Desk Copy

ECE 300
Spring Semester, 2008
HW Set #7

Due: March 6, 2008
wlg

Name wlg
Print (last, first)

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.**

Check according to your section: __________ 8:10 AM; _________ 11:10 AM

From the text:

7.42 Ans: (a) $v_o(t) = 8(1-e^{-0.25t})\ u(t)$ V — problem is 20%
(b) $v_0(t) = 8e^{-t/12}\ u(t)$ V

7.44 Ans: $i(t) = -3e^{-0.25t}\ u(t)$ V — problem is 15%

7.53 Ans: (a) $5e^{-t/2}\ u(t)$ A — problem is 20 %
(b) $I(t) = 6e^{-2t/3}\ u(t)$ A

7.59 Ans: $v_0(t) = 6e^{-4t}\ u(t)$ V — problem is 20%

In Addition: Use p-spice to obtain a plot of $v_o(t)$ out to six time constants. Submit your circuit (your p-spice circuit) with your name in the caption. Also, submit the time response curve for $v_o(t)$ as obtained from p-spice.

wlg

ECE 300
HW # 7
SPRING 2008

7.42

(a) The switch in the circuit below has been open for a long time and is closed at $t=0$. Find $V_o(t)$.

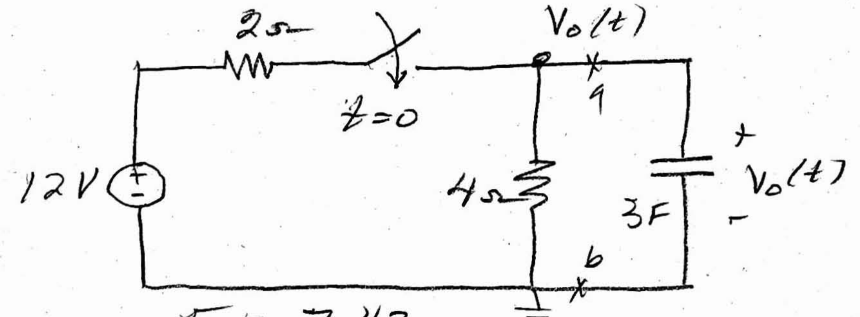


Fig 7.42

With the switch open, $V_o(0^-) = 0$.

After the switch is closed we can write the following nodal equation.

$$\frac{V_o - 12}{2} + \frac{V_o}{4} + 3\frac{dV_o}{dt} = 0$$

$$3\frac{dV_o}{dt} + \frac{V_o}{2} + \frac{V_o}{4} = 6$$

$$3\frac{dV_o}{dt} + \frac{3}{4}V_o = 6$$

$$\frac{dV_o}{dt} + 0.25V_o = 2$$

$$V_o(t) = V_p(t) + V_c(t)$$

7.42 cont.

$$V_c = k_c e^{-0.25t}$$

$$V_p = k_p$$

$$\frac{dk_p}{dt} + 0.25 k_p = 2$$

$$k_p = 8$$

$$\therefore \quad V_o(t) = k_c e^{-0.25t} + 8$$

$$V_o(0^+) = V_o(0^-) = 0 = k_c + 8$$

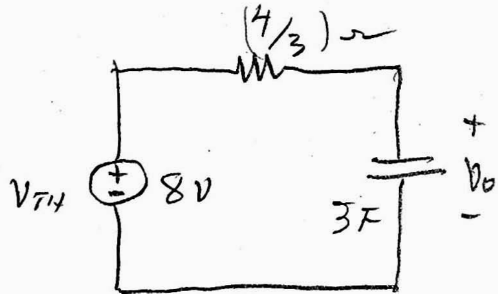
so

$$0 = k_c + 8$$

$$k_c = -8$$

$$V_o(t) = 8(1 - e^{-0.25t})\, u(t) \text{ V}$$

Also, after the switch is closed you can make a Thevenin equivalent to the left of a-b. This gives



$$R_{eq}C = \tau = \frac{4}{3} \times 3 = 4 \text{ sec}$$

$$K_{ss} = 8 \text{ V}$$

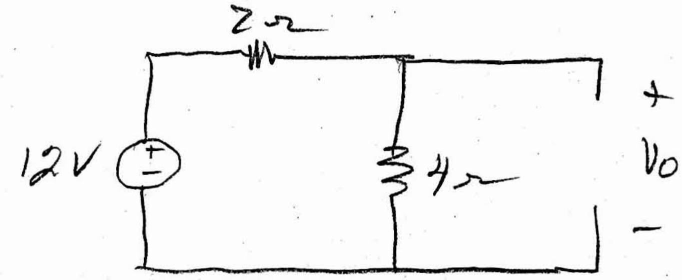
$$\therefore \quad V_o(t) = 8 - 8e^{-0.25t}\, u(t) \text{ V}$$

