Desk Copy

ECE 300
Spring Semester, 2008
HW Set #11

Due: April 3, 2008

Name wlg
Print (last, first)

wlg

Check according to your section: ________ 8:10 AM; ________ 11:10 AM

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations. Be sure to show how you got your answers.** Each problem counts 10%, except problem (10.55) counts 20%.

This is a minimum problem set. You will probably want to work other problems to refine your uderstanding of A.C. circuit analysis.

(9.42) Basic Concepts: Voltage Division
Ans: $v_o(t) = 17.14\cos(200t)$ V

(9.47) Basic Concepts: Impedance
Ans: $i_s(t) = 460.7\cos(2000t + 52.6^\circ)$ mA

(9.50) Basic Concepts: Current Division
Ans: $v_x(t) = 50\cos(100t - 50^\circ)$ V

(9.60) Basic Concepts: Impedance (calculator exercise)
Ans: $\mathbf{Z_{in}} = (51.1 + j9.88)\ \Omega$

(9.64) Basic Concepts: Impedance
Ans: $\mathbf{Z_T} = (19 - j5)\ \Omega$, $\mathbf{I} = 1.53\angle 104.7$ A

✓(10.14) Nodal Analysis
Ans: $\mathbf{V_1} = 28.9\angle 135.4^\circ$ V; $\mathbf{V_2} = 49.18\angle 124.1^\circ$ V

✓(10.17) Nodal Analysis
Ans: $\mathbf{I_o} = 9.25\angle\text{-}162.1$ A

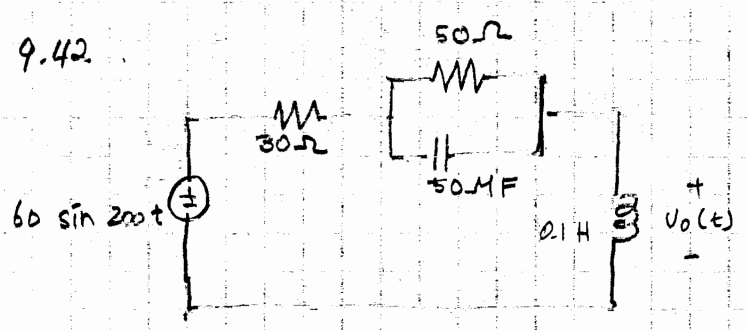
✓(10.31) Mesh Analysis
Ans: $\mathbf{I_o} = 2.18\angle 61.4^\circ$ A

✓(10.38) Mesh Analysis
Ans: $\mathbf{I_o} = 3.35\angle 174.3$ A

✓(10.55) Thevenin and Norton
In finding the Norton circuit, you are to do so by finding the open-circuit voltage and short-circuit Current. Draw both the Thevenin and Norton equivalent circuits.
Ans: (a) $\mathbf{Z_{TH}} = \mathbf{Z_N} = 22.36\angle\text{-}63.4\ \Omega$; $\mathbf{V_{TH}} = \text{-}50\angle 30^\circ$ V; $\mathbf{I_N} = 2.24\angle 273.4^\circ$ A
(b) $\mathbf{Z_{TH}} = \mathbf{Z_N} = 10\angle 26^\circ\ \Omega$; $\mathbf{V_{TH}} = 33.92\angle 58^\circ$ V; $\mathbf{I_N} = 3.39\angle 32^\circ$ A

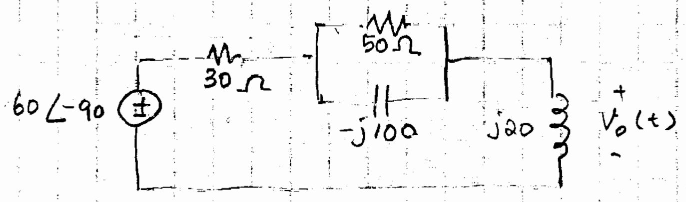
HW # 11 Solution
ECE 300 Sp 08

\# 9.42



Calculate $v_o(t)$

In phasor representation : $\omega = 200$



$$50 // -j100 = \frac{(50)(-j100)}{50-j100} = 40 - j20$$

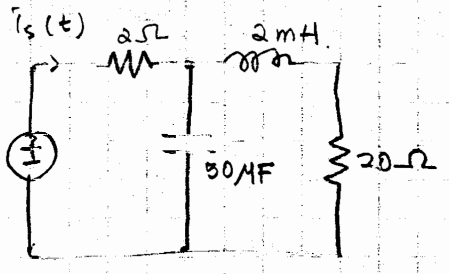
$$V = \frac{j20}{30 + j20 + 40 - j20}(60\angle -90)$$

$$= \frac{j20}{70}(60\angle -90^\circ)$$

$$= 17.14\angle 0^\circ$$

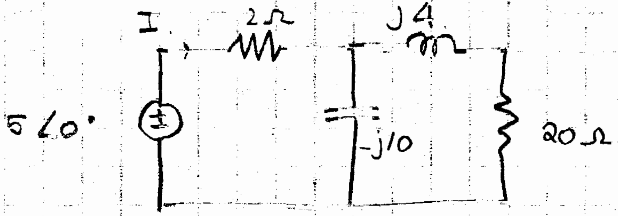
$$\therefore\ V_o(t) = 17.14 \cos(200t)\ \text{V}$$

