Due: February 26, 2008

Name__________________

Print (last, first)

Check according to your section: ________8:10 AM; ________11:10 AM

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers. Each problem counts 10 points.

From the text:

PP 6.7

6.13 Ans: $v_1 = 30 \text{ V}$, $v_2 = 40 \text{ V}$

6.26 Ans: (a) $C_{eq} = 35 \mu F$; (b) $Q_1 = 0.75 \text{ mC}$, $Q_2 = 1.5 \text{ mC}$, $Q_3 = 3 \text{ mC}$; (c) $W = 393.8 \text{ mJ}$

6.46 Ans: $L = 2 \text{ A}$; $v_e = 0 \text{ V}$; $W_L = 1 \text{ J}$; $W_C = 0 \text{ J}$

6.61 Ans: (a) $L_{eq} = 6.67 \text{ mH}$, $i_{1(0)} = e^{-t} \text{ mA}$, $i_d(t) = 2e^{-t} \text{ mA}$; (b) $v_d(t) = -20e^{-t} \mu V$; (c) $W = 1.35 \text{ nJ}$
Find voltage across each capacitor.

Solution:

First, we need to find \( C_{eq} \).

\[
\frac{1}{C_{eq}} = \frac{1}{C_{1}} + \frac{1}{C_{2}} + \frac{1}{C_{3}}
\]

\[
C_{eq} = \frac{1}{\frac{1}{C_{1}} + \frac{1}{C_{2}} + \frac{1}{C_{3}}}
\]

\[
C_{eq} = \frac{1}{\frac{1}{10\,\text{uF}} + \frac{1}{20\,\text{uF}} + \frac{1}{20\,\text{uF}}} = 10\,\text{uF}
\]

The total charge is

\[
Q = V \cdot C_{eq} = 60 \times 10^{-6} \cdot C = 12 \times 10^{-6} \cdot C
\]

At \( C_{1} \): capacitor,

\[
Q = \left( 12 \times 10^{-6} \right) \cdot C \Rightarrow \frac{V_{1}}{C_{1}} = \frac{12 \times 10^{-6}}{4 \times 10^{-6}} \Rightarrow 50V
\]
\[ V_1 = 10 \text{ V} \]

Applying KVL:

\[ -50 + V_1 + V_{1,3,4} = 0 \]
\[ V_{1,3,4} = 50 \text{ V} = V_1 \]
\[ V_1 = 50 \text{ V} \]

Since \( C_3 \) is parallel with \( C_3 \) and \( C_4 \),

\[ V_L = V_{1,3,4} = 50 \text{ V} \]

At capacitors \( C_3 \) and \( C_4 \):

\[ \frac{Q_3}{Q_4} = \frac{C_3}{C_4} \]

Where \( Q_3 = C_3 V_3 \)

\[ \frac{(12 - 6) \times 10^{-6}}{60 \times 10^{-6}} \]

\[ = 0.5 \times 10^{-6} \times 30 \]

\[ = 0 \times 10^{-4} \text{ C} = 0 \times 10^{-4} \text{ C} \]

So, since \( C_3 \) and \( C_4 \) are in series, \( Q_3 = Q_4 = Q_4 \)

\[ V_3 = \frac{Q_3}{C_3} \]

\[ = \frac{0 \times 10^{-6}}{60 \times 10^{-6}} = 0 \text{ V} = V_3 \]

\[ V_4 = \frac{Q_4}{C_4} \]

\[ = \frac{0 \times 10^{-6}}{60 \times 10^{-6}} = 0 \text{ V} = V_4 \]
0.13 Find the voltage across the capacitors in the circuit under dc condition.

Solution

At dc condition, the capacitors are open.

