Lesson 8
A.C. Circuits
Examples

Nodal Analysis

Nodal analysis, as explained in DC circuits, can also apply to A.C. Circuits. The difference being that we are dealing with phasors and impedance.

Example 8.1

Use nodal analysis to find $V_1(t)$ and $V_2(t)$ in the following circuit,

![Circuit Diagram]

Figure 8.1.1 Circuit for Example 8.1

Three points of note here:

1. We establish the impedance of the capacitors and inductors using the formula $Z = \omega L$.

2. Rather than solve for $V_1(t)$, $V_2(t)$, $V_x(t)$ we use phasor voltages then change to the steady state time domain.
(3) We leave the source as a sine reference. Most of the time the reference is given as cosine.

Impedance

\[ Z = \frac{-j}{2x,2} = -j2.5 \Omega \]

\[ 2H \quad j \times 2 = j4 \Omega \]

**Draw the Phase Circuit**

![Diagram of a phase circuit](image)

**Figure 8.2: Phase Circuit for Example 8.1**

At node \( V_1 \):

\[ \frac{V_1}{2} + \frac{V_1 - V_2}{-j2.5} = 10 \]

At node \( V_2 \):

\[ \frac{V_2 - V_1}{-j2.5} + \frac{V_2}{j4} + \frac{V_2 - 3V_1}{4} = 0 \]
\[
\left( \frac{1}{2} - \frac{1}{j2.5} \right) \hat{V}_1 + \frac{\hat{V}_2}{j2.5} = 10
\]

\[
\left( -\frac{3}{4} + \frac{1}{j2.5} \right) \hat{V}_1 + \left( \frac{1}{4} + \frac{1}{j4} - \frac{1}{j2.5} \right) \hat{V}_2 = 0
\]

\[
(0.5 + j0.4) \hat{V}_1 - j0.4 \hat{V}_2 = 10
\]

\[
(-0.75 - j0.4) \hat{V}_1 + (0.25 + j0.15) \hat{V}_2 = 0
\]

\[
\begin{bmatrix}
(0.5+j0.4) & (0-j0.4) \\
(-0.75-j0.4) & (0.25+j0.15)
\end{bmatrix}
\begin{bmatrix}
\hat{V}_1 \\
\hat{V}_2
\end{bmatrix}
= \begin{bmatrix}
10 \\
0
\end{bmatrix}
\]

\[
\hat{V}_1 = 11.3 / 60^\circ \text{ V} \quad \hat{V}_2 = 33 / 57.1 \text{ V}
\]

\[
\therefore \hat{V}_1(t) = 11.3 \sin(2t + 60^\circ) \text{ V} \quad \hat{V}_2(t) = 33 \sin(2t + 57.1) \text{ V}
\]

Example 4.2

This problem illustrates mesh analysis of a 2 mesh circuit that does not have current sources or dependent sources. We will later work a problem with current sources.
Consider the circuit below. Find $I_1$, $I_2$, and $I_3$.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{circuit.png}
\caption{Circuit for Example 8.2.}
\end{figure}

**Method #1**

\[
20 I_1 - j15(I_1 - I_2) + j8(I_2 - I_3) = 12 e^{j45}\pi
\]

\[
(28 - j15) I_1 - 8 I_2 + j15 I_3 = 12 e^{j45}\pi
\]

**Method #2**

\[
8(I_2 - I_1) + j16(I_2 - I_3) - j25 I_2 = 0
\]

\[
-8 I_1 + (8 - j9) I_2 - j16 I_3 = 0
\]

**Method #3**

\[
-j15(I_3 - I_1) + 10 I_2 + j14(I_2 - I_3) = 0
\]

\[
-j15 I_1 - j16 I_2 + (10 + j1) I_3 = 0
\]

\[
\begin{bmatrix}
28-j15 & -8 & j15 \\
-8 & 8-j9 & -j16 \\
j15 & -j16 & 10+j1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
= 
\begin{bmatrix}
12 e^{j45}\pi \\
0 \\
0
\end{bmatrix}
\]

\[
I_1 = 0.38 \angle 9.6^\circ \ A, \ I_2 = 0.344 \angle 24.4^\circ \ A
\]

\[
I_3 = 0.146 \angle -60.4^\circ \ A
\]
Example 8.3

This problem illustrates the mesh analysis method when both independent and dependent sources are present. Consider the circuit of Figure 8.4.