7.42 cont

(b) Consider that the switch in Fig 7.42 has been closed for a very long time and is opened at $t=0$. Find $V_o(t)$.

For $t<0$

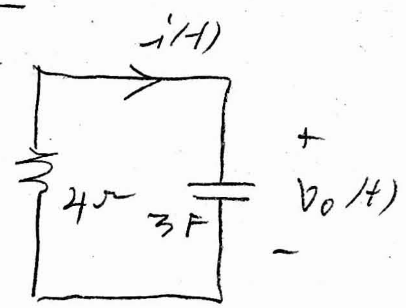
We have



$$V_o(0^-) = \frac{12\times 4}{4+2} = 8V$$

Now $\underline{V_o(0^+)} = V(0^-) = \underline{8V}$ I.C.

For $t>0$



The solution is of the form

$$4i + V_o = 0$$

$$4\times 3\frac{dV_o}{dt} + V_o = 0$$

$$\frac{dV_o}{dt} + \frac{V_o}{12} = 0$$

7.42 cont,

so

$$V_0 = K e^{-\frac{t}{12}}$$

since $V_0(0^+) = 8$, $K = 8$

$$V_0(t) = 8 e^{-\frac{t}{12}} u(t) \text{ V}$$

$$\boxed{V_0(t) = 8 e^{-\frac{t}{12}} u(t) \text{ V}}$$

7.44

The switch in the following diagram has been in position a for a very long time and at $t=0$ is moved to position b. Find $i(t)$ for $t>0$.

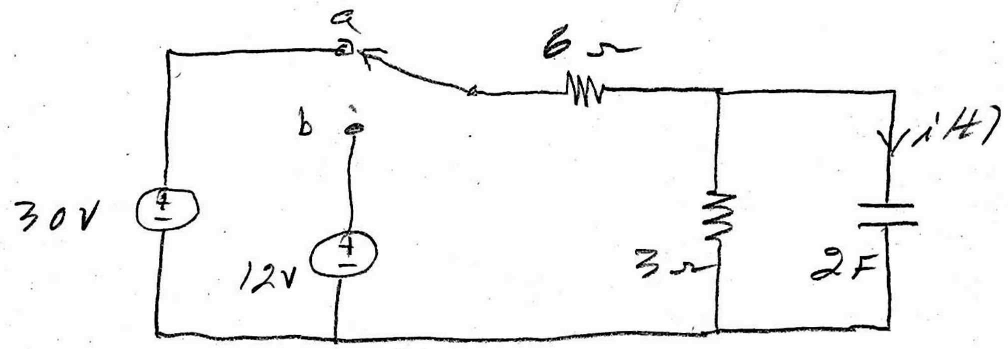
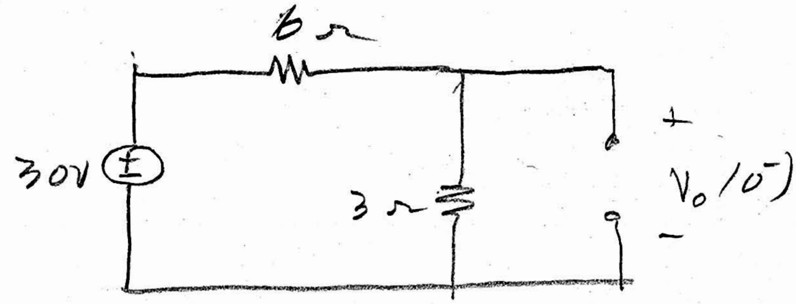


Fig 7.44 (a)

