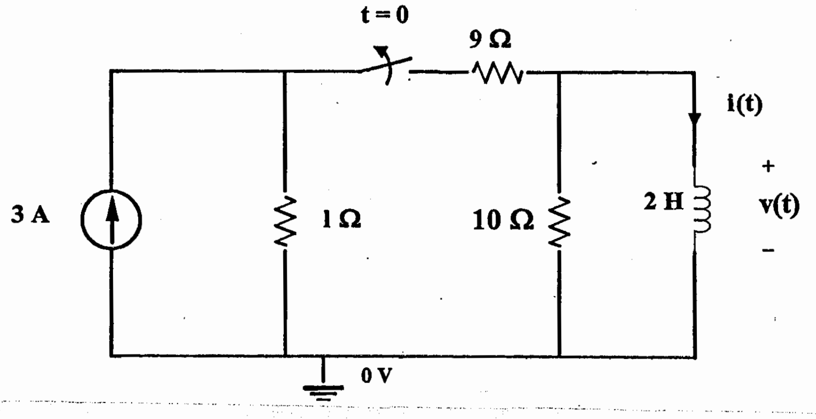
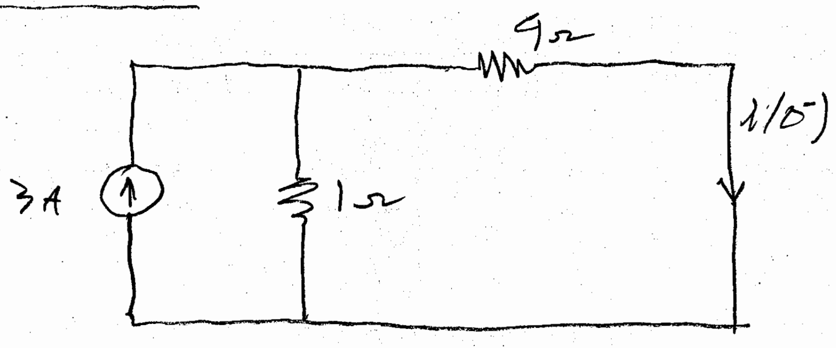
ECE 300
Spring Semester, 2008
Test #3

wlg: **Test B : Section II: 8:10 AM**

(1) The switch in the circuit of Figure 1 has been closed for a very long time and is opened at t = 0.
(a) Find the voltage v(t).
(b) At what time (in milliseconds) will v(t) equal to 50% of the initial value of $v(0^+)$?



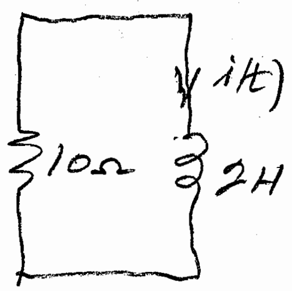
1(a) For $t<0$



$$i(0^-) = \frac{3\times 1}{10} = 0.3A$$

$$i(0^+) = i(0^-) = 0.3A \qquad (1)$$

$t>0$



Test B

$i(\infty) = 0$

$\tau = \frac{L}{R} = \frac{2}{10} = 0.2$

$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-\frac{t}{\tau}}$

$i(t) = 0.3 e^{-5t} A \quad u(t)$

One could solve directly for $v(t)$.

$v(t) = L \frac{di}{dt} = 2 \left[-0.3 \times 5 e^{-5t} \right]$

$v(t) = -3 e^{-5t} V$

or $v(t) = -iR = -i \times 10 = -3 e^{-5t} V$

(b)