9.47



Calculate $i_s(t)$

In phasor representation: $\omega = 2000$



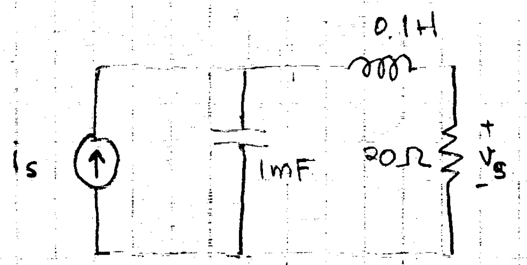
$$-j10 \,//\, (20 + j4) = \frac{(-j10)(20+j4)}{20 - j6} = 4.59 - j8.62$$

$$Z = 2 + (4.59 - j8.62)$$
$$= 6.59 - j8.62$$

$$I = \frac{V}{Z} = \frac{5\angle 0}{6.59 - j8.62} = 0.28 + j0.37$$
$$= 0.46\angle 52.6°$$

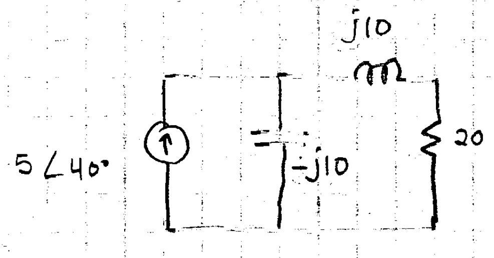
$$\therefore \; i_s(t) = 0.46 \cos(2000t + 52.6°) \text{ A}$$

#9. 50



Determine v_s,
given $i_s(t) = 5\cos(100t + 40°)$ A

In phasor : $\omega = 100$



I flows through j10 and 20 Ω $= (5\angle 40°)\dfrac{(-j10)}{-j10 + j10 + 20}$

$= (5\angle 40°)\dfrac{-j10}{20}$

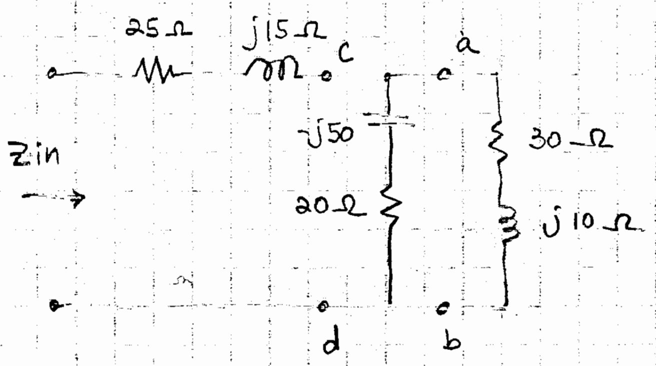
$= 2.5\angle -50$

$V_{20} = 20(-2.5\angle -50)$

$= 50\angle -50$

$\therefore\ v_s(t) = 50\cos(100t - 50°)$ V

9.60



Calculate Z_{in}

$$Z_{ab} = (30 + j10)\,\Omega$$

$$Z_{cd} = (30 + j10) \,//\, (20 - j50)$$

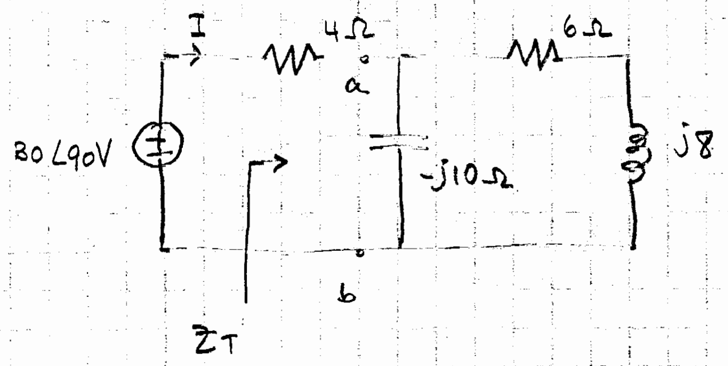
$$= \frac{(30 + j10)(20 - j50)}{(30 + j10) + (20 - j50)}$$