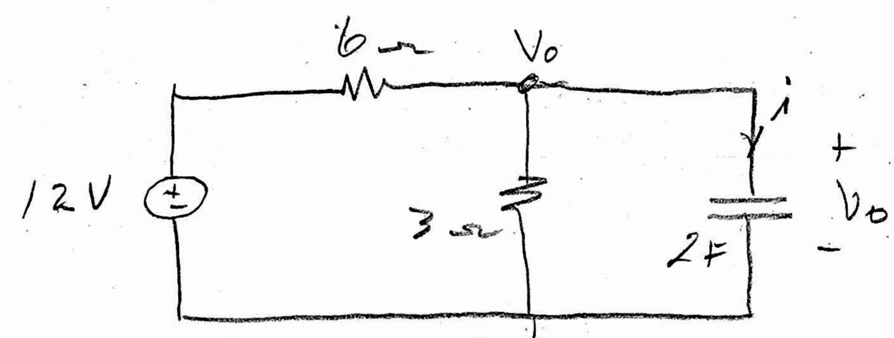
For $t<0$



$$V_o(0^-) = \frac{30 \times 3}{3+6} = 10\,V$$

So $V_o(0^+) = V_o(0^-) = 10V$

For $t>0$



7.44

Writing a node equation at V_0 gives

$$C\left(\frac{V_0 - 12}{6} + \frac{V_0}{3} + i = 0\right)$$

$$V_0 - 12 + 2V_0 + 6i = 0$$

$$3V_0 + 6i = 36$$

$$V_0 = \frac{1}{C}\int_0^t i(t)\,dt + i(0^+)$$

$$\frac{d}{dt}\left[\frac{3}{2}\int_0^t i\,dt + i(0^+) + 6i = 12\right]$$

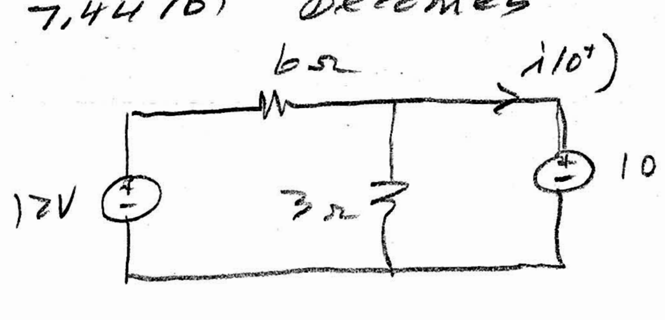
$$\frac{3}{2}i(t) + 6\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{1}{4}i(t) = 0$$

$$i(t) = K e^{-0.25t} \qquad (A)$$

$$K = i(0^+)$$

18/6 Since there is an initial voltage of 10V on the capacitor, the circuit of Fig 7.44(b) becomes



7.44 cont.

Making a Thevenin equivalent in Figure 7.44 (c) gives

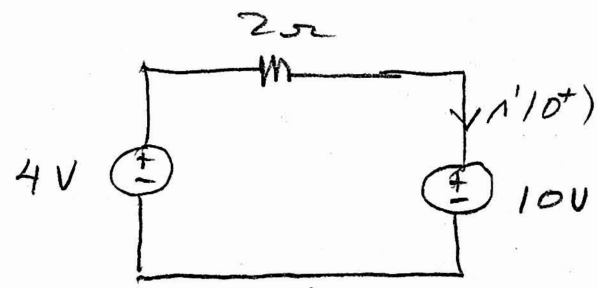


Fig 7.44 (d)

We have

$$-4 + 2i'(0^+) + 10 = 0$$

$$i'(0^+) = -3\ A$$

so, back to (A)

$$i(t) = -3e^{-0.25t}\,u(t)\ A$$

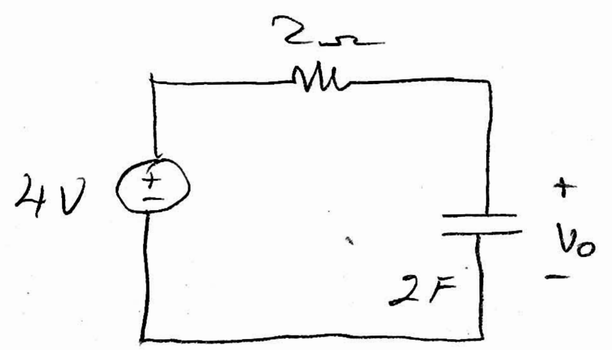
It would have been an easier solution to have solved for $V_o(t)$ then use

$$i(t) = C\,\frac{dV_o}{dt}$$

A procedure for solving for $V_o(t)$ is now given.

