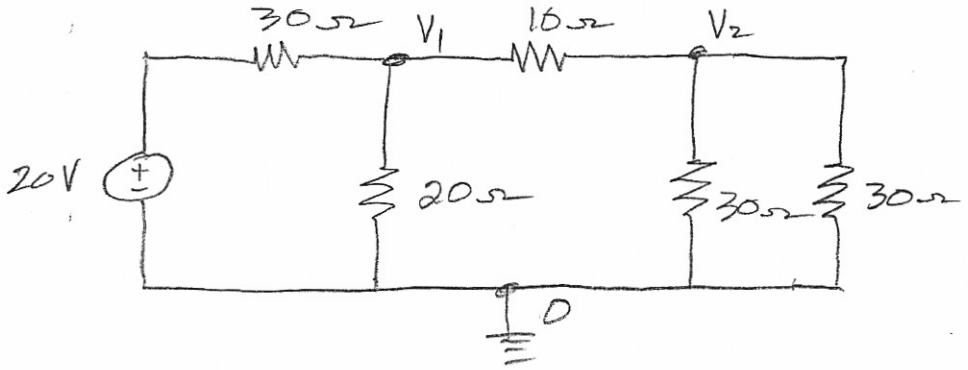
WRG

ECE 301
HW # 2

Desk Solutions

(1) 3.2 from text.

Find voltages V_1 and V_2 for the following circuit.



At V_1

$$\frac{V_1 - 20}{30} + \frac{V_1}{20} + \frac{V_1 - V_2}{10} = 0$$

60

$$2V_1 - 40 + 3V_1 + 6V_1 - 6V_2 = 0$$

$$\boxed{11V_1 - 6V_2 = 40}$$

At V_2

$$\frac{V_2 - V_1}{10} + \frac{V_2}{30} + \frac{V_2}{30} = 0$$

$$3V_2 - 3V_1 + V_2 + V_2 = 0$$

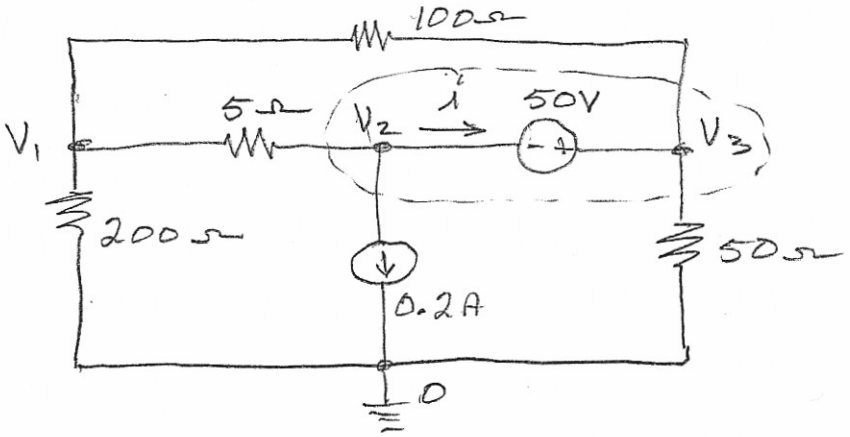
$$\boxed{-3V_1 + 5V_2 = 0}$$

$$\begin{bmatrix} 11 & -6 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \end{bmatrix}$$

$V_1 = 5.41\ V \qquad V_2 = 3.24\ V$

(2) Problem 3.9

For the following circuit find V_1, V_2, V_3, and i using nodal analysis



Method 1: Supernode

At V_1

$\times 200$ $\quad \frac{V_1}{200} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{100} = 0$

$$V_1 + 40V_1 - 40V_2 + 2V_1 - 2V_3 = 0$$

$$43V_1 - 40V_2 - 2V_3 = 0 \qquad (1)$$

At The Supernode

$\times 100$ $\quad \frac{V_2 - V_1}{5} + 0.2 + \frac{V_3}{50} + \frac{V_3 - V_1}{100} = 0$

$$20V_2 - 20V_1 + 2V_3 + V_3 - V_1 = -20$$

$$-21V_1 + 20V_2 + 3V_3 = 0$$

(2) cont. Method 1:

Constraint Equation

$$V_2 + 50 - V_3 = 0$$

$$0V_1 + V_2 - V_3 = -50 \quad (3)$$

From (1), (2) & (3) IN MATRIX FORM

$$\begin{bmatrix} 43 & -40 & -2 \\ -21 & 20 & 3 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \\ -50 \end{bmatrix}$$

Calculator solution;

$V_1 = -45.23\,V$, $V_2 = -48.69\,V$, $V_3 = 1.31\,V$

At V_2

$$\frac{V_2 - V_1}{5} + 0.2 = i$$

$$\frac{-48.69 + 45.23}{5} + 0.2 = i$$

$$i = -0.492\,A$$

Method 2; Direct Method

At V_1 (SAME AS before)

$$43V_1 - 40V_2 - 2V_3 + 0i = 0 \quad (4)$$

(2) continued method 2

At V_2

$\times 5$ $\quad \frac{V_2 - V_1}{5} + 0.2 + i = 0$

$V_2 - V_1 + 1 + 5i = 0$

$$-V_1 + V_2 + 0V_3 + 5i = -1 \qquad (5)$$

At V_3

$\times 100$ $\quad \frac{V_3 - V_1}{100} + \frac{V_3}{50} - i = 0$

$V_3 - V_1 + 2V_3 - 100i = 0$

$$-V_1 + 0V_2 + 3V_3 - 100i = 0 \qquad (6)$$

Constraint Equation

$$0V_1 + V_2 - V_3 + 0i = -50 \qquad (7)$$

From (4), (5), (6) and (7)