$$= (26.1 - j\,5.12)\,\Omega$$

$$\therefore \quad Z_{in} = (25 + j15) + (26.1 - j\,5.12)\,\Omega$$

$$= (51.1 + 9.88)\,\Omega$$

9.64 FIND Z_T and I.



$$Z_{ab} = (-j10) /\!/ (6+j8) = \frac{(-j10)(6+j8)}{6+j8-j10}$$

$$= (15 - j5)\,\Omega$$

$$\therefore \quad Z_T = 4 + (15 - j5)$$

$$= (19 - j5)\,\Omega$$

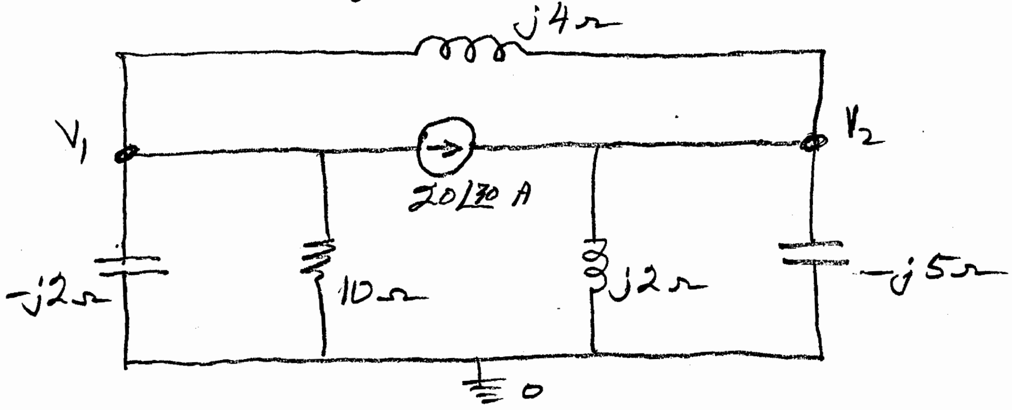
$$I = \frac{V}{R} = \frac{30\angle 90^\circ}{19 - j5} =$$

$$= (-0.388 + 1.48)\ A$$

$$= 1.53 \angle 104.74\ A$$

wlg

(10.14) Determine node voltages V_1 & V_2 in the following circuit.



At V_1

$$-\frac{V_1}{j2} + \frac{V_1}{10} + \frac{V_1 - V_2}{j4} = -20\angle 30^\circ$$

$$j0.5V_1 + 0.1V_1 - j0.25V_1 + j0.25V_2 = -20\angle 30$$

$$(0.1 + j0.25)V_1 + j0.25V_2 = -20\angle 30$$

At V_2

$$\frac{V_2}{j2} - \frac{V_2}{j5} + \frac{V_2 - V_1}{j4} = 20\angle 30$$

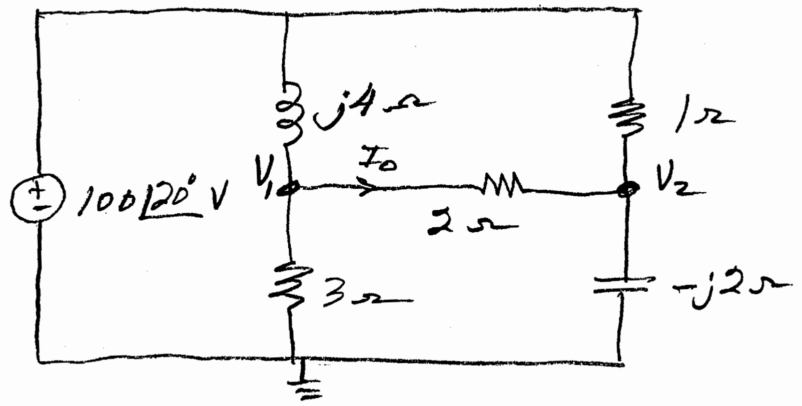
$$-j0.5V_2 + j0.2V_2 - j0.25V_2 + j0.25V_1 = 20\angle 30$$

$$j0.25V_1 - j0.55V_2 = 20\angle 30$$

$$\begin{bmatrix} (0.1+j0.25) & j0.25 \\ j0.25 & -j0.55 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20\angle -150 \\ 20\angle 30 \end{bmatrix}$$

$V_1 = 28.9\angle 135.4^\circ$ V $\quad V_2 = 49.17\angle 124.1^\circ$ V

(10.17) Use nodal analysis to find current I_o in the following circuit.



At V_1

$$\frac{V_1 - 100\angle 20}{j4} + \frac{V_1}{3} + \frac{V_1 - V_2}{2} = 0$$

$$-j0.25V_1 + j25\angle 20 + 0.3333V_1 + 0.5V_1 - 0.5V_2 = 0$$

$$(0.833 - j0.25)V_1 - 0.5V_2 = -j25\angle 20 = 25\angle -70^\circ$$

At V_2

$$\frac{-V_2}{j2} + \frac{V_2 - 100\angle 20}{1} + \frac{V_2 - V_1}{2} = 0$$

$$j0.5V_2 + V_2 - 100\angle 20 + 0.5V_2 - 0.5V_1 = 0$$

$$-0.5V_1 + (1.5 + j0.5)V_2 = 100\angle 20$$

(10.17) cont

$$\begin{bmatrix} (0.833 - j0.25) & -0.5 \\ -0.5 & 1.5 + j0.5) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 25\angle -70^\circ \\ 100\angle 20^\circ \end{bmatrix}$$