7.44 cont

We make a Thevenin equivalent of Figure 7.44 (b)



For this circuit we have

$$\frac{dv_o}{dt} + \frac{v_o(t)}{RC} = \frac{V_i}{RC}$$

$R = 2, \quad C = 2, \quad RC = 4 \quad V_i = 4$

$$v_o(t) = k_c e^{-\frac{t}{4}} + 4$$

at $t = 0^+$

$$v_o(0^+) = 10 = k_c + 4$$

$$k_c = 6$$

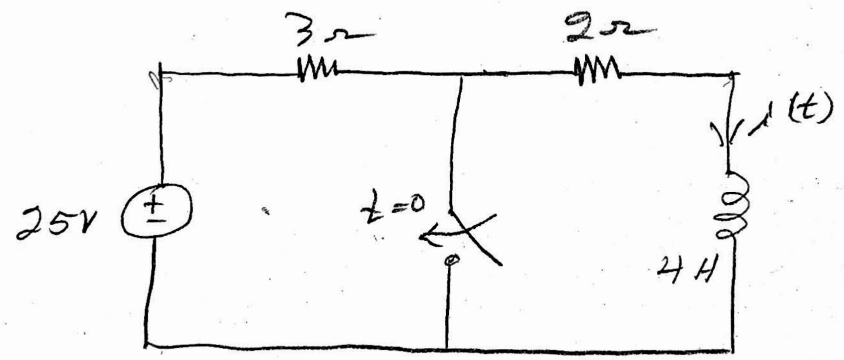
$$v_o = 6e^{-\frac{t}{4}} + 4$$

$$i = 2 \times \frac{dv_o}{dt} = 2 \times 6 \left(-\frac{1}{4}\right) e^{-0.25t}$$

$$\boxed{i(t) = -3e^{-0.25t} u(t) \text{ A}}$$

7.53

(a) Determine the inductor current $i(t)$ for both $t<0$ and $t>0$ for the following circuit.



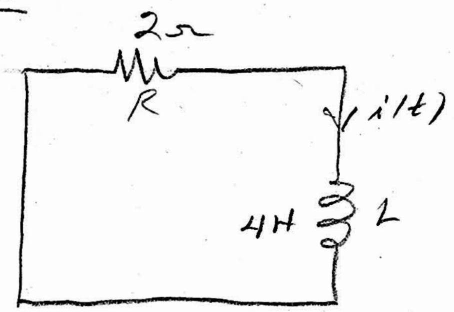
FOR $t<0$

The inductor looks like a short circuit for steady state so

$$i(0^-) = \frac{25}{5} = 5A = i(0^+)$$

i(t) thru L can't inst.

FOR $t>0$



$$L\frac{di}{dt} + Ri = 0$$

$$4\frac{di}{dt} + 2\,i(t) = 0$$

$$\frac{di}{dt} + \frac{1}{2}\,i(t) = 0$$

(A)

7.53 cont.

The solution for (A) is

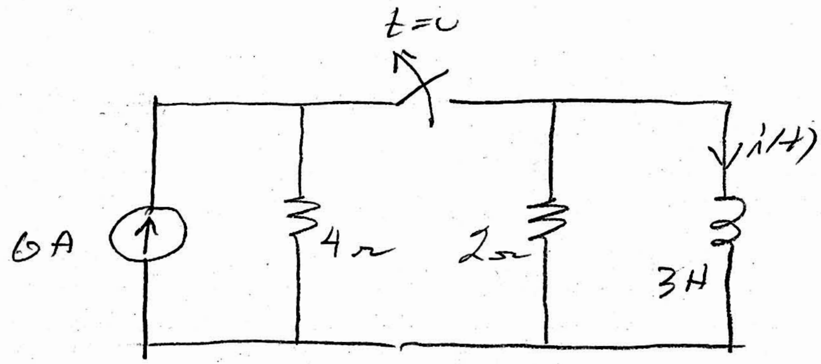
$$i(t) = i(0^+) e^{-0.5t} u(t) \text{ A}$$