KCL at node Va:

\[
\begin{align*}
V_a + V_1 &= 60 + V_4 - V_2 = 0 \\
10 &\quad 20 &\quad 60 \\
&\quad 0 &\quad 10 &\quad 20 &\quad 60 \\
&= 0 \\
&= 10 \cdot (V_a - V_1) + 5 \cdot (V_a - V_4) + 2 \cdot (V_a - V_2) = 0
\end{align*}
\]

At node V1:

\[
\begin{align*}
V_1 + V_a &= 0 \\
0 &\quad 30 &\quad 60 \\
&\quad 0 &\quad 30 &\quad 60 \\
&= 0 \\
&= 30 \cdot V_a + 60 \cdot V_2 = 0
\end{align*}
\]

At node V2:

\[
\begin{align*}
V_2 - V_a &= 0 \\
60 &\quad 0 &\quad 30 \\
&\quad 0 &\quad 60 &\quad 30 \\
&= 0 \\
&= -60 \cdot V_2 + 30 \cdot V_4 = 0
\end{align*}
\]

Solving these equations, we have:

\[
\begin{align*}
V_a &= 40 \text{ V} \\
V_1 &= 30 \text{ V} \\
V_2 &= 40 \text{ V} \\
V_4 &= 50 \text{ V}
\end{align*}
\]
The capacitors $C_1 = 2 \mu F$, $C_2 = 10 \mu F$, $C_3 = 20 \mu F$ are connected in parallel across a 150 V voltage source.

Find:
(a) total capacitance
(b) charge of each capacitor
(c) total energy stored in the parallel combination

Solution:

(a) $C_{eq} = C_1 + C_2 + C_3 = (2 + 10 + 20) \mu F = 32 \mu F$

(b) $q_1 = C_1 V_1 = (2 \times 10^{-6})(150) = 3 \times 10^{-4} C = 0.35 \text{ mC}$

$\quad q_2 = C_2 V_2 = (10 \times 10^{-6})(150) = 1.5 \text{ mC}$

$\quad q_3 = C_3 V_3 = (20 \times 10^{-6})(150) = 3 \text{ mC}$

(c) Total energy stored

$W = \frac{1}{2} C_{eq} V^2$

$\quad = \frac{1}{2} (32 \times 10^{-6})(150)^2$

$\quad = 0.39575 \text{ J}$

$\quad = 0.39575 \text{ mJ}$
Find $v_c$, $i_L$, and the energy stored in the capacitor & inductor
in the circuit below, under the condition

\[ V_n = 6 \text{V}, \quad L = 14 \text{H}, \quad C = 6 \text{F} \]

\[ B = E = 8.5 \text{H} \]

\[ \delta A = 1 \]

Solution: Under the condition, capacitor is replaced by open circuit &
inductor by short circuit.

\[ V_n = 6 \]

\[ B = E = 8.5 \]

\[ \delta A = 1 \]

By observation, node C is connected directly to the battery (4V).
Hence $v_c = 0$.

Since there is no current flows through $E_m$, we can apply current
division:

\[ i_m = \frac{1}{2} i_L - \frac{1}{2} (3 A) \]

\[ i_L = \frac{2 A}{3} \]

Energy stored:

\[ W_c = \frac{1}{2} C V^2 = \frac{1}{2} (2 \text{F})(0)^2 = 0 \text{J} \]

\[ W_L = \frac{1}{2} L i^2 = \frac{1}{2} (8.5 \text{H})(2)^2 = 14 \text{J} \]
Solution

(a) \[ I_{dc} = \frac{20 \times 10^{-3}}{20 + 5} = 0.667 \text{ mA} \]

By current division:

\[ I_1(t) = \frac{I_{dc}}{2 + 5} (\text{mA}) \]
\[ I_2(t) = \frac{3e^{-5t}}{20} \text{ mA} \]

(b) \[ V_{dc}(t) = \frac{20 \times 10^{-3}}{20 - 5} (e^{-t} \times 10^{-3}) \]
\[ V_{dc}(t) = -20e^{-t} \text{ V} \]

(c) Energy stored in \( 20 \text{ mH} \) at \( t = 1 \text{ sec} \):

\[ W = \frac{1}{2} \int_{t=0}^{1} (I(t))^2 dt = \frac{1}{2} \left( 20 \times 10^{-3} \right)^2 (e^{-1} \times 10^{-3}) \]
\[ W = 1.35 \times 10^{-9} \text{ J} = 135 \mu\text{J} \]