ECE 301
Fall Semester, 2005
HW #6

You are given the circuit of Figure 1.

(a) If the phasor current \( I_{ad} = 0.5 \angle 0 \) A, find the phasor current \( I_{ca} \). Ans: \( 0.236 \angle -45^\circ \) A.

(b) Find the phasor source voltage, \( V_s \). \( 1.57 \angle -58^\circ \) V.

(2) Find the impedance seen by the source voltage \( (Z_{cb}) \) of Figure 1. Ans: Verify by using \( V_s \) and \( I_{ca} \) of problem 1.

(3) Determine \( Y_{eq} \) for the circuit of Figure 3. Ans: \( 0.521 \angle 24.9^\circ \) S
(4) Given the circuit of Figure 4. Use mesh analysis to find the phasor currents $I_1$ and $I_2$. 
Ans: $I_1 = 0.27\angle15.8\, \text{A}$, $I_2 = 2.09\angle152.9\, \text{A}$.

Figure 4: Circuit for problem 4.

(5) You are given the circuit of Figure 6. Find $V_o$ using nodal analysis. Ans $V_o = 4.04\angle70.35\, \text{V}$

Figure 5: Circuit for problem 5.

(6) Find the Thevenin equivalent circuit to the left of a-b of Figure 6. Ans: $V_{TH} = -50\angle30\, \text{V}$
$Z_{TH} = (10 - j20)\, \text{Ω}$

Figure 6: Circuit for problem 6.
(1) You are given the circuit of Figure 1.
(a) If the phasor current $I_{ad} = 0.5 \angle 0 \, \text{A}$, find the phasor current $I_{ca}$. Ans: $0.236 \angle -45^\circ \, \text{A}$.
(b) Find the phasor source voltage, $V_s$. $1.57 \angle -58^\circ \, \text{V}$.

12) $V_{ab} = I_{ad} (1-j2) = 0.5 (1-j2) = 1.12 \angle -63.4^\circ \, \text{V}$

$\bar{I}_{ab} = \frac{\bar{V}_{ab}}{j\omega} = \frac{(1.12 \angle -63.4^\circ)}{3 \angle 90^\circ} = 0.373 \angle -153.4^\circ \, \text{A}$

So, $\bar{I}_{ca} = I_{ad} + \bar{I}_{ab}$

$\bar{I}_{ca} = 0.5 + (0.373 \angle -153.4^\circ)$

$\bar{I}_{ca} = 0.236 \angle -45^\circ \, \text{A}$

(b) $\bar{V}_b = 2 \times \bar{I}_{ca} + \bar{V}_{ab}$

$\bar{V}_b = 2 \times (0.236 \angle -45^\circ) + (1.12 \angle -63.4^\circ)$

$\bar{V}_b = 1.57 \angle -58^\circ \, \text{V}$
(2) Find the impedance seen by the source voltage \( Z_{ab} \) of Figure 1. Ans: Verify by using \( \mathbf{V}_s \) and \( \mathbf{I_{ca}} \) of problem 1.

\[
Z_{ab} = \frac{(3 \angle 90^\circ) \times (1 - j2)}{j3 + 1 - j2} = 4.5 - j1.5
\]

Then

\[
Z_{cb} = 2 + Z_{ab} = 2 + 4.5 - j1.5
\]

\[
Z_{cb} = 6.5 - j1.5 = 6.67 \angle -13^\circ \text{ ohms}
\]