we know $i(0^+) = 5$

so

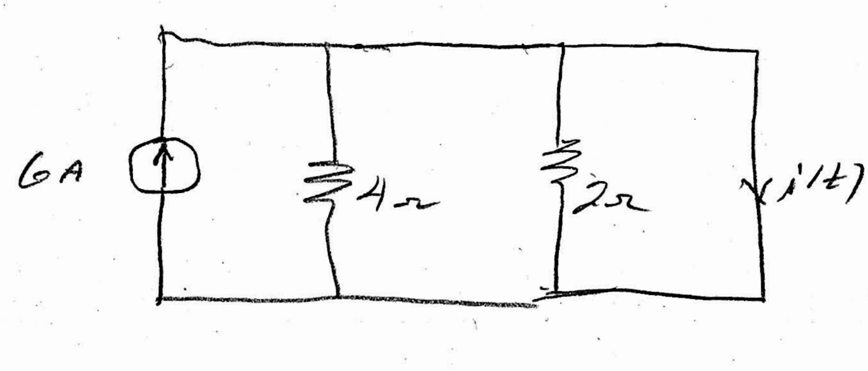
$$i(t) = 5 e^{-0.5t} u(t) \text{ A}$$

(b) Determine the inductor current $i(t)$ for both $t<0$ and $t>0$ for the following circuit



For $t<0$

we have

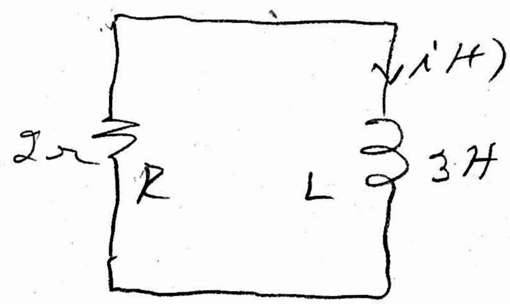


7.53

both the 2Ω and 4Ω resistors are shorted. Therefore

$$i(0^-) = 6A = i(0^+)$$ i cannot change inst. thru inductor

FOR $t>0$



We have

$$Ri + L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L}i(t) = 0 \qquad R = 2\Omega,\ L = 3\Omega$$

$$\frac{di}{dt} + \frac{2}{3}i(t) = 0$$

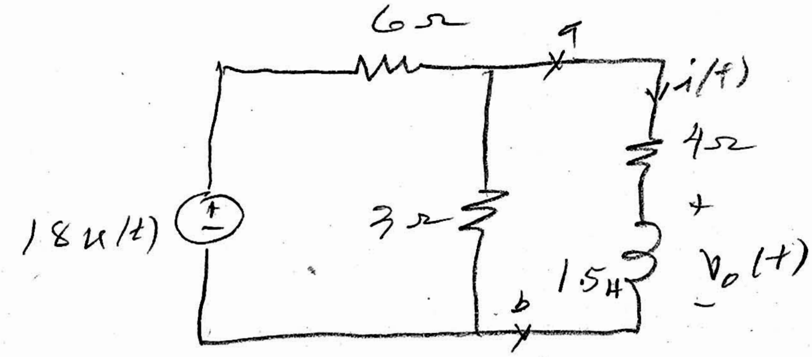
$$i(t) = i(0^+)e^{-\frac{2}{3}t}u(t)\ A$$

$$i(0^+) = 6A$$

$$\therefore \boxed{i(t) = 6e^{-\frac{2}{3}t}u(t)\ A}$$

7.59

Determine the step response for $v_o(t)$ if $v_s = 18u(t)$.

