Desk Copy

ECE 300
Spring Semester, 2008
HW Set #12

Due: April 17, 2008

Name wlg
Print (last, first)

wlg

Check according to your section: ________ 8:10 AM; ________ 11:10 AM

Use engineering paper. Work only on one side of the paper. Use this sheet as your cover sheet, placed on top of your work and stapled in the top left-hand corner. Number the problems at the top of the page, in the center of the sheet. **Do neat work. Underline your answers. Show how you got your equations.** **Be sure to show how you got your answers.**

(1) Consder the AC circuit below. Both sources operate at the same frequency.
  (a) Find the average real power delivered to R_1 and R_2.
  (b) What is the average real power supplied by V_1? What is the average real power supplied by V_2?
  (c) Show that the average real power supplied to the capacitor is 0 W.

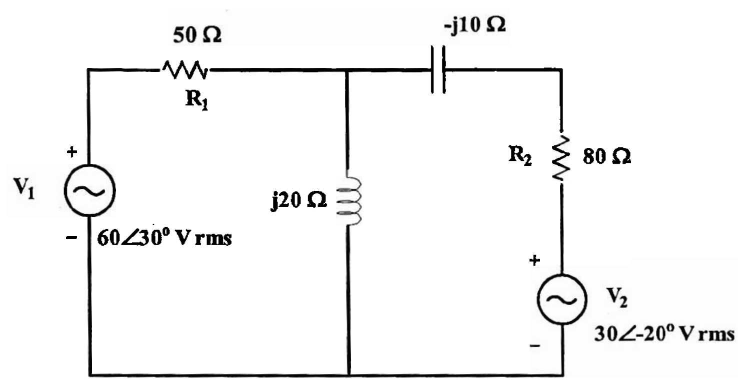

Figure 1: Circuit for Problem 1.

(2) Consider the following load for a system.
  (a) Determine the power factor for this load (include lead or lag).
  (b) If a voltage is connected to this load, will the resulting current **I** lead or lag this voltage? Explain.

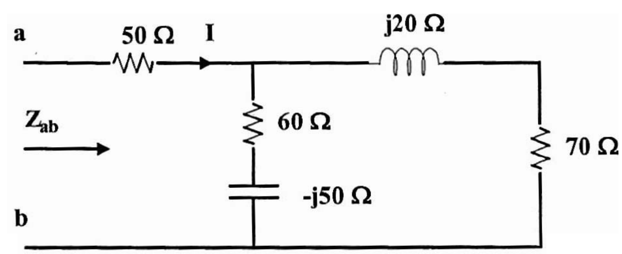

Figure 2: Circuit for Problem 2.

(3) Consider the circuit shown below.
(a) Determine the average real power delivered to each of the boxed networks in the circuit.
(b) Check your answer by making a power balance check.

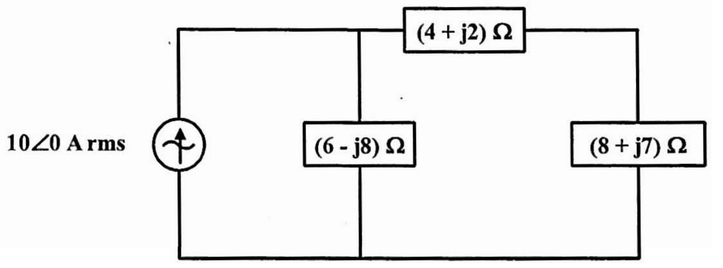

Figure 3: Circuit for Problem 3.

(4) Consider the network given below.
(a) What impedance Z_L should be connected as a load so that maximum power will be absorbed by it? (28.8 –j38.4) Ω
(b) What is this maximum power? 250 W

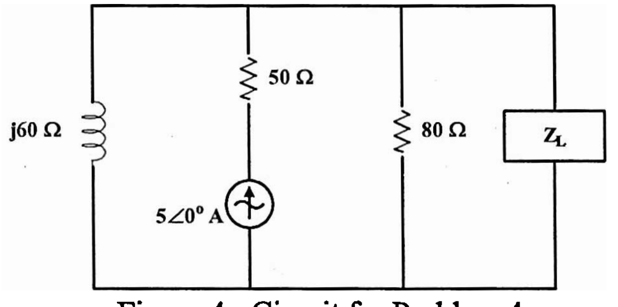

Figure 4: Circuit for Problem 4.

(5) Find the effective value of the following periodic voltage waveform.

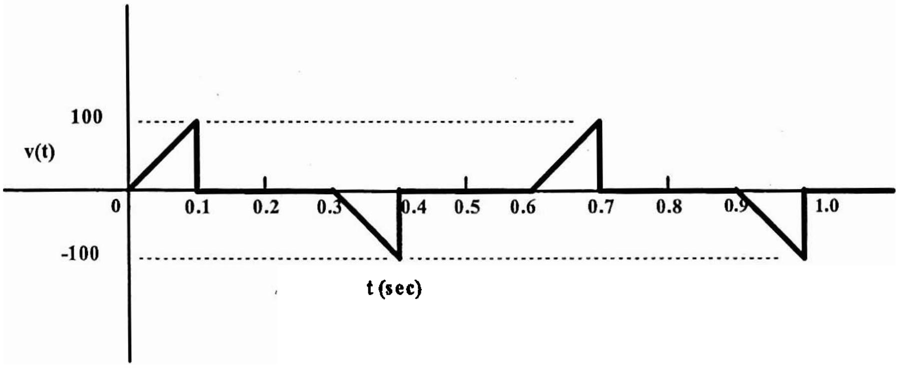

Figure 5: Waveform for Problem 5.

