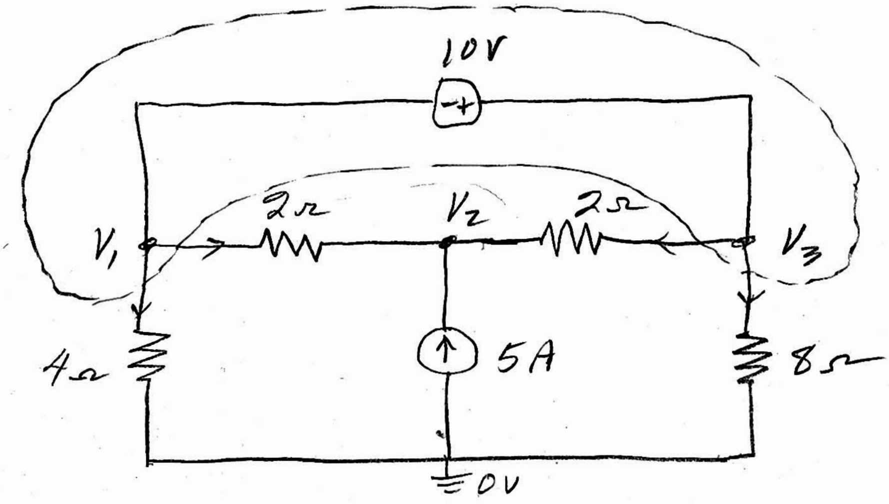
$P_{1\Omega} = (V_o-36)^2 \times 1 = \underline{292.4\,W}$ close enough

$P_{2\Omega} = \frac{V_o^2}{2} = \underline{178.6\,W}$

$P_{4\Omega} = \frac{(V_o+12)^2}{4} = \frac{V^2}{R} = \underline{238.7\,W}$

3.18

Use nodal analysis to determine the node voltages in the following circuit.



There is a super node as indicated.

Σi leaving; using voltage rises.

At the Super node:

$$8\left(\frac{V_1}{4} + \frac{V_1 - V_2}{2} + \frac{V_3 - V_2}{2} + \frac{V_3}{8} = 0\right)$$

$$2V_1 + 4V_1 - 4V_2 + 4V_3 - 4V_2 + V_3 = 0$$

$$\boxed{6V_1 - 8V_2 + 5V_3 = 0}$$

At V_2:

$$2\left(\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} - 5 = 0\right)$$

$$V_2 - V_1 + V_2 - V_3 - 10 = 0$$

$$\boxed{-V_1 + 2V_2 - V_3 = 10}$$

3.18 continued

Constraint Equation

$$V_1 + 10 - V_3 = 0$$

$$\boxed{V_1 + 0V_2 - V_3 = -10}$$

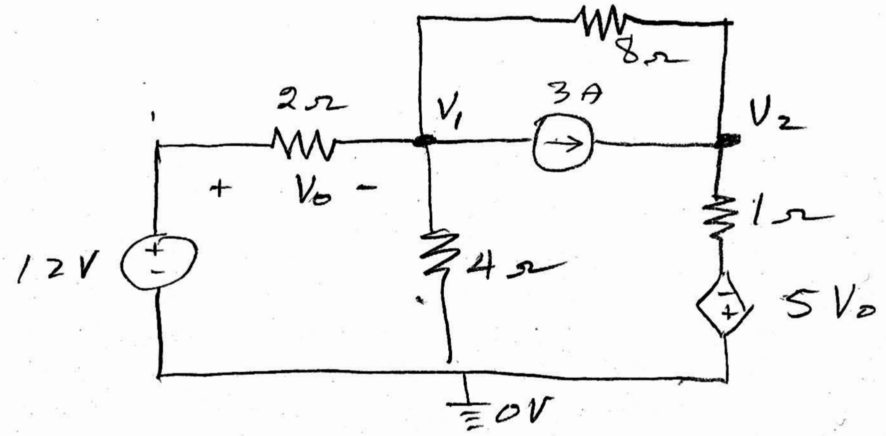
$$\begin{bmatrix} 6 & -8 & 5 \\ -1 & 2 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ -10 \end{bmatrix}$$

$V_1 = 10\,V$; $V_2 = 20\,V$; $V_3 = 20\,V$

3.22

Determine V_1 and V_2 in the following circuit.

Will use nodal analysis.