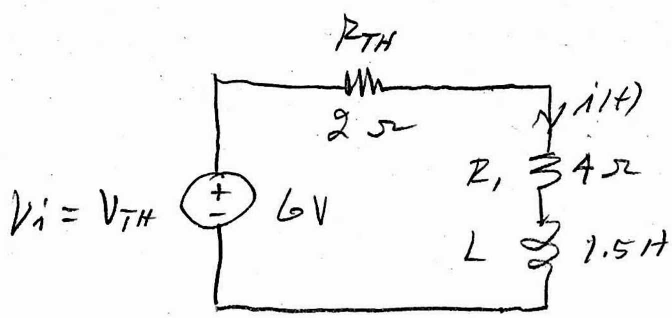


<u>For $t<0$:</u>

The circuit is at rest. $i(0^-)=0$. Since current thru an inductor cannot change instantaneously, $i(0^+)=i(0^-)=0$

<u>For $t>0$</u>

It looks like the easiest thing to do is make a Thevenin equivalent to the left of a-b. This gives



We have

$$-V_i + i(t)\left(R_{TH}+R_1\right) + L\frac{di}{dt} = 0$$

7.59 cont.

putting in numbers,

$$1.5\frac{di}{dt} + 6i(t) = 6$$

$$\frac{di}{dt} + 4i(t) = 4$$

$$i(t) = i_p + i_c$$

$$i_c = K_c e^{-4t}$$

$$i_p = K_p$$

$$4K_p = 4$$

$$K_p = 1$$

$$i(t) = K_c e^{-4t} + 1$$

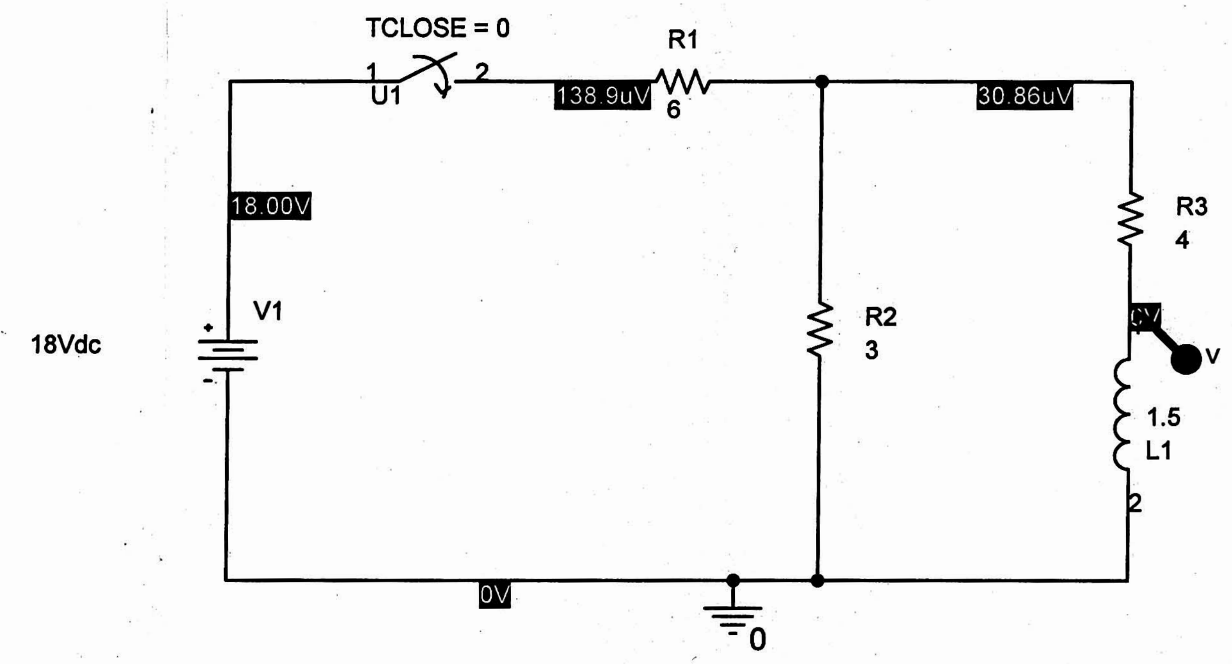
$$i(0^-) = 0 = K_c + 1$$

$$K_c = -1$$

$$i(t) = 1 - e^{-4t}$$

$$V_o = L\frac{di}{dt} = 1.5(-)(-4)e^{-4t}$$

$$\boxed{V_o(t) = 6e^{-4t}u(t)\ \text{V}}$$



Circuit Diagram for Problem 7_59. Spring 2008 W. Green: Under HW7_59 Office Laptop.