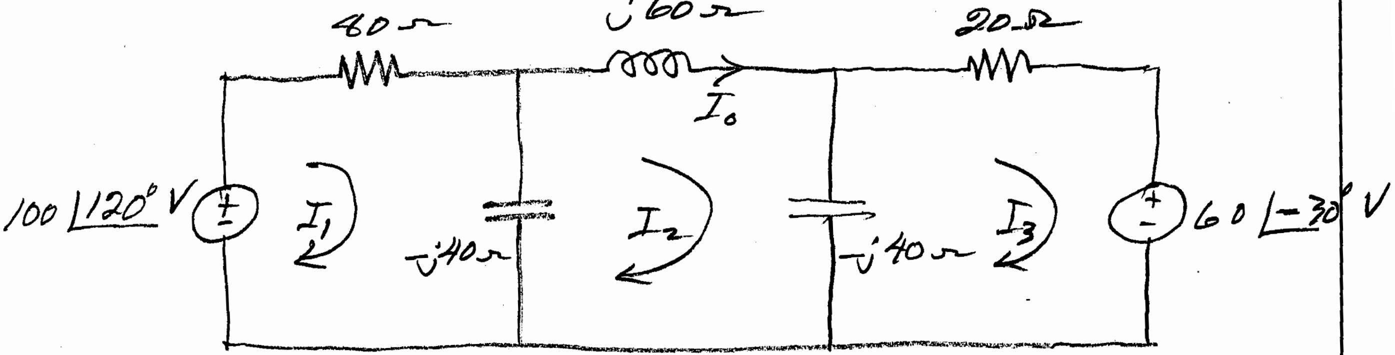
$V_1 = 63.066 - j14.657$ V $V_2 = 80.676 - j8.976$ V

$V_1 = 64.75\angle -13.08$ V $V_2 = 81.17\angle -6.35^\circ$ V

$$I_o = \frac{V_1 - V_2}{2} = 9.25\angle -162.13^\circ \text{ A}$$

(10.31)

Use mesh analysis to determine the current I_o in the following circuit.



Assign mesh currents $\vec{I}_1$, $\vec{I}_2$ and $\vec{I}_3$. Write the 3 mesh equation. Solve for $\vec{I}_1$, $\vec{I}_2$ and $\vec{I}_3$. Then $\vec{I}_o = \vec{I}_2$

<u>mesh 1</u>

$$(80-j40)I_1 + j40I_2 = 100\angle 120^\circ \qquad (1)$$

<u>mesh 2</u>

$$-j40(\vec{I}_2-\vec{I}_1) + j60I_2 - j40(\vec{I}_2-\vec{I}_3) = 0$$

$$j40I_1 - j20I_2 + j40I_3 = 0 \qquad (2)$$

<u>Mesh 3</u>

$$-j40(\vec{I}_3-\vec{I}_2) + 20\vec{I}_3 + 60\angle -30^\circ$$

$$j40I_2 + (20-j40)I_3 = -60\angle -30 \qquad (3)$$

(10.31) cont

From equations (1), (2), (3)

$$\begin{bmatrix} 80-j40, & j40, & 0 \\ j40, & -j20, & j40 \\ 0, & j40, & 20-j40 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \end{bmatrix} = \begin{bmatrix} 100\angle 120^\circ \\ 0 \\ 60\angle 150 \end{bmatrix}$$

