ECE 300
Spring Semester, 2008
HW Set #5

Due: February 26, 2008 | **Name**__________________________
wlg | Print (last, first)

Check according to your section: __________ 8:10 AM; __________ 11:10 AM

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 10 points.

From the text:

PP 6.7

6.13 Ans: $v_1 = 30$ V, $v_2 = 40$ V

6.26 Ans: (a) $C_{eq} = 35\ \mu F$; (b) $Q_1 = 0.75$ mC, $Q_2 = 1.5$ mC, $Q_3 = 3$ mC; (c) $W = 393.8$ mJ

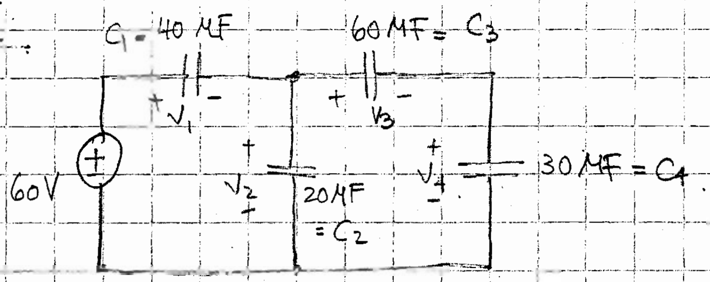
6.46 Ans: $I_L = 2$ A; $v_c = 0$ V; $W_L = 1$ J; $W_C = 0$ J

6.61 Ans: (a) $L_{eq} = 6.67$ mH, $i_{1}(t) = e^{-t}$ mA, $i_2(t) = 2e^{-t}$ mA; (b) $v_o(t) = -20e^{-t}\ \mu V$; (c) $W = 1.35$ nJ

EE 300 SPRING 08

HW #5 Solution

PP 6.7



Find voltage across each capacitor.

Solution:

First, we need to find Ceq

$$\frac{1}{C_{eq_{3,4}}} = \frac{1}{60\,\mu F} + \frac{1}{30\,\mu F} \Rightarrow C_{eq_{3,4}} = 20\,\mu F$$

$$C_{eq_{2,3,4}} = 20\,\mu F + 20\,\mu F = 40\,\mu F$$

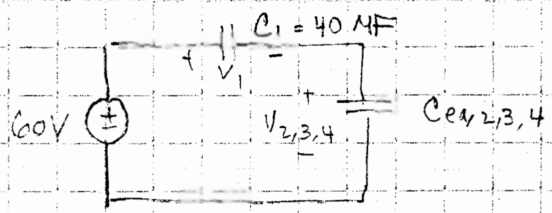
$$\frac{1}{C_{eq}} = \frac{1}{40\,\mu F} + \frac{1}{40\,\mu F} \Rightarrow C_{eq} = 20\,\mu F$$

The total charge is

$$q = V \cdot C_{eq} = 60 \times 20\times10^{-6}\ C = 12\times10^{-4}\ C$$

At C_1: capacitor;

$$q_1 = q = 12\times10^{-4}\ C \Rightarrow V_1 = q/C_1 = \frac{12\times10^{-4}\,C}{40\times10^{-6}\,F} = 30\ V$$



Apply KVL :

$$-60 + V_1 + V_{2,3,4} = 0$$

$$V_{2,3,4} = 60\,V - V_1$$

$$= 60\,V - 30\,V$$

$$= 30\,V$$

At C_2 :

Since C_2 is parallel with C_3 and C_4 ,

$$V_2 = V_{2,3,4} = \underline{\underline{30\,V}}$$

At capacitors C_3 and C_4 :

$$q_{3,4} = q_1 - q_2 \quad , \text{ where } q_2 = C_2 V_2$$

$$= (12 - 6) \times 10^{-4}\,C \qquad = 20 \times 10^{-6} \times 30$$

$$= 6 \times 10^{-4}\,C \qquad = 6 \times 10^{-4}\,C$$

So, since C_3 and C_4 are in series, $q_{3,4} = q_3 = q_4$:

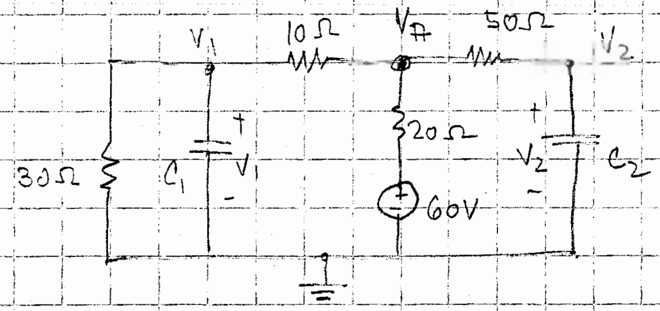
$$V_3 = q_3 / C_3$$

$$= \frac{6 \times 10^{-4}}{60 \times 10^{-6}} = \underline{\underline{10\,V}} = V_3$$

$$V_4 = q_4 / C_4$$

$$= \frac{6 \times 10^{-4}}{60 \times 10^{-6}} = \underline{\underline{20\,V}} = V_4$$

6.13 Find the voltage across the capacitors in the ckt under dc condition.



Solution:

At dc condition, the capacitors are open ckts.

KCL at node V_A:

$$\frac{V_A - V_1}{10} + \frac{V_A - 60}{20} + \frac{V_A - V_2}{50} = 0$$

$$\Rightarrow 10(V_A - V_1) + 5(V_A - 60) + 2(V_A - V_2) = 0$$

$$17V_A - 10V_1 - 2V_2 = 300 \quad \text{--- (1)}$$

at node V_1:

$$\frac{V_1 - V_A}{10} + \frac{V_1}{30} = 0$$

$$\Rightarrow 40V_1 - 30V_A = 0 \quad \text{--- (2)}$$

at node V_2

$$\frac{V_2 - V_A}{50} = 0$$

$$\Rightarrow -V_A + V_2 = 0 \quad \text{--- (3)}$$

Solving eqns (1), (2), & (3), we have

$V_A = 40$ V

$V_1 = 30$ V

$V_2 = 40$ V.

$\therefore$ $V_1 = 30$ V

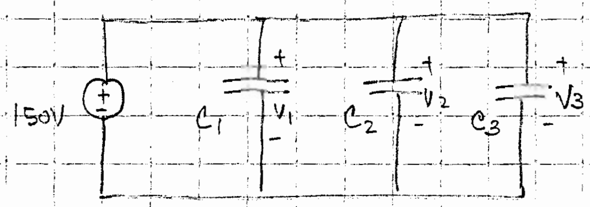
$V_2 = 40$ V

6.26

Three capacitors $C_1 = 5\,\mu F$, $C_2 = 10\,\mu F$, $C_3 = 20\,\mu F$ are connected in parallel across a 150 V voltage source.
Find (a) total capacitance
(b) charge of each capacitor
(c) total energy stored in the parallel combination

Solution :

(a) $C_{eq} = C_1 + C_2 + C_3$
$= (5+10+20)\,\mu F$
$= 35\,\mu F$



(b) $q_1 = C_1 V_1$, $V_1 = V_2 = V_3 = V$ since they are in parallel
$= (5\times10^{-6})(150)$
$= 7.5\times10^{-4}\,C = 0.75\,mC$

$q_2 = C_2 V_2$
$= (10\times10^{-6})(150)$
$= 15\times10^{-4}\,C = 1.5\,mC$

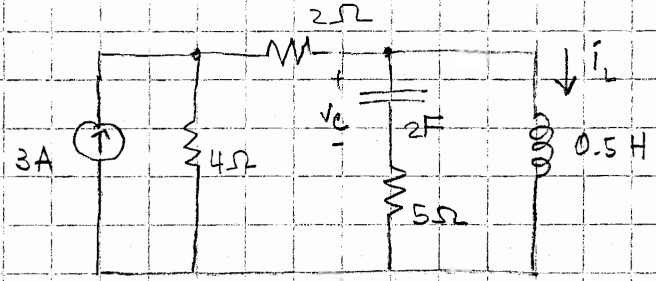
$q_3 = C_3 V_3$
$= (20\times10^{-6})(150)$
$= 30\times10^{-4}\,C = 3\,mC$