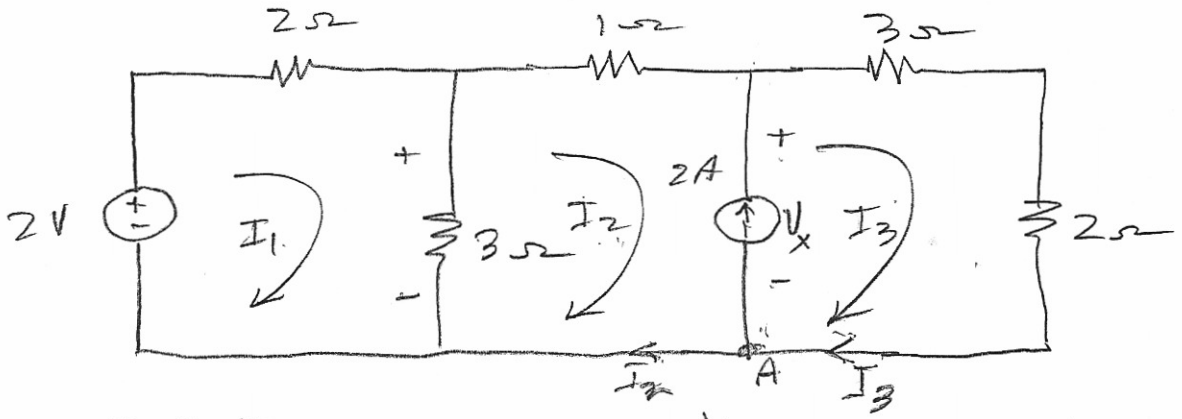
$$\begin{bmatrix} 43 & -40 & -2 & 0 \\ -1 & 1 & 0 & 5 \\ -1 & 0 & 3 & -100 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -50 \end{bmatrix}$$

$V_1 = -45.23\ V$, $V_2 = -48.69\ V$, $V_3 = 1.31\ V$

$i = 0.492\ A$ (same as method 1)

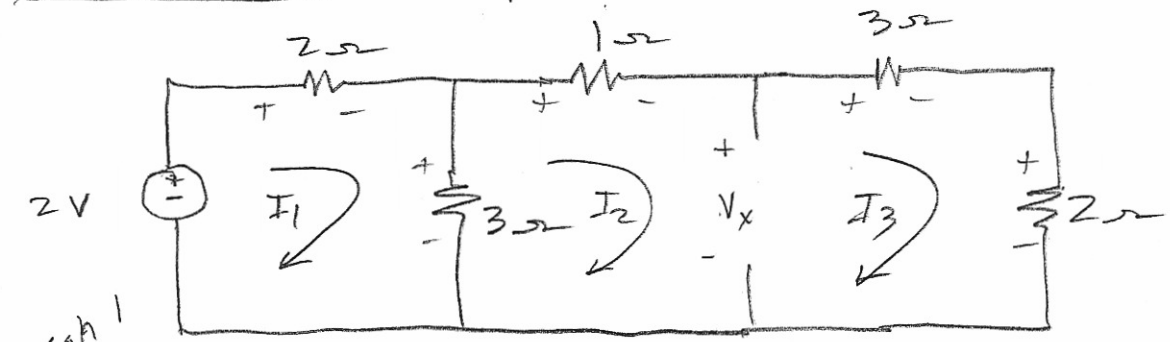
check

(4) Prob 3.17



Find I_1, I_2, I_3 and V_x

Method 1 Super mesh



mesh 1

$$-2 + 2I_1 + 3(I_1 - I_2) = 0$$

$$\boxed{5I_1 - 3I_2 + 0I_3 = 2} \quad (1)$$

super mesh

$$-3(I_1 - I_2) + 1 \times I_2 + 3I_3 + 2I_3 = 0$$

$$\boxed{-3I_1 + 4I_2 + 5I_3 = 0} \quad (2)$$

Constraint Equation (At A)

$$I_3 = 2 + I_2$$

$$\boxed{0I_1 - I_2 + I_3 = 2} \quad (3)$$

(4). Prob 3,17 cont

From (1), (2), (3)

$$\begin{bmatrix} 5 & -3 & 0 \\ -3 & 4 & 5 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$I_1 = -0.33\,A$, $I_2 = -1.22\,A$, $I_3 = 0.778\,A$

$V_x = 5 I_3 = 3.89\,V$

Method 2: Direct

Mesh 1 is the same as for method 1.

$$5I_1 - 3I_2 + 0I_3 + 0V_x = 2 \quad (4)$$

mesh 2

$$-3(I_1 - I_2) + 1 \times I_2 + V_x = 0$$

$$-3I_1 + 4I_2 + 0I_3 + V_x = 0 \quad (5)$$

mesh 3

$$-V_x + 3I_3 + 2I_3 = 0$$

$$0I_1 + 0I_2 + 5I_3 - V_x = 0 \quad (6)$$

From the current constraint (same as before)

$$0I_1 - I_2 + I_3 + 0V_x = 2 \quad (7)$$

(4) Prob 3.17 cont
method 2

From (4), (5), (6), (7)