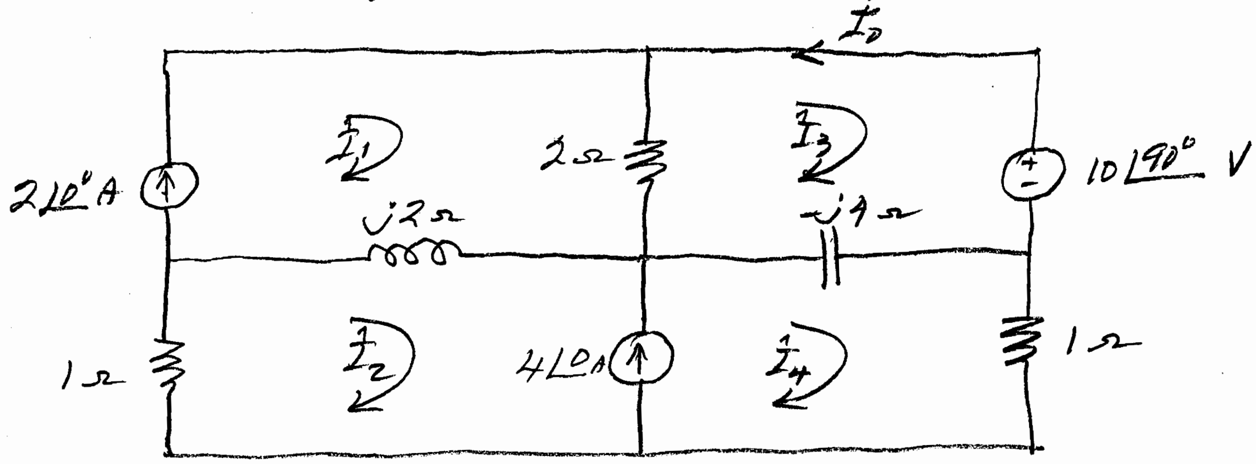
$\vec{I}_1 = (0.0412 + j0.5819)\,A$

$\vec{I}_2 = (1.0424 + j1.914)\,A = 2.179\angle 61.43^\circ\ A$

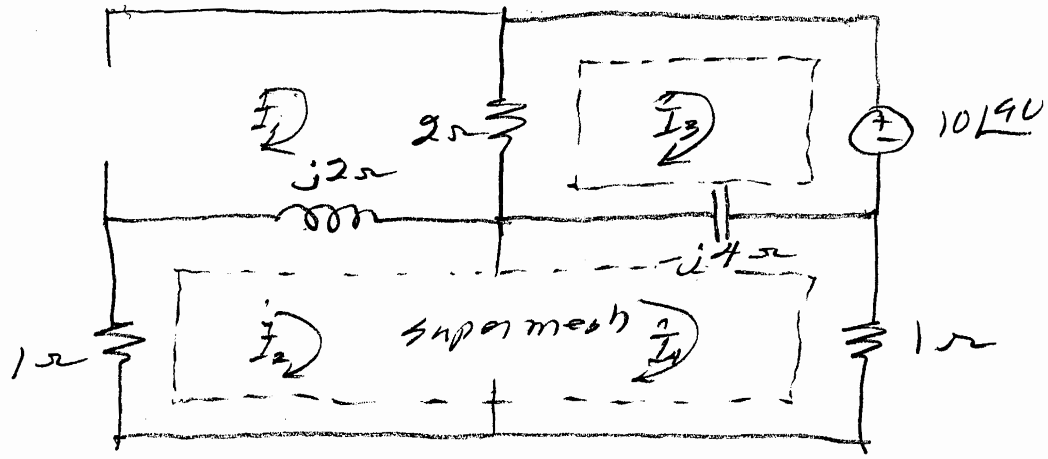
$\vec{I}_3 = (0.4800 + j0.3752)\,A$

(10.38)

Use mesh analysis to find current $\bar{I}_o$ in the following circuit.



Assign the mesh currents as shown above, Remove current sources $2\angle 0$ A and $4\angle 0^\circ$ A. Redraw the circuit.



Write mesh equation for mesh 3 and the supermesh.

Impose the two constraint equations

$$\bar{I}_1 = 2\angle 0 \qquad (1)$$

$$\bar{I}_4 - \bar{I}_2 = 4\angle 0 \qquad (2)$$

(10.38)

Mesh 3

$$2(I_3 - I_1) + 10\angle 90 - j4(I_3 - I_4) = 0$$

$$-2I_1 + 0I_2 + (2-j4)I_3 + j4I_4 = -10\angle 90^\circ \quad (3)$$

Supermesh

$$I_2 + j2(I_2 - I_1) - j4(I_4 - I_3) + I_4 = 0$$

$$-j2I_1 + (1+j2)I_2 + j4I_3 + (1-j4)I_4 = 0 \quad (4)$$

Combine (3) and (4) with (1) and (2)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -2, & 0, & 2-j4, & j4 \\ -j2, & 1+j2, & j4 & 1-j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 2\angle 0 \\ 4\angle 0 \\ 10\angle -90 \\ 0 \end{bmatrix} \begin{matrix} (1) \\ (2) \\ \\ \\ \end{matrix}$$

Left my TI 86 at home. TI 68 not enough memory.

Solved by using MATLAB. Program on the following page.

(10.38) cont 3


```
% Homework problem 10.38 from Alexander
% March 31, 2008.  Office PC: Program HW10_38.m
% By: W. Green for ECE 300 Spring 2008

B = [2;4;-j*10;0];

A = [1, 0, 0, 0;0, -1, 0, 1;-2, 0, 2-j*4 ,j*4; -j*2, 1+j*2, j*4, 1-j*4];

I = inv(A)*B;

I1 = I(1)
I2 = I(2)
I3 = I(3)
I4 = I(4)

Io = -I3
magIo = abs(Io)          %gives the magnitude of Io
angIo = 57.3*phase(Io)   %angle of Io converted from radians to degrees
```

```
>> HW10_38

I1 =

     2


I2 =

  -3.0000 + 0.3333i


I3 =

   3.3333 - 0.3333i


I4 =

   1.0000 + 0.3333i


Io =

  -3.3333 + 0.3333i


magIo =

    3.3500


angIo =

  174.3022
```

(10.55)

Find the Thevenin and Norton for the following circuit.