Find \( V_o \).

\[
\begin{align*}
\text{Figure 8.41 Circuit for Example 8.3}.
\end{align*}
\]

We assign mesh currents as shown. We open those sources that are current sources. We note the relationships between the mesh currents and the current sources. Here,

\[
I_1 = 4 \angle 90^\circ \text{ A}, \quad I_3 = -2 \text{ A}
\]

We redraw the circuit as shown in Figure 8.5.
\[ \begin{align*}
J_2 & = 2 + 3 \quad \text{V}_2^- = 12 \quad \text{V} \\
J_2 & = 2 - 2 \quad \text{V}_2^+ = 0 \\
\text{Figure 8.5: Circuit for analysis, Ex 8.3} \\
\end{align*} \]

We write

\[ 2 (\frac{1}{J_2} - \frac{1}{J_1}) - J^3 J_2 + 12 L_0 + 2 (J_2 - J_3) = 0 \]

or

\[ -2 \frac{1}{J_1} + (4 - J^3) \frac{1}{J_2} - 2 \frac{1}{J_3} = -12 L_0 \]

\[ \begin{align*}
\text{V}_1 & = 4 \text{ L}, \\
\text{I}_2 & = -2 \\
-2 (J^4) + (4 - J^3) \frac{1}{J_2} - 2 (J - 2) & = -12 L_0 \\
(4 - J^3) \frac{1}{J_2} & = -12 + J^4 = -16 + j8 \\
\text{I}_2 & = \frac{-16 + j8}{4 - j3} = -3.52 - j0.64 \\
\text{V}_2 & = (\frac{1}{J_1} - \text{I}_2) \text{V} = (4 + 3.52 + j0.64) \text{V} \\
\text{V}_2 & = (3.52 + j4.64) \text{V} \\
\text{V}_0 & = 11.65 \text{ L}2.8^\circ \text{V} \quad \text{Ans.}
\end{align*} \]
Example 8.4

This example illustrates how to find the Thévenin equivalent circuit of a AC circuit. Consider the circuit shown in Figure 8.1. Find the Thévenin and Norton circuits looking in terminals a-b. How much current will flow through a 1Ω resistor placed between a-b?

![Circuit Diagram](image)

Figure 8.1: Circuit for Example 8.4.

\[ Z_{TH} \]

We replace the 60 V source with a short and find the impedance looking in a-b.

\[ Z_{TH} = Z + (120)(5-j10) \]
\[ = Z + \frac{(0+j20)(5-j10)}{\sqrt{20} + 5-j10} \]
\[ Z_{TH} = Z + \frac{(20+j0)(5-j10)}{5+j10} \]
\[ Z_{TH} = 2 + (16 - j12) \]

\[ Z_{TH} = 16 - j12 = 21.6\angle -33.7^\circ \Omega \]

To find \( V_{TH} \) we apply the voltage division rule. Thus,

\[ V_{TH} = \frac{(60 \angle 120^\circ)(j20)}{5 - j10 + j20} \]

\[ V_{TH} = \frac{1}{5 + j10} \]

Note:

\( V_{TH} \) is the open circuit voltage across terminals \( a-b \). Since no current flows through the 2 \( \Omega \) resistor when \( a-b \) is open, there is then no voltage (0 volts) across the 2 \( \Omega \) resistor. Thus, \( V_{TH} \) becomes the voltage across the inductor. Therefore we apply the voltage divider rule to find the voltage across the inductor and hence \( V_{TH} \).
\[ V_{TH} = 107.33 \angle 146.6^\circ \text{ V} \]

The Thévenin circuit is shown below.

Figure 8.7: Thévenin circuit for Example 8.4.

When we place a 10 ohm resistor across \( a-b \) we solve for the current \( I \), which is the current in the 10 ohm resistor.

\[ V_{TH} = 107.33 \angle 146.6^\circ \text{ V} \]

Figure 8.8: Circuit of Example 8.4 with a 10 ohm load.

\[ I = \frac{107.33 \angle 146.6^\circ}{21.6 \angle -33.7^\circ + 10} = 3.53 \angle 169.8^\circ \text{ A} \]
To find the Norton circuit, we can divide $V_{TH}/I_{TH} = I_N$

$$I_N = \frac{167.33 \angle 146.6^\circ}{21.6 \angle -33.7^\circ}$$

$$I_N = 4.47 \angle -179.7^\circ \ A$$

Then the Norton circuit is as shown in Figure 8.9.

![Diagram of Norton circuit](image)

Figure 8.9: The Norton equivalent circuit for Example 8.4.

We also know that the Norton current source is the short circuit that flows from a to b in the circuit shown in Figure 8.10.
In this case, a source transformation has been made on the 60/120°V source and the \((5-j10)\)Ω impedance. This will allow direct application of the current splitting rule to find \(I_N\).

\[
\begin{align*}
5.37/176.6 & \\
\end{align*}
\]

\[
\begin{align*}
5-10 & 3+j20
\end{align*}
\]

Figure 8.10: Circuit for finding \(\hat{I}_N\) in Example 8.4.

\[
\hat{I}_N = \frac{(5.37/176.6)(5-j10)(j20)}{2 + (5-j10)(j20)}
\]

\[
\hat{I}_N = \frac{5.37/176.6(16-j12)}{2 + 16-j12}
\]

\[
\hat{I}_N = 4.97/179.8° \ A
\]

This agrees with the previous clot, \(QED\).