$v(0^+) = -3$

$50\% \, v(0^+) = -1.5$

$-1.5 = -3 e^{-5t}$

$0.5 = e^{-5t}$

$\ln 0.5 = -5t \ln e = -5t$

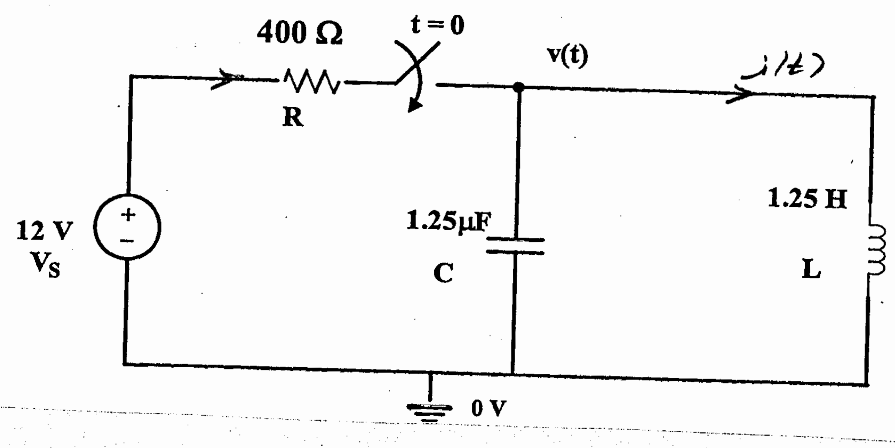
$5t = 0.693$

$\boxed{t = 0.139 \text{ sec}}$

$\boxed{t = 139 \text{ msec}}$

Test B

(2) There is no energy stored in the circuit of Figure 2 when the switch is closed at t = 0. Find v(t) for $t \ge 0$.



For $t<0$

$v(0^-)=0 \quad \therefore \quad v(0^+) = 0 \qquad (1)$

$i(0^+) = 0$

For $t>0$

$$\frac{v-V_s}{R} + C\frac{dv}{dt} + \frac{1}{L}\int_0^t v\,dt + i(0^+) = 0 \qquad (2)$$

Take the $\frac{d}{dt}[\cdot]$ of (2)

$$\frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{v}{L} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad (3)$$

Char. Eq

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} \qquad (4)$$

With numbers

$$s^2 + 2{,}000s + 640{,}000 = 0 \qquad (5)$$

Roots of char. eq.

$(s+400)(s+1600)$

$$v(t) = A_1 e^{-400t} + A_2 e^{-1600t} \qquad (6)$$

$v(0^+) = 0$

$$0 = \left(A_1 e^{-400t} + A_2 e^{-1600t}\right)\Big|_{t=0}$$

$$\boxed{0 = A_1 + A_2}$$

From (2) at $t = 0^+$

$$C\frac{dv(0^+)}{dt} = \frac{V_s}{R} - \cancel{\frac{v(0^+)}{R}}^{\text{from (1)}} - \cancel{i(0^+)}^{0}$$

$$\frac{dv(0^+)}{dt} = 24{,}000$$

Take $\frac{d[\cdot]}{dt}$ of (6)

$$\frac{dv(t)}{dt} = -400 A_1 e^{-400t} - 1600 A_2 e^{-1600t} \qquad (7)$$

Evaluate (7) at $t = 0^+$

$$\frac{dv(0^+)}{dt} = 24{,}000 = -400 A_1 - 1600 A_2$$

or $\quad 60 = -A_1 - 4A_2$

$$\begin{bmatrix} 1 & 1 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 60 \end{bmatrix} \qquad \begin{matrix} A_1 = 20 \\ A_2 = -20 \end{matrix}$$