At V_1: Σi leaving = 0; use voltage rises.

$$8\left(\frac{V_1-12}{2} + \frac{V_1-V_2}{8} + 3 + \frac{V_1}{4} = 0\right.$$

$$4V_1 - 48 + V_1 - V_2 + 24 + 2V_1 = 0$$

$$\boxed{7V_1 - V_2 = +24}$$

At V_2

$$8\left(\frac{V_2-V_1}{8} + \frac{V_2+5V_0}{1} - 3 = 0\right)$$

$$V_2 - V_1 + 8V_2 + 40V_0 - 24 = 0$$

But $V_1 + V_0 - 12 = 0$

$$V_0 = 12 - V_1$$

3.22 cont.

$$-V_1 + 9V_2 + 40(12 - V_1) = 24$$

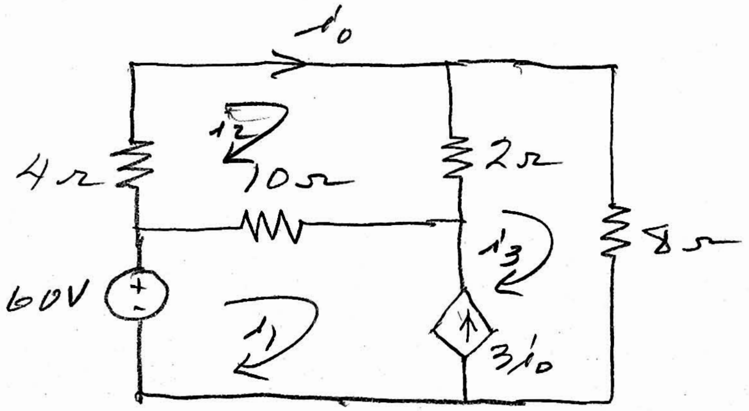
$$-41V_1 + 9V_2 = -456$$

$$\begin{bmatrix} 7 & -1 \\ -41 & 9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} +24 \\ -456 \end{bmatrix}$$

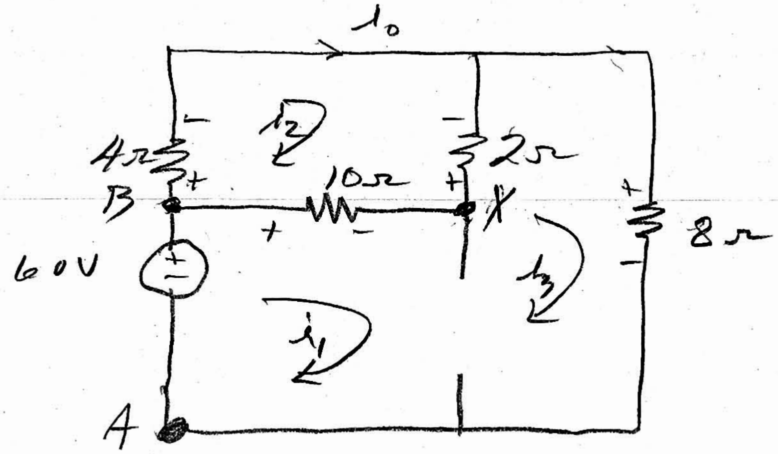
$V_1 = -10.91\,V$; $V_2 = -100.36\,V$

3.50

Use mesh analysis to find current i_o in the circuit below.



Assign mesh current as shown above. Then redraw as shown below with current source omitted.



Write the following mesh equations.

At A: Σ drops = 0, C.W.

$$-60 + 10(i_1 - i_2) + 2(i_3 - i_2) + 8i_3 = 0$$

$$10i_1 - 12i_2 + 10i_3 = 60$$

3.50 cont.

At B

Σ Drops = 0, C.W.

$$4i_2 - 2(i_3 - i_2) - 10(i_1 - i_2) = 0$$

$$\boxed{-10i_1 + 16i_2 - 2i_3 = 0}$$

At X: KCL: Σi entering = 0

$$i_2 - i_3 + i_1 - i_2 + 3i_0 = 0$$

$$i_1 - i_3 + 3i_0 = 0$$

but $i_0 = i_2$

$$\boxed{i_1 + 3i_2 - i_3 = 0}$$

$$\begin{bmatrix} 10 & -12 & 10 \\ -10 & 16 & -2 \\ 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

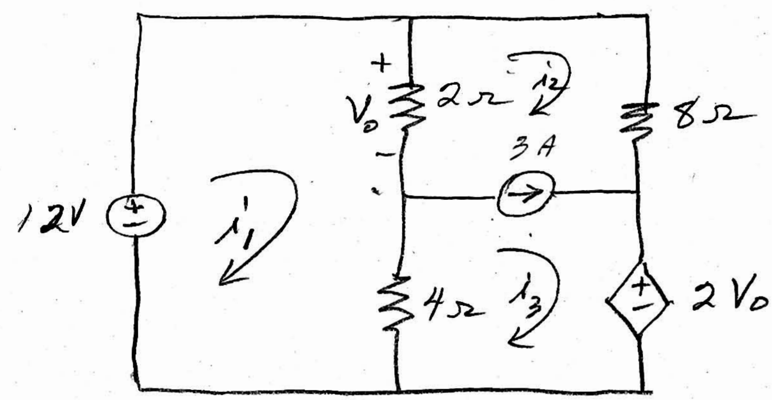
$i_1 = 1.44A$; $i_2 = 1.73A$; $i_3 = 6.63A$