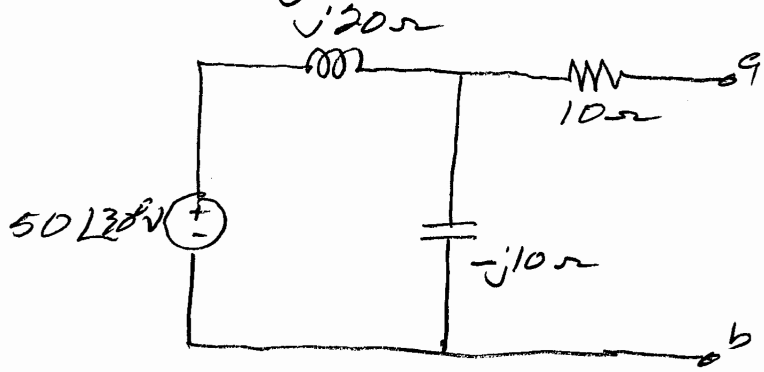


Figure 1

Thevenin Solution

V_{ab} open circuit $= V_{TH}$.

(a) V_{TH} is the voltage across the $-j10\Omega$ impedance of Figure 1.

$$V_{TH} = \frac{(50\angle 30)\times(-j10)}{-j10+j20} = \frac{(50\angle 30)(10\angle -90)}{10\angle 90}$$

$$V_{TH} = 50\angle -150 = -50\angle 30^\circ \text{ V}$$

Find Z_{TH} from the following

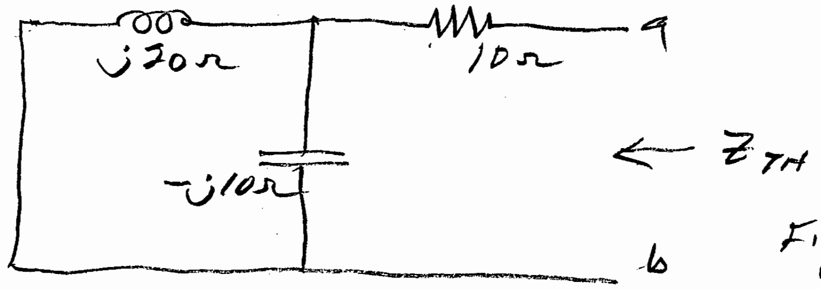


Figure 2

$$Z_{TH} = 10 + \frac{(20\angle 90)(10\angle -90)}{20\angle 90 + 10\angle -90} = 22.36\angle -63.4^\circ\ \Omega$$

(10.55)

(a) cont

The Thevenin circuit is given below

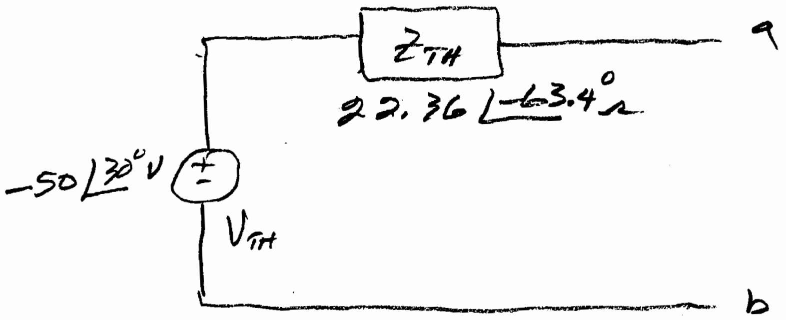


Figure 3
Thevenin circuit for part (a)

(a) Norton:

Find $I_{sc} = I_N$ and V_{oc}

Referring to Figure 1 with terminal a-b shorted,

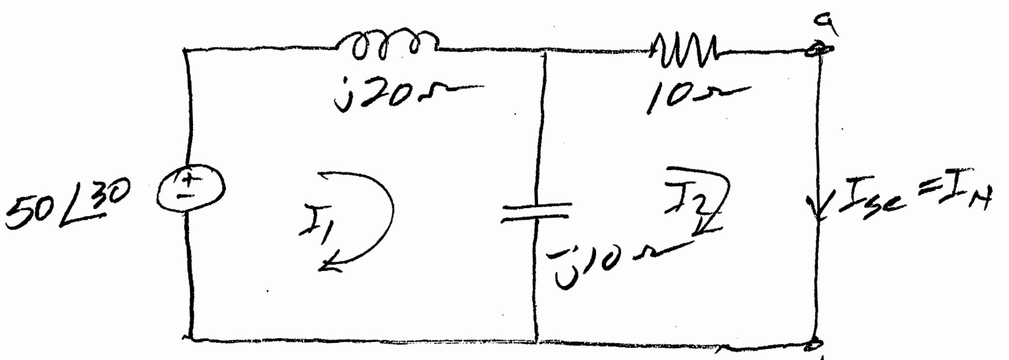


Figure 4: Circuit for finding I_N, part (a)

$$j10 I_1 + j10 I_2 = 50\angle -150^\circ \qquad (1)$$

$$j10 I_1 + (10 - j10) I_2 = 0 \qquad (2)$$

(10.55) cont,

solving (1) and (2)

$$\begin{bmatrix} j10 & j10 \\ j10 & 10-j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50\angle 30 \\ 0 \end{bmatrix}$$