(c) Total energy stored:

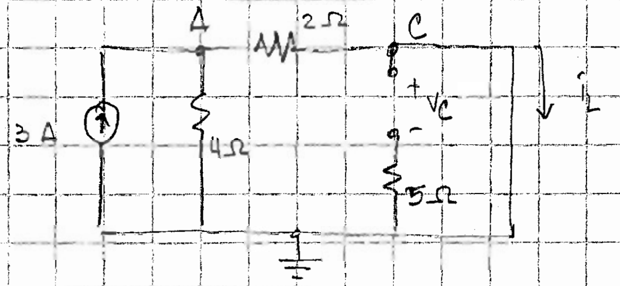
$$W = \frac{1}{2} C_{eq} V^2$$
$$= \frac{1}{2}(35\times10^{-6})(150)^2$$
$$= 0.39375\,J$$
$$\approx 393.75\,mJ$$

6.46.

Find V_c, i_L and the energy stored in the capacitor & inductor in the circuit below under dc condition



Solution   Under dc condition, capacitor is replaced by open ckt & inductor by short ckt



By observation, node C is connected directly to the reference (gnd) therefore $V_c = 0$

Since there is no current flows through 5Ω, we can apply current division:

$$i_L = i_{2\Omega} = (3\,A)\frac{4}{4+2}$$

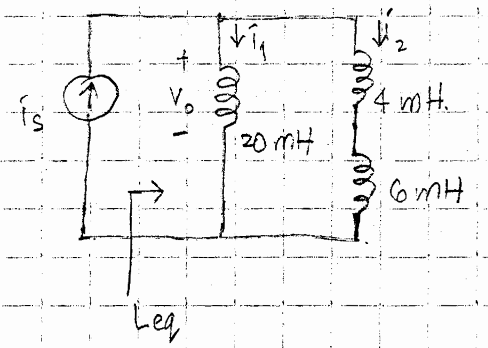
$$i_L = 2\,A$$

Energy stored:

$$W_c = \frac{1}{2} C V_c^2 = \frac{1}{2}(2)(0)^2 = 0\,J$$

$$W_L = \frac{1}{2} L i_L^2 = \frac{1}{2}(0.5)(2)^2 = 1\,J$$

6.61



Find :

(a) L_{eq}, $i_1(t)$, $i_2(t)$ if $i_s = 3e^{-t}$ mA

(b) $V_o(t)$

(c) energy stored in the 20 mH at t = 1 sec.

Solution.

(a) $L_{eq} = 20\text{ mH} // (4+6)\text{mH}$

$$= \frac{20 \times 10}{20+10} = \underline{6.667 \text{ mH}}$$

By current division :

$$i_1(t) = i_s \frac{(4+6)}{(4+6+20)}$$

$$= (3e^{-t} \text{ mA})(10/30)$$

$$i_1(t) = \underline{e^{-t} \text{ mA}}$$

$$i_2(t) = (3e^{-t} \text{ mA}) \frac{20}{30}$$

$$= \underline{2e^{-t} \text{ mA}}$$

(b) $$V_o(t) = L_{20} \frac{di_1(t)}{dt} = 20 \times 10^{-3} \frac{d}{dt}(e^{-t} \times 10^{-3})$$

$$= -20\, e^{-t} \times 10^{-6} \text{ V}$$

$$V_o(t) = \underline{-20\, e^{-t}\, \mu F}$$

(c) Energy stored at t = 1sec in 20 mH inductor :

$$W_{20} = \frac{1}{2} L_{20} (i_1(t))^2 \Big|_{t=1} = \frac{1}{2}(20 \times 10^{-3})(e^{-} \times 10^{-3})^2$$

$$= 1.35 \times 10^{-9}\text{ J} = \underline{1.35 \text{ nJ}}$$