(6) For the circuit below let $I = 4\angle 35°$ A rms.
Find the average power being supplied;

(a) by the source; 655 W
(b) to the 20 Ω resistor; 320 W
(c) to the load; 335 W

Find the apparent power being supplied

(d) by the source; 800 VA
(e) to the 20 Ω resistor; 320 VA
(f) to the load; 568 VA
(g) what is the load PF? 0.590 lagging

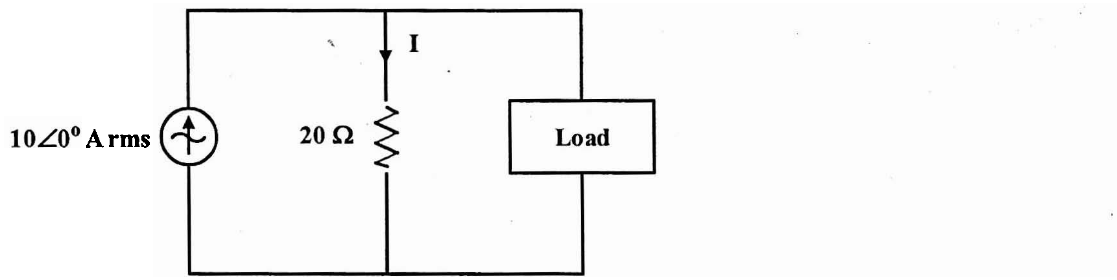

Figure 6: Circuit for Problem 6.

(7) The load in the diagram below draws 10 kVA at PF 0.8 lagging. If $|I_L| = 40$ A rms, what must be the value of C to cause the source to operate at PF = 0.9 lagging? 79.48 μF.

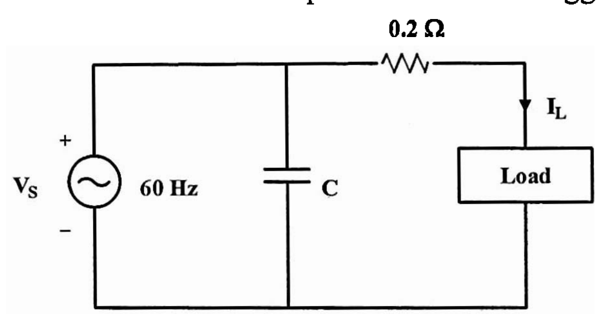

Figure 7: Circuit for Problem 7.

(8) Both sources in the following circuit are operating at the same frequency. Find the complex power generated by each source and the complex power absorbed by each passive circuit element.

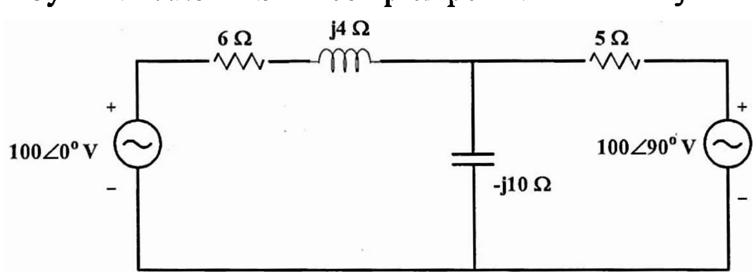

Figure 8: Circuit for Problem 8.

(9) Consider the circuit shown below.

(a) Find the complex power delivered to each passive element in the circuit. $S_{20} = 37.83$ kVA $S_{250} = 483.3$ kVA, $S_C = 49.57\angle{-90^\circ}$ kVA, $S_L = 77.34\angle 90^\circ$ kVA;

(b) Show that the sum of those values is equal to the complex power generated bythe source. $S_{source} = 521.9\angle 3.05^\circ$ kVA;

(c) Is the result true for the values of apparent power?

(d) What is the average power delivered by the source? 521.2 kW

(e) What is the reactive power delivered by the source? 27.76 kVA (inductive)

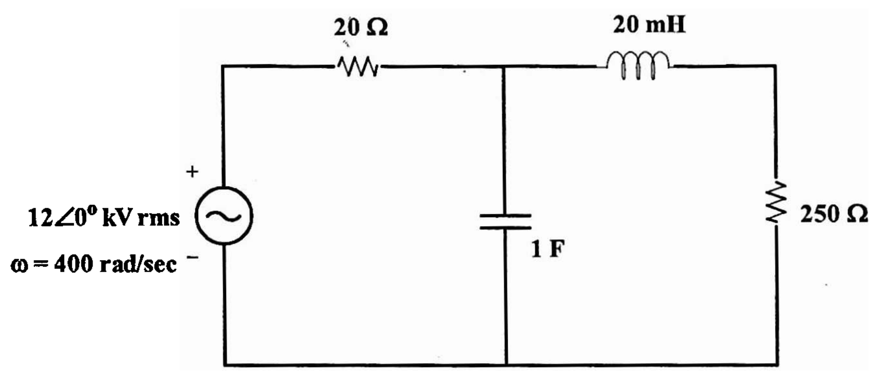

Figure 9: Circuit for Problem 9.

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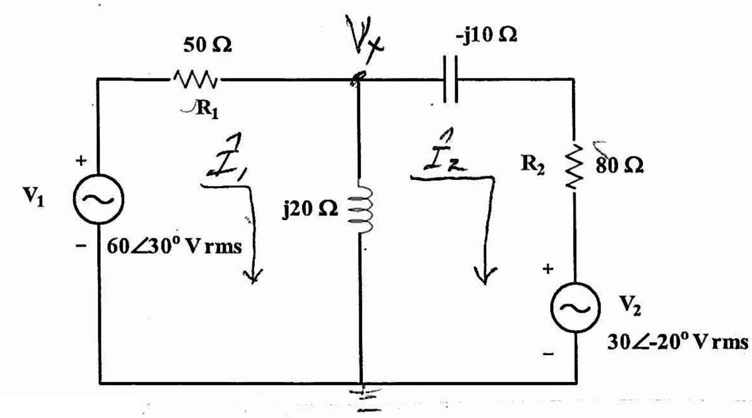
Spring, 2008

(1) Consder the AC circuit below. Both sources operate at the same frequency.
   (a) Find the average real power delivered to R_1 and R_2.
   (b) What is the average real power supplied by V_1? What is the average real power supplied by V_2?
   (c) Show that the average real power supplied to the capacitor is 0 W.



Using mesh analysis;

1a)

$$-60\angle 30^\circ + (50+j20)\vec{I}_1 - j20\vec{I}_2 = 0$$

OR

$$(50+j20)\vec{I}_1 - j20\vec{I}_2 = 60\angle 30^\circ \qquad (1.1)$$

Also,

$$j20(\vec{I}_2 - \vec{I}_1) + (80-j10)\vec{I}_2 = -30\angle -20 \qquad (1.2)$$

From (1.1) and (1.2)

$$\begin{bmatrix} 50+j20 & -j20 \\ -j20 & 80+j10 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 60\angle 30 \\ 30\angle 160 \end{bmatrix} \qquad (1.3)$$

$\vec{I}_1 = 0.926\angle 6.26^\circ$ A $\qquad \vec{I}_2 = 0.517\angle 129.4^\circ$ A

(1) cont

(a)

$$\_\_ = I_1^2 \times R_1 = (0.926)^2 \times 50 = \underline{42.87W}$$

$$P_{R_2} = I_2^2 R_2 = (.517)^2 80 = \underline{21.38\ W}$$

(b)

$$P_{sup,V_1} = V_s I_1 \cos(\theta_V - \theta_I)$$

$$P_{sup,V_1} = 60 \times (0.926) \cos(30 - 6.26)$$

$$P_{sup,V_1} = 50.86\,W \qquad (1.5)$$

#

$$P_{sup,V_2} = -V_2 \times I_2 \cos(\theta_V - \theta_I)$$

$$P_{sup,V_2} = -30 \times 0.517 \cos(-20 - 129.4)$$

$$P_{sup,V_2} = 13.35\,W \qquad (1.6)$$

To check:

$$P_{sup,V_1} + P_{sup,V_2} = P_{R_1} + P_{R_2}$$

$$50.86 + 13.35 = 42.87 + 21.28$$

$$64.21 \stackrel{?}{=} 64.15 \quad \text{(good within roundoff)}$$

(c) continued on next page

(1) cont.

$$P_{cap} = Re[|\vec{I}_2|^2 \times (-j10)] = Re[0.517^2 \times (-j10)]$$

$$P_{cap} = Re[0 - j2.67] = 0$$

$$\underline{Q_{CAP} = -2.67 \text{ VARs}} \quad (1.7)$$

As a check on things (not required)

$$\vec{S}_{coil} = |(\vec{I}_1 - \vec{I}_2)|^2 (j20) = (1.283)^2(j20) \text{ VA}$$

$$\vec{S}_{coil} = [0 + j32.92 \text{ VA}$$

$$\underline{Q_{coil} = 32.92 \text{ VARs}}$$

$$\underline{Q_{coil} + Q_{CAP}} = 32.92 - 2.67 = \underline{30.25} \text{ VARs}$$

$$\vec{S}_{source1} = \vec{V}_1 \vec{I}_1^* = (60\angle 30)(0.926\angle{-6.26^\circ}$$

$$S_{source1} = 55.56\angle 23.74 = (50.86 + j22.37) \text{ VA}$$

$$Q_{source1} = 22.37$$

$$\vec{S}_{source2} = -\vec{V}_2 \times \vec{I}_2^* = 30\angle 160 \times 0.517\angle{-129.4^\circ}$$

$$\vec{S}_{source2} = (13.35 + j7.9) \text{ VA}$$

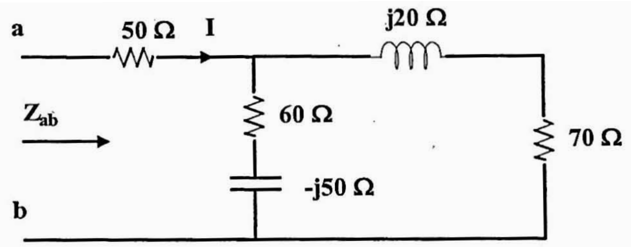
$$Q_{source2} = 7.9 \text{ VARs}$$

$$Q_{source1} + Q_{source2} \stackrel{?}{=} Q_{CAP} + Q_{coil}$$

$$22.37 + 7.9 \stackrel{?}{=} 30.25$$

$$\underline{30.27} = 30.25 \quad \text{(close, round-off)}$$

(2) Consider the following load for a system.
(a) Determine the power factor for this load (include lead or lag).
(b) If a voltage is connected to this load, will the resulting current **I** lead or lag this voltage? Explain.



(a) The approach: Find Zab. The angle of Zab is the power factor angle.

$$\bar{Z}_{ab} = 50 + \frac{(70+j20)\times(60-j50)}{70+j20+60-j50}$$

$$\bar{Z}_{ab} = 50 \quad \frac{(70+j20)(60-j50)}{130-j30}$$

$$\bar{Z}_{ab} = 92.20\angle -5^{\circ}\ \Omega$$

$P.f. = \cos(-5^{\circ}) = 0.9962$ leading

(when the angle of $\bar{Z}$ is Negative, this means the angle of $\bar{I}$ is greater than the angle of $\bar{V}$. In such case I leads V so leading P.f.

(b) The power factor is leading, as explained above. Therefore I leads V; leading P.f.

(3) Consider the circuit shown below.
(a) Determine the average real power delivered to each of the boxed networks in the circuit.
(b) Check your answer by making a power balance check.

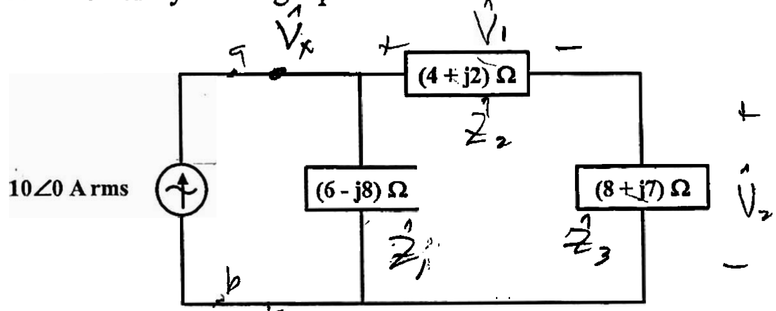


Figure 3: Circuit for Problem 3.

(a) Approach: Find $\hat{V}_x$. Do this by find equivalent $\hat{Z}_{ab}$: $\hat{V}_x = Z_{ab}(10\angle 0)$. From there, find each individual voltage and use voltage to find power.

$$Z_{ab} = (6-j8) \parallel [(4+j2) + (8+j7)]$$

$$Z_{ab} = \frac{(6-j8) \times (12+j9)}{(6-j8) + (12+j9)}$$

$$Z_{ab} = 8.32\angle -19.44\ \Omega$$

$$\hat{V}_x = (8.32\angle -19.44)(10\angle 0) = \underline{83.2\angle -19.44\ V}$$

$$P_{Z_1} = Re\left[\frac{[V_x]^2}{\hat{Z}_1^*}\right] = Re\left[\frac{(83.2)^2}{6+j8}\right] = Re[415.33 - j553.8]\ W$$

$$P_{Z_1} = \underline{415.33}\ W$$

continued on next page

(3) cont.

$$\hat{V}_1 = \frac{\hat{V}_x\,(4+j2)}{(4+j2)+(8+j7)} = \frac{(83.2\angle -19.44)(4+j2)}{(12+j9)}$$

$$\hat{V}_1 = 24.81\angle -29.7^\circ\ V$$

$$P_{Z_2} = Re\left[\frac{V_1^2}{Z_2^*}\right] = \frac{Re\,24.81^2}{4-j2} = Re\left[123.11+j61.55\right]$$

$$\underline{P_{Z_2} = 123.11\ W}$$

$$\hat{V}_2 = \frac{\hat{V}_x \times (8+j7)}{(4+j2)+(8+j7)} = \frac{(83.2\angle -19.44)(8+j7)}{12+j9}$$

$$\hat{V}_2 = 58.96\angle -15.12^\circ\ V$$

$$P_{Z_3} = Re\left[\frac{[V_2^2]}{Z_3^*}\right] = Re\left[\frac{58.96^2}{(8-j7)}\right]$$

$$P_{Z_3} = Re\left[246.12\ j215.3\right]$$

$$\underline{P_{Z_3} = 246.12\ W}$$

Total power to loads

$$\Sigma P_{Loads} = P_{Z_1} + P_{Z_2} + P_{Z_3} = 415.33 + 123.11 + 246.12$$

$$\boxed{\Sigma P_{LOADS} = 787.57\ W}$$

(3) cont.

$$P_{source} = V_x I_s \cos(\theta_V - \theta_I)$$

$$= 83.2 \times 10 \cos(-15.44 - 0)$$

$$P_{source} = 784.57\ W$$

$$\Sigma P_{LOADS} = 787.57\ W \quad \text{(ROUND-off error)}$$

(4) Consider the network given below.

(a) What impedance Z_L should be connected as a load so that maximum power will be absorbed by it? $(28.8 - j38.4)\ \Omega$

(b) What is this maximum power? 250 W

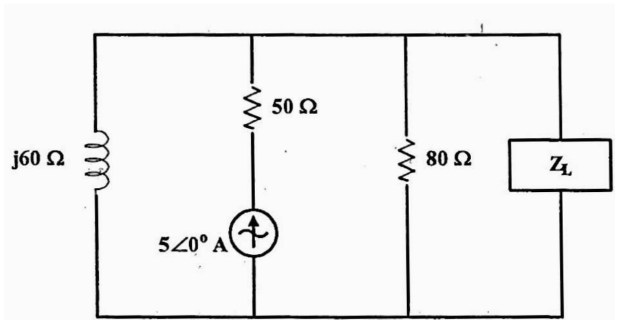
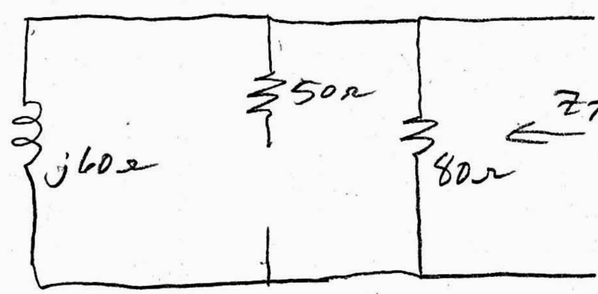


Figure 4: Circuit for Problem 4.

(a) Find Z_{TH} for the following circuit.



$$\hat{Z}_{TH} = \frac{80 \times 60\angle 90}{80 + j60} = 28.8 + j\,38.4\ \Omega$$

$$\hat{Z}_L = \hat{Z}_{TH}^{*} = (28.8 - j\,38.4)\ \Omega$$

(b) Find V_{TH}

$$V_{TH} = \left(\frac{5 \times j60}{80 + j60}\right) \times 80 = 240\angle 53.13^\circ\ V$$

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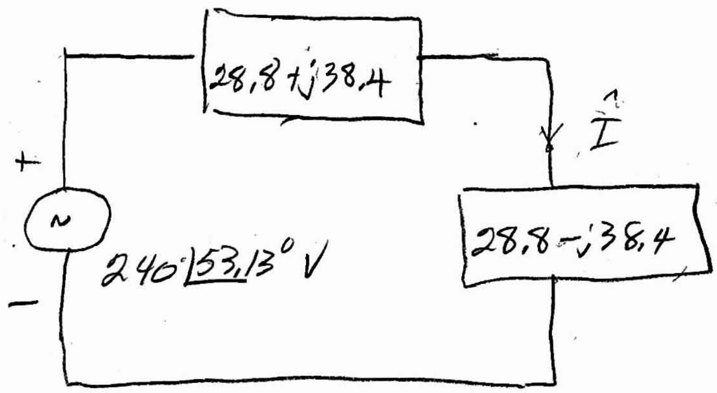
(4) cont.



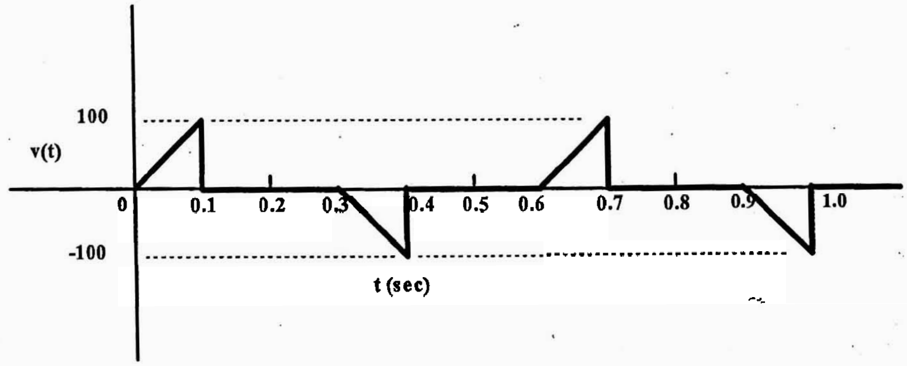
$$\hat{I} = \frac{240\angle 53.13}{2\times 28.8} = 4.167\angle 53.13^\circ\ A$$

$$P_{LOAD} = Re\left[\frac{I^2}{2}\times(28.8 - j38.4)\right]$$

$$= \frac{(4.167)^2}{2}\times 28.8$$

$$P_{LOAD} = 250\ W$$

(5) Find the effective value of the following periodic voltage waveform.



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$v(t) = \frac{100}{.1}t = 1000t \qquad (0 \le t \le .1) \quad \text{zero otherwise}$$

$$v^2(t) = 1\times10^6 t^2$$

$$V_{rms} = \sqrt{\frac{1}{.3}\int_0^{.1} 1\times10^6 \frac{t^3}{3}\Big|_0^{.1}} = \sqrt{\frac{1\times10^6 \times (.1)^3}{.3 \times 3}}$$

$$V_{rms} = 1\times10^3\sqrt{\frac{1\times10^{-3}}{.9}} = 1\times10^3\sqrt{\frac{.001}{.9}}$$

$$V_{rms} = 1\times10^3\sqrt{\frac{0.01}{9}} = \frac{1\times10^2}{3}$$

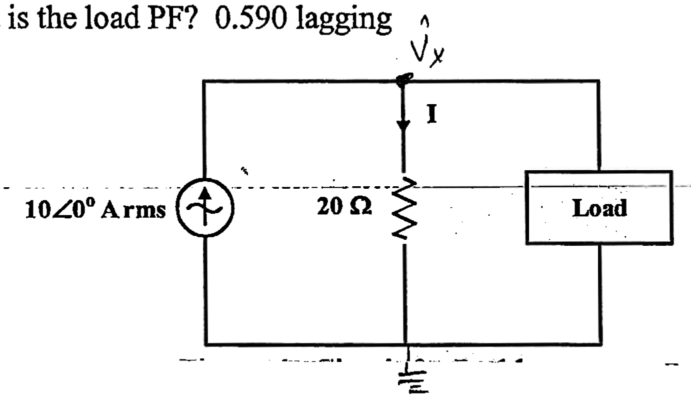
$$V_{rms} = \frac{100}{3} = 33.3 \text{ Volts}$$

(6) For the circuit below let $I = 4\angle 35^\circ$ A rms.
Find the average power being supplied;

(a) by the source; 655 W
(b) to the 20 Ω resistor; 320 W
(c) to the load; 335 W

Find the apparent power being supplied

(d) by the source; 800 VA
(e) to the 20 Ω resistor; 320 VA
(f) to the load; 568 VA
(g) what is the load PF? 0.590 lagging



(a) Find $\hat{V}_x$ then use

$$\hat{S} = \hat{V}_x \hat{I}_s^*$$

$$\hat{V}_x = 20 \times 4\angle 35 = 80\angle 35^\circ \text{ V}$$

$$\hat{S} = 80\angle 35 \times 10 = 800\angle 35^\circ \text{ VA}$$

$$\hat{S} = (655.32 + j458.87) \text{ VA}$$

$$P_{source} = \text{Re}\, \hat{S} = 655.32 \text{ W}$$

(b) $P_{20} = I^2 \times 20 = 4^2 \times 20$

$$P_{20} = 320 \text{ W}$$

(6) cont.

(c) to the load

Since the source supplies

$P_{source} = 655.32\,W$

and the 20Ω resistor uses 320W of this 655.32W, it follows that

$P_L = P_{source} - P_{20}$

$P_L = 655.32 - 320W$

$P_L = 335.32\ W$

(d)

$S_{source} = I_s \times V_x = 10 \times 80 = 800\ VA$

(e) $S_{20} = I \times V_x = 4 \times 80 = 320\ VA$

(f) $\hat{I}_L = I_s - I = 10\angle 0 - 4\angle 35 = 7.1\angle -18.84\ A$

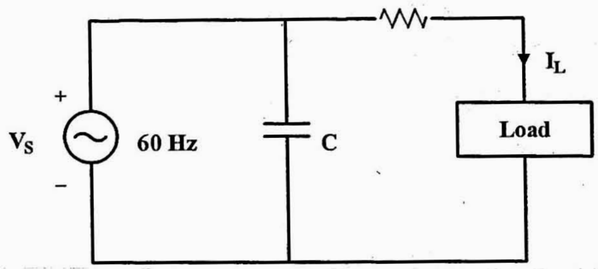
$I_L = 7.1$

$S_{LOAD} = V_x I_L = 80 \times 7.1 = 568\ VA$

(g) $Z_{IN} = \frac{\vec{V}_x}{I_s} = \frac{80\angle 35}{10\angle 0} = 8\angle 35$

$p.f. = \cos 35^\circ = 0.8192$ lagging

(7) The load in the diagram below draws 10 kVA at PF 0.8 lagging. If $|I_L| = 40$ A rms, what must be the value of C to cause the source to operate at PF = 0.9 lagging? 79.48 μF.



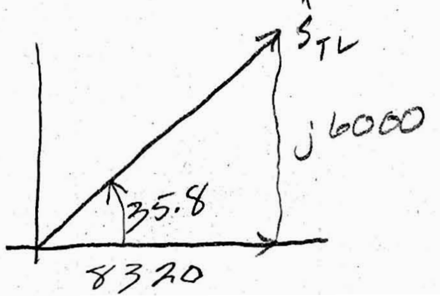
$\hat{Z}_L, \hat{S}_L$

First find $\hat{S}_{TL}$ including the 0.2 Ω resistor.

Use $\hat{Z}_{TL} = (\hat{Z}_L + 0.2)\,\Omega$

$$\hat{S}_{TL} = 0.2(40)^2 + \underbrace{8000 + j\,6000}_{\hat{S}_L}$$

$$\hat{S}_{TL} = 8320 + j\,6000 = 10257.8\angle 35.8^\circ \text{ VA}$$



$\theta = 35.8 = \theta_{OLD}$

For a p.f. = 0.9 lagging, $\theta = 25.84 = \theta_{NEW}$

$$C = \frac{P[\tan\theta_{OLD} - \tan\theta_{NEW}]}{\omega V_{rms}^2}$$

$$C = \frac{8320[\tan 35.8 - \tan 25.84]}{2\pi \times 60 \times V_{Rms}^2}$$

We need V_{rms}

17) cont.

We know

$$\vec{S}_{TL} = \frac{V_{rms}^2}{Z_{TL}^*} \quad (A)$$

We need Z_{TL}^*

$$\vec{S}_{TL} = |I|^2 \vec{Z}_{TL}$$

$$\vec{Z}_{TL} = \frac{10257.8 \angle 35.8}{(40)^2} = 6.41 \angle 35.8 \ \Omega$$

From (A)

$$V_{rms} = \sqrt{\vec{S}_{TL} \times Z_{TL}^*} = \sqrt{10257.8 \angle 35.8^\circ \, 6.41 \angle -35.8}$$

$$V_{rms} = 256.4$$

$$C = \frac{8320 \left[0.72122 - 0.48428\right]}{2\pi \times 60 \times 256.4^2}$$

$$C = 79.54 \mu F$$ close

(8) Both sources in the following circuit are operating at the same frequency. Find the complex power generated by each source and the complex power absorbed by each passive circuit element.

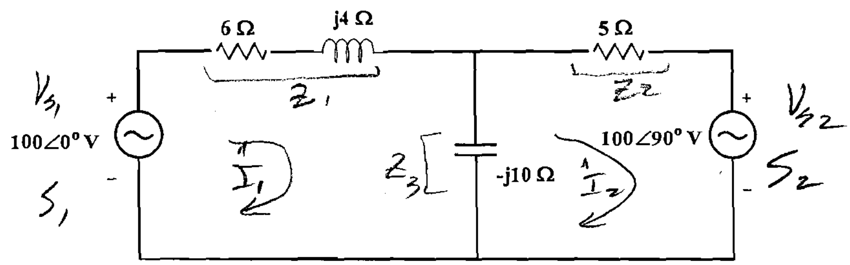


Figure 8: Circuit for Problem 8.

First find mesh currents $\hat{I}_1$ and $\hat{I}_2$

By inspection

$$\begin{bmatrix} 6-j6 & j10 \\ j10 & 5-j10 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ -100\angle 90 \\ 100\angle -90 \end{bmatrix}$$

$\hat{I}_1 = 9.81\angle -64.44^\circ$ A $\quad \hat{I}_2 = 14.99\angle -58.4^\circ$ A

$$\hat{S}_1 = \frac{\hat{V}_{s_1}\hat{I}_1^*}{2} = \frac{100 \times (9.81\angle 64.44)}{2}$$

$\boxed{\hat{S}_1 = 490.5\angle 64.40^\circ \text{ VA}} = (215.02 + j440.86)$ VA

$$\hat{S}_2 = -\frac{\hat{V}_{s_2} \times \hat{I}_2^*}{2} = \frac{-100\angle 90 \times (14.99\angle 58.4)}{2}$$

$\boxed{\hat{S}_2 = 749.5\angle -31.6^\circ \text{ VA}} = (638.37 - j392.73)$ VA

$\boxed{\Sigma S_{sup} = (853.39 + j48.13) \text{ VA}}$

18) cont

$$P_{6\Omega} = \frac{I_1^2}{2} \times 6 = \frac{(9.81)^2}{2} \times 6$$

$$* \boxed{S_{6\Omega} = P_{6\Omega} = S_{6\Omega} = 288.7 \text{ W}}$$

$$\hat{S}_{j4} = \frac{I_1^2}{2} Z_{j4} = \frac{(9.81)^2}{2} \times j4$$

$$* \boxed{\hat{S}_{j4} = j192.47 \text{ VARs}}$$

$$\hat{S}_{5\Omega} = \frac{I_2^2}{2} \times 5 = \frac{(14.99)^2}{2} \times 5$$

$$* \boxed{\hat{S}_{5\Omega} = 561.75 \text{ W}}$$

$$\hat{S}_{-j10} = \frac{|\hat{I}_1 - \hat{I}_2|^2 \times 10\angle{-90}}{2}$$

$$= \frac{|(9.81\angle{-64.44}) - (14.99\angle{-58.4})|^2}{2} \times 10\angle{-90}$$

$$= \frac{(5.33)^2 \times 10\angle{-90}}{2}$$

$$\boxed{\hat{S}_{-j10} = -j142.04}$$

$$\hat{S}_{LOADS} = 288.7 + j192.47 + 561.75 - j142.04$$

$$\hat{S}_L = (850.45 + j50.4) \text{ VA}$$

$$\hat{S}_{SOURCES} = (853.39 + j48.13) \text{ VA}$$

round-off errors

(9) Consider the circuit shown below.

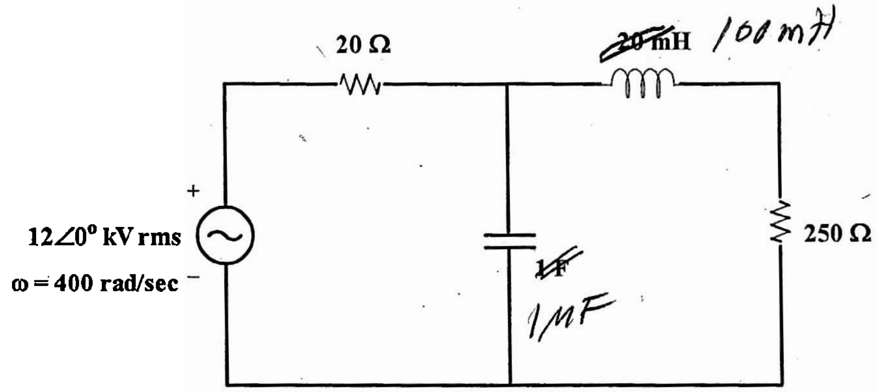
(a) Find the complex power delivered to each passive element in the circuit. $S_{20} = 37.83$ kVA $S_{250} = 483.3$ kVA, $S_C = 49.57\angle\text{-}90°$ kVA, $S_L = 77.34\angle 90°$ kVA;

(b) Show that the sum of those values is equal to the complex power generated bythe source. $S_{source} = 521.9\angle 3.05°$ kVA;

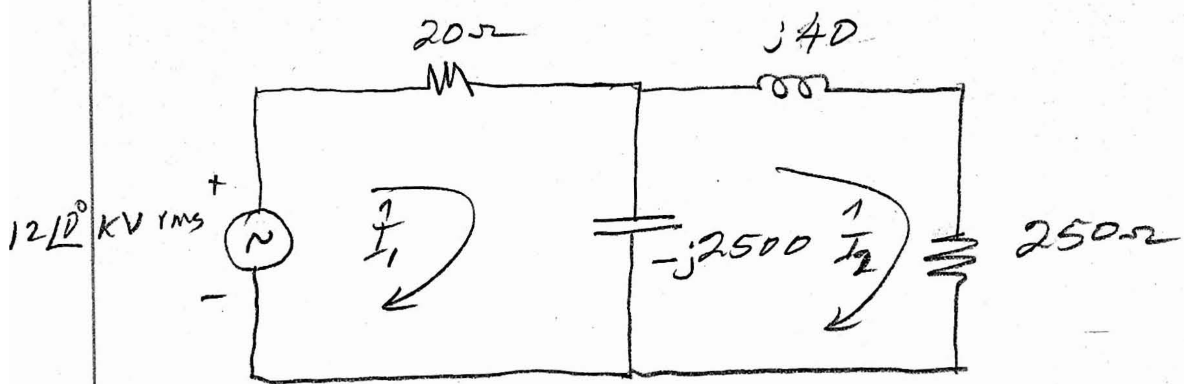
(c) Is the result true for the values of apparent power?

(d) What is the average power delivered by the source? 521.2 kW

(e) What is the reactive power delivered by the source? 27.76 kVA (inductive)



Draw the phasor circuit.



Find $\vec{I}_1$ and $\vec{I}_2$ first

(9) cont

By inspection

$$\begin{bmatrix} 20-j2500 & j2500 \\ j2500 & 250-j2460 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 12\times10^3\angle 0 \\ 0 \end{bmatrix}$$

$\hat{I}_1 = 43.49\angle -3.05^\circ\ A_{rms}$ $\quad \hat{I}_2 = 43.97\angle -8.85^\circ\ A_{rms}$

$$\hat{S}_{20} = I_1^2 \times 20 = 43.49^2 \times 20$$

$$\boxed{\hat{S}_{20} = 37.83\ KVA}$$

$$S_{250} = I_2^2 \times 250 = (43.97)^2 \times 250$$

$$\boxed{S_{250} = 483.3\ KVA}$$

$$\hat{S}_c = |\hat{I}_1 - \hat{I}_2|^2(-j2500) = (4.45)^2(-j2500)$$

$$\boxed{\hat{S}_c = (49.51\angle -90)\ KVA}$$

$$\hat{S}_L = |I_2|^2\, 40\angle 90 = (43.97)^2 \times 40\angle 90$$

$$\boxed{\hat{S}_L = 77.33\angle 90\ KVA}$$

(4) cont

$$\sum \hat{S}_{LOADS} = (37.83 + 483.3 + j77.33 - j49.51)\ KVA$$

$$\sum \hat{S}_{LOADS} = (521.13 + j27.82)\ KVA$$

$$\sum \hat{S}_{LOADS} = 521.87 \angle 3.06^\circ\ KVA$$

From the source

$$\hat{S}_{source} = 12\times10^3 \times \hat{I}_1^* = 12\times10^3 (43.49 \angle +3.05$$

$$\hat{S}_{source} = \underline{521.88 \angle 3.05^\circ}\ KVA$$

compared to } check

$$\hat{S}_{LOADS} = 521.87 \angle 3.06^\circ\ KVA$$

(c) $S_{20\ Apparent} = 37.83\ KVA$

$S_{250\ apparent} = 483.3\ KVA$

$S_{C\ APPARENT} = |S_C| = 49.51\ KVA$

$S_{L\ apparent} = |S_L| = 77.33\ KVA$

$$\hat{S}_{LOAD\ APPARENT} = 37.83 + 483.3 + 49.51 + 77.33$$

$$S_{LOAD\ APPARENT} = 647.97\ KVA$$

(9) cont

$$S_{source_{APPARENT}} = |\dot{S}_{source}| = |521.88\angle 3.05| \text{ kVA}$$

$$S_{source_{APPARENT}} = 521.88 \text{ kVA}$$

$$S_{LOADS_{APPARENT}} = 647.97 \text{ kVA}$$

They are not the same. The answer is <u>NO</u>.

(d)

$$P_{source} = Re(\dot{S}_{source}) = Re(521.88\angle 3.05°)$$

$$P_{source} = Re[521.1 + j27.77]$$

$$P_{source} = 521.1 \text{ kW}$$

(e)

$$Q_{source} = Im(\dot{S}_{source}) = 27.77 \text{ kVARs inductive}$$