$I_1 = -2.366 + j2.098$ A

$I_2 = I_N = 2.24 \angle -86.6^\circ$

$\therefore I_N = 2.24 \angle 273.4^\circ$

Z_{TH} same as above.

As a check $Z_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{V_{TH}}{I_N} = \frac{-50\angle 30}{2.24\angle 273.4}$

$Z_{TH} = 22.3 \angle -63.4^\circ\ \Omega$ check

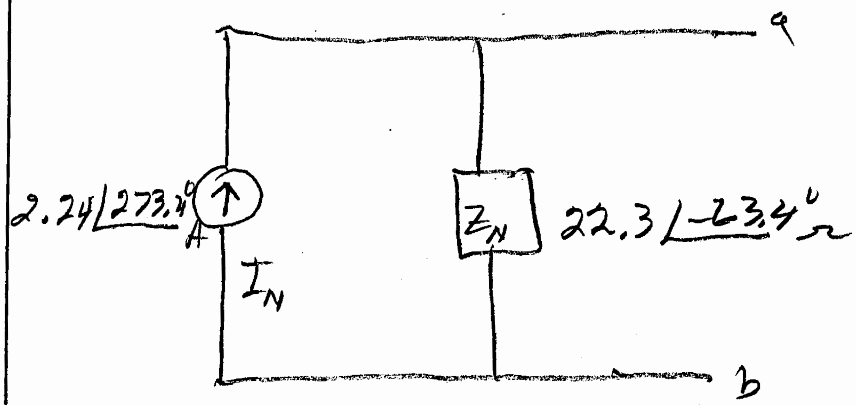


Figure 5: Norton equivalent ckt, part (a)

(10.55) cont

Part (b)

Find the Thevenin and Norton equivalent circuits for the circuit shown below.

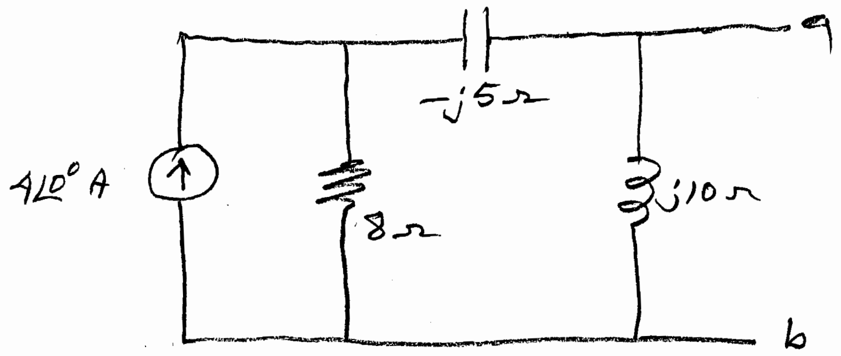


Figure 6: Circuit for part (b)

Thevenin Part (b)

Using current division;

$$V_{ab} = V_{TH} = \left(\frac{(4\angle 0)(8\angle 0)}{8 - j5 + j10}\right)(j10)$$

$$V_{TH} = 33.92\angle 58^\circ \ V$$

Use the following circuit to find Z_{TH}.

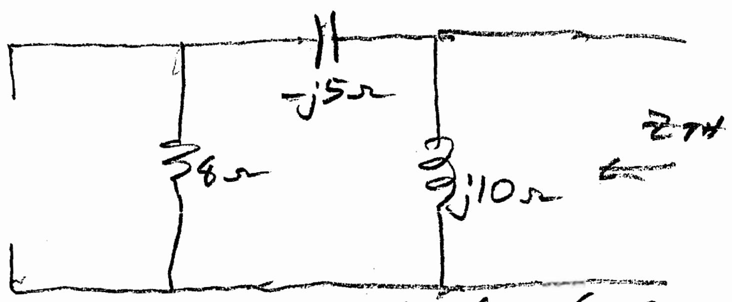


Figure 7: Circuit for finding Z_{TH}, part (b)