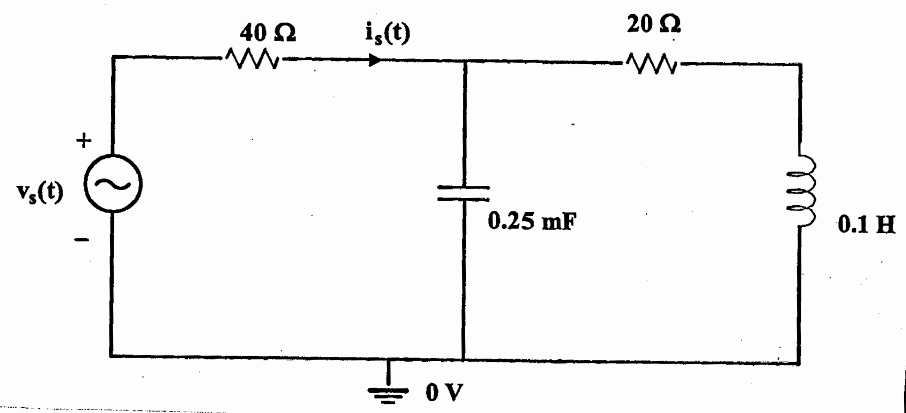
$$\boxed{v(t) = \left[20 e^{-400t} - 20 e^{-1600t}\right] u(t) \ \text{V}}$$

(3) The circuit in Figure 3 is in steady state. $v_s = 60\sin(200t + 40^\circ)$ V.

(a) Determine the steady state current $i_s(t)$ using a cosine reference.

(b) Draw a phasor diagram showing $\mathbf{V_S}$ and $\mathbf{I_S}$.

(c) Does $\mathbf{V_S}$ lead $\mathbf{I_S}$ or $\mathbf{I_S}$ lead $\mathbf{V_S}$ and give the angle?



$V_s = 60\sin(200t + 40^\circ)$ V take to cosine reference

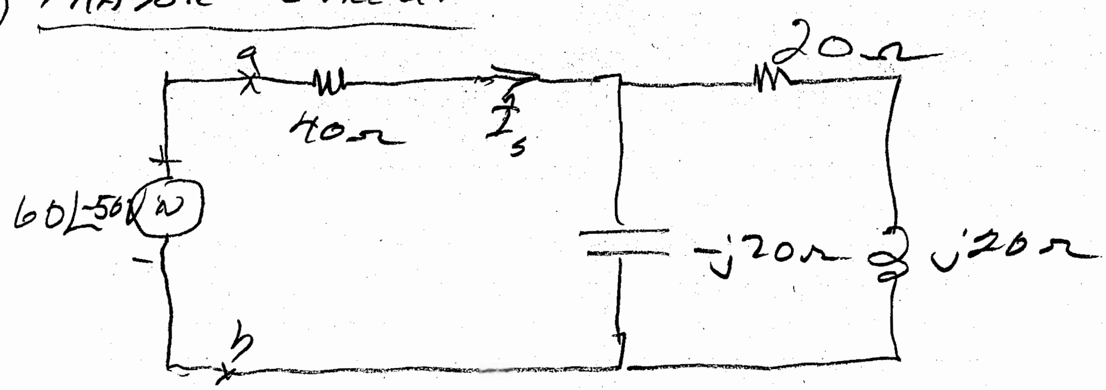
$V_s = 60\cos(200t - 50^\circ)$ V

$\hat{V}_s = 60\angle{-50}$

$0.1H \rightarrow j\omega H \rightarrow j200 \times 0.1 \rightarrow j20\,\Omega$

$0.25C \rightarrow \dfrac{-j}{\omega C} \rightarrow \dfrac{-j}{200 \times .25 \times 10^{-3}} \rightarrow -j20$