$$\boxed{i_0 = i_2 = 1.73A}$$

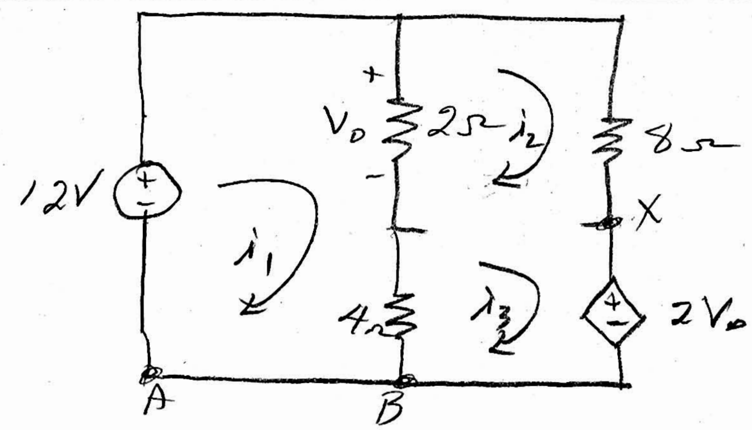
3.52

Use mesh analysis to find i_1, i_2, i_3 in the following circuit.

First assign mesh currents as shown below.



Now remove the current source and write the equations.



At A: KVL; use drops, c.w.

$$-12 + 2(i_1 - i_2) + 4(i_1 - i_3) = 0$$

$$6i_1 - 2i_2 - 4i_3 = 12$$

3.52 cont,

At B: KVL; Σ drops = 0, c.w.

$$4(i_3 - i_1) + 2(i_2 - i_1) + 8i_2 + 2V_0 = 0$$

$$-6i_1 + 10i_2 + 4i_3 + 2V_0 = 0$$

but $V_0 = 2(i_1 - i_2)$

$$-6i_1 + 10i_2 + 4i_3 + 2 \times 2(i_1 - i_2) = 0$$

$$\boxed{-2i_1 + 6i_2 + 4i_3 = 0}$$

At X: KCL: Σ i entering = 0

$$i_2 + 3 - i_3 = 0$$

$$\boxed{0i_1 + i_2 - i_3 = -3}$$

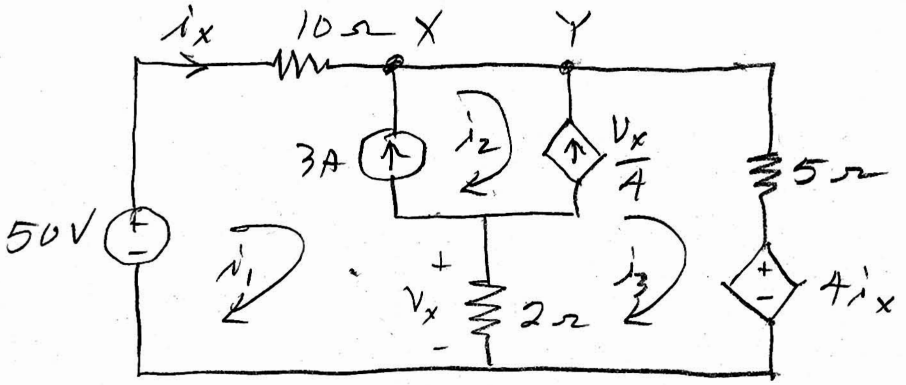
$$\begin{bmatrix} 6 & -2 & -4 \\ -2 & 6 & 4 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -3 \end{bmatrix}$$