To verify

\[
Z_{cb} = \frac{\mathbf{V}_b}{\mathbf{I_{ca}}}
\]

\[
Z_{cb} = \frac{1.57 \angle -58^\circ}{0.236 \angle -45^\circ}
\]

\[
Z_{cb} = 6.65 \angle -13^\circ \text{ check}
\]
(3) Determine $Y_{eq}$ for the circuit of Figure 3. Ans: $0.521 \angle 24.9^\circ \ \Omega$

$$Y_{eq} = \left( \frac{1}{-j4} \right) + \frac{1}{(3+j1)} + \frac{1}{(5-j2)}$$

$$Y_{eq} = j0.25 + 0.3-j0.1 + 1.172 + j0.069$$

$$Y_{eq} = (1.3 + 1.172) + j(0.25-0.1+0.069)$$

$$Y_{eq} = 0.472 + j0.219$$

$$Y_{eq} = 0.521 \angle 24.9^\circ \ \Omega$$
(4) Given the circuit of Figure 4. Use mesh analysis to find the phasor currents $I_1$ and $I_2$.

Ans: $I_1 = 0.27\angle15.8^\circ$ A, $I_2 = 2.09\angle152.9^\circ$ A.

\[ 10\frac{d}{dt}I_1 + (15 + j20)(I_1 - I_2) - 60\angle130 = 0 \]

\[ (25 + j20)I_1 + (-15 - j20)I_2 = 60\angle130 \]

\[ (15 + j20)(I_2 - I_1) + (-40 - j20)I_2 + 25I_2 + 40\angle-50 \]

\[ (-15 - j20)I_1 + (40 - j20)I_2 = -90\angle-50 \]

\[
\begin{bmatrix}
25 + j20 & -15 - j20 \\
-15 - j20 & 40 - j20
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
60\angle130 \\
-90\angle-50
\end{bmatrix}
\]

$\vec{I}_1 = 0.27\angle15.8^\circ$ A

$\vec{I}_2 = 2.09\angle152.9^\circ$ A
(5) You are given the circuit of Figure 6. Find \( V_o \) using nodal analysis. Ans \( V_o = 4.04 \angle 70.35^\circ \) V

\[
\frac{V_o}{2} + \frac{V_o - V_2}{j2} + \frac{V_1 - V_2}{j4} + \frac{V_1 - V_2}{4} = 0
\]

\[
0.5 V_o - j0.5 V_o + j0.5 V_2 + j0.25 V_1 + 0.25 V_1 - 0.25 V_2 = 0
\]

\[
(0.5 - j0.5) V_o + (0.25 + j0.25) V_1 + (-0.25 + j0.5) V_2 = 0
\]

\[
\frac{V_2 - V_1}{4} + \frac{V_2 - V_o}{j2} + 0.2 V_0 = 0
\]

\[
0.25 V_2 - 0.25 V_1 - j0.5 V_2 + j0.5 V_0 + 0.2 V_0 = 0
\]

\[
(0.25 + j0.5) V_0 - 0.25 V_1 + (0.25 - j0.5) V_2 = 0
\]

\[
V_o - 12 - V_1 = 0
\]

\[
V_o - V_1 + 0.2 V_2 = 12
\]

\[
\begin{bmatrix}
0.5 - j0.5 & 0.25 + j0.25 & -0.25 + j0.15 \\
0.2 + j0.5 & -0.25 & 0.25 - j0.5 \\
1 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
V_o \\
V_1 \\
V_2
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
12
\end{bmatrix}
\]

\[
V_o = 4.04 \angle 70.35^\circ \text{ V}
\]
(6) Find the Thevenin equivalent circuit to the left of a-b of Figure 6. Ans: \( V_{TH} = -50 \angle 30^\circ \text{ V} \)
\( Z_{TH} = (10 - j20) \ \Omega \)

\[ Z_{TH} = \frac{(10 + j90)(20 - j90)}{-j10 + j20} + 10 \ \Omega \]

\[ Z_{TH} = (-j20 \ \Omega + 10) \ \Omega = (10 - j20) \ \Omega \]

\[ V_{TH} = \frac{(50 + j30)(10 - j90)}{j20 - j10} = \frac{500 - 60}{10 + j90} \]

\[ V_{TH} = 50 - 150^\circ \text{ V} = -50 + 150^\circ \text{ V} \]

Thevenin Circuit