** Profile: "SCHEMATIC1-Bias10" [C:\ORCAD\ORCAD_10.0_DEMO\HW7_59-PSpiceFiles\SCHEMATIC1\Bias10.sim]
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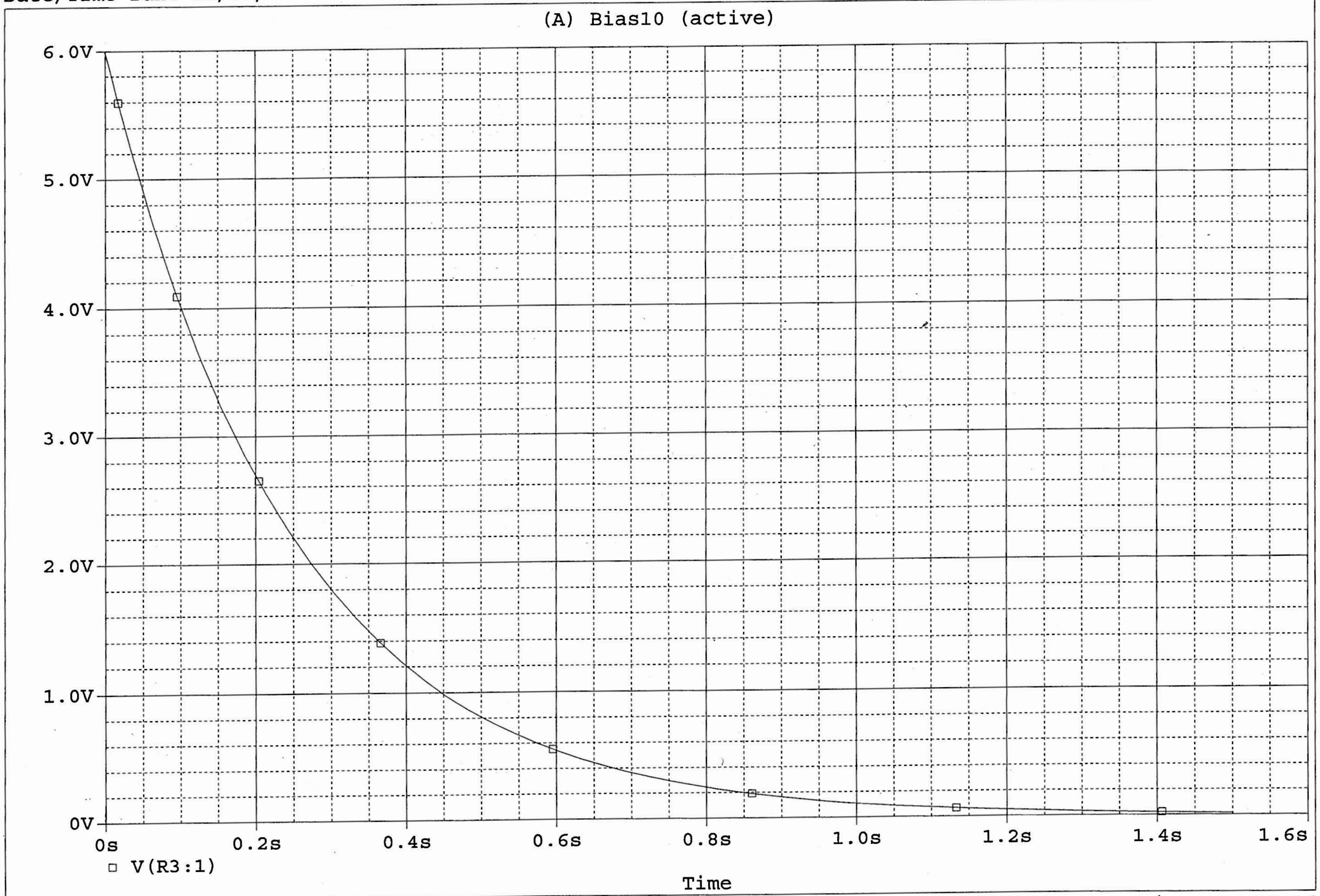

Date: March 05, 2008 Page 1 Time: 10:58:46