$\boxed{i_1 = 3.5\ A}$; $\boxed{i_2 = -0.5\ A}$; $\boxed{i_3 = 2.5\ A}$

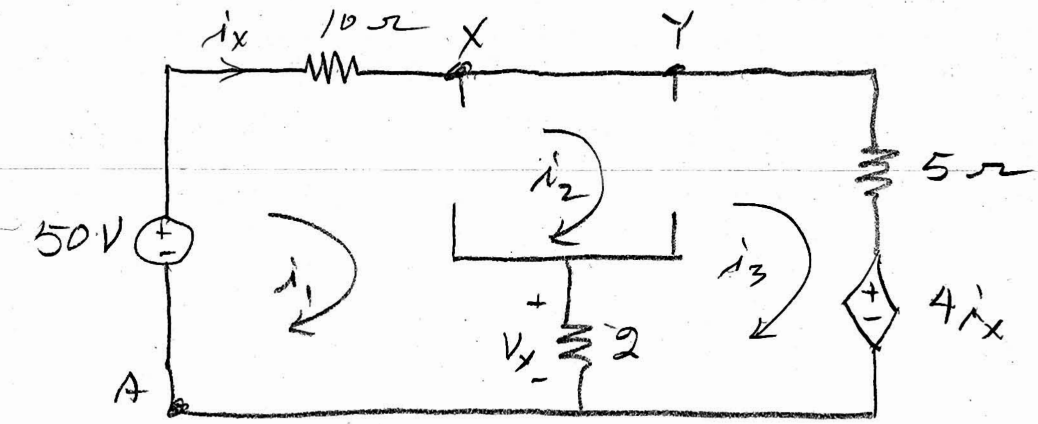
3.63

Find V_x and i_x in the following circuit using mesh analysis.

Assign mesh currents as follows:



Now redraw with current sources removed.



At A: KVL: Σ drops = 0, C.W.

$$-50 + 10i_1 + 5i_3 + 4i_1 = 0 \quad (\text{note}; i_x = i_1)$$

$$14i_1 + 0i_2 + 5i_3 = 50$$

At X: Σ i entering = 0

$$i_1 - i_2 + 3 = 0$$

$$i_1 - i_2 + 0i_3 = -3$$

3.63 cont.

At Y; Σi entering $= 0$

$$i_2 + \frac{V_x}{4} - i_3 = 0.$$

$$V_x = 2(i_1 - i_3)$$

$$i_2 + \frac{2(i_1 - i_3)}{4} - i_3 = 0$$

$$i_2 + 0.5 i_1 - 0.5 i_3 - i_3 = 0$$

$$\boxed{0.5 i_1 + i_2 - 1.5 i_3 = 0}$$

$$\begin{bmatrix} 14 & 0 & 5 \\ 1 & -1 & 0 \\ 0.5 & 1 & -1.5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -3 \\ 0 \end{bmatrix}$$

$i_1 = 2.11\,A$; $i_2 = 5.11\,A$; $i_3 = 4.11\,A$

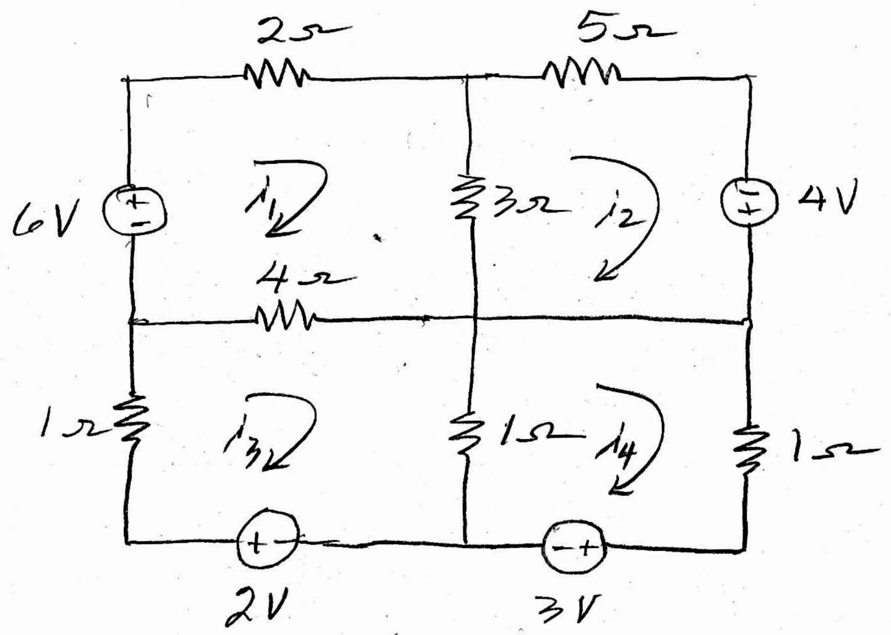
$$\boxed{i_x = i_1 = 2.11\,A}$$

$$V_x = 2(i_1 - i_3) = 2(2.11 - 4.11)$$

$$\boxed{V_x = -4\,V}$$

3.73

Use inspection to write and solve for the mesh currents as shown in the following circuit.



$$\begin{bmatrix} 9 & -3 & -4 & 0 \\ -3 & 8 & 0 & 0 \\ -4 & 0 & 6 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \\ -3 \end{bmatrix}$$

$i_1 = 1.58A$; $i_2 = 1.09A$

$i_3 = 1.24A$; $i_4 = -0.88A$