(10.55) cont
part 1b)

$$Z_{TH} = \frac{(j10)(8-j5)}{j10+8-j5} = 10\angle 26^\circ \ \Omega$$

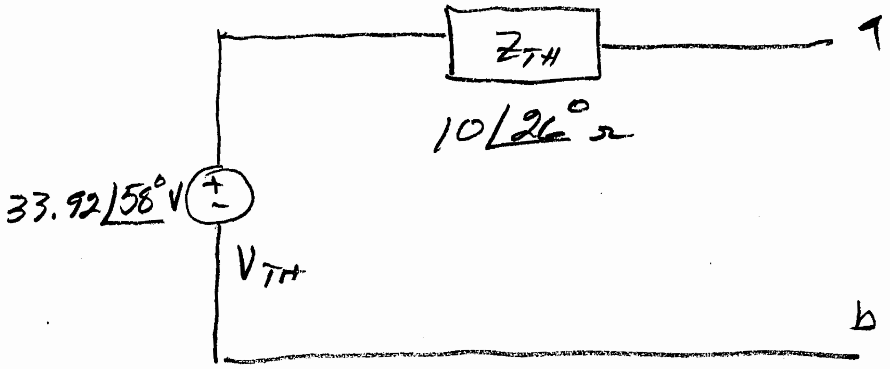
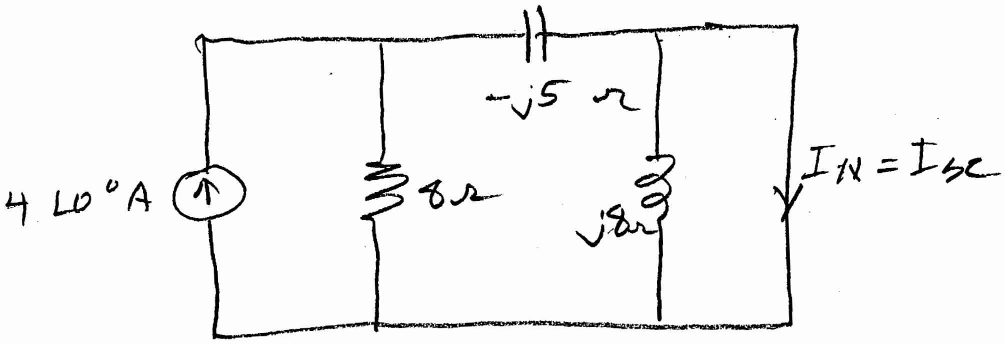


Figure 8: Thevenin Circuit for part (b)

1b) Norton Circuit
To get I_N, find I_{sc} below



The $j8\Omega$ coil is shorted out.
Find I_N then as shown in Figure 9.

(10.55) cont.
part (b)

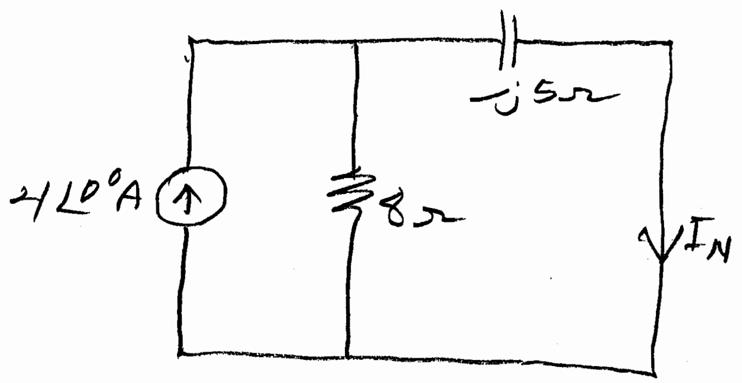


Figure 9: Circuit for finding I_N, part (b)

$$I_N = \frac{4 \times 8}{8 - j5} = 3.39\angle 32^\circ \text{ A}$$

$$Z_N = Z_{TH} = 10\angle 26^\circ\ \Omega$$

As a check

$$Z_N = Z_{TH} = \frac{V_{TH}}{I_N} = \frac{33.92\angle 58}{3.39\angle 32}$$

$$Z_N = Z_{TH} = 10\angle 26^\circ\ \Omega \quad \text{check}$$

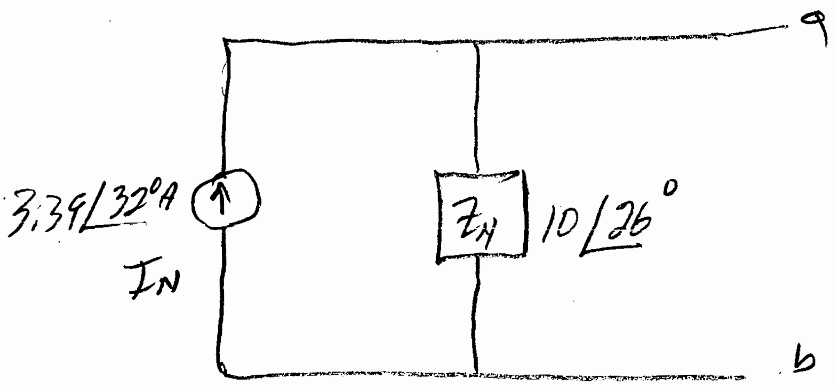


Figure 10: Norton Circuit, part (b)