(a) Phasor circuit



$$Z_{ab} = 40 + \frac{(20 + j20) \,//\, (-j20)}{20 + j20 - j20}$$

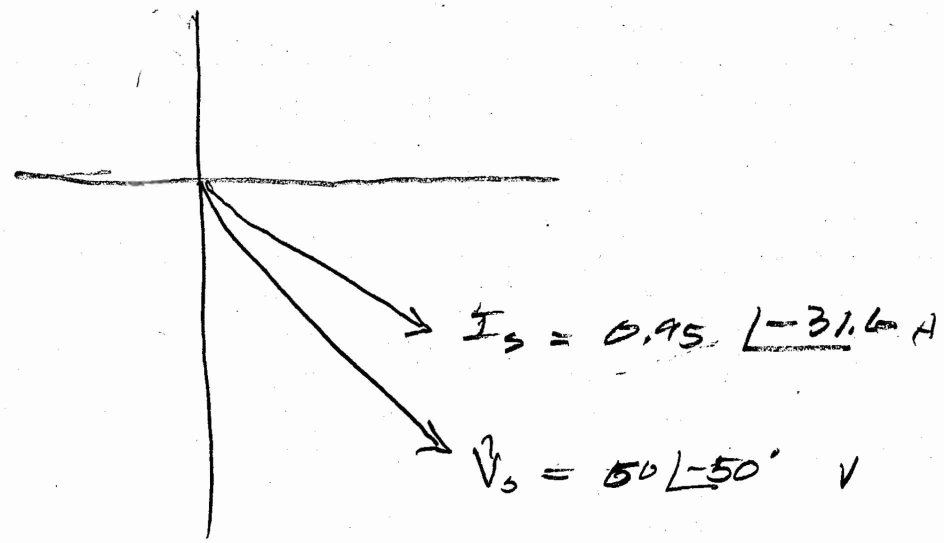
(3) cont.

$$Z_{ab} = 63.25 \angle{-18.4^\circ}\ \Omega$$

$$\hat{I}_s = \frac{60\angle{-50}}{63.25\angle{-18.4}} = 0.95\angle{-31.6}\ A$$

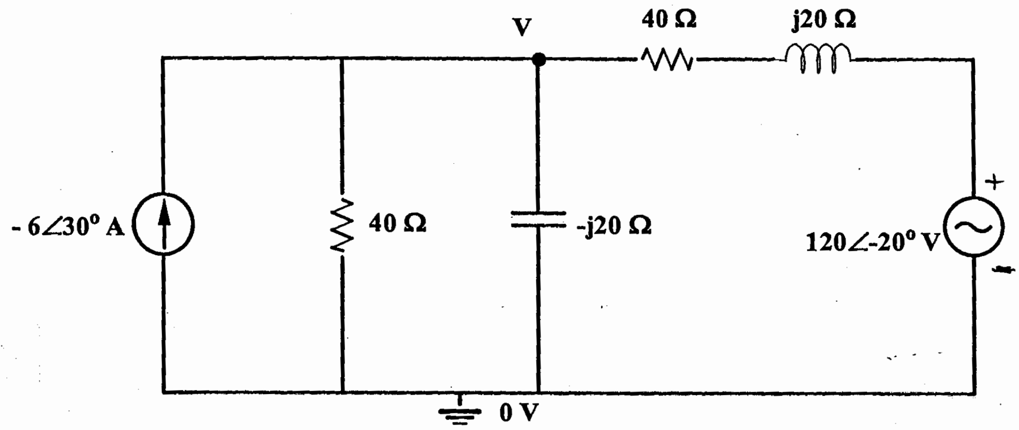
$$\boxed{i_s(t) = 0.95\cos(200t - 31.6^\circ)\ A}$$

(b)



(c) $\hat{I}_s$ leads $\hat{V}_s$ by $50 - 31.6 = 18.4^\circ$

(4) Use nodal analysis to find the phasor voltage V in Figure 4.



At $\hat{V}$

$$\frac{\hat{V} - 120\angle{-20}}{40+j20} - \frac{j\hat{V}}{20} + \frac{\hat{V}}{40} = -6\angle{30}$$

$$(0.02 - j0.01)\hat{V} + 2.68\angle{133.4} + j0.05\hat{V} + 0.025\hat{V} = -6\angle{30}$$

$$(0.045 + j0.04)\hat{V} = -2.68\angle{133.4} - 6\angle{30}$$

$$\boxed{\hat{V} = 99.3\angle{-165.8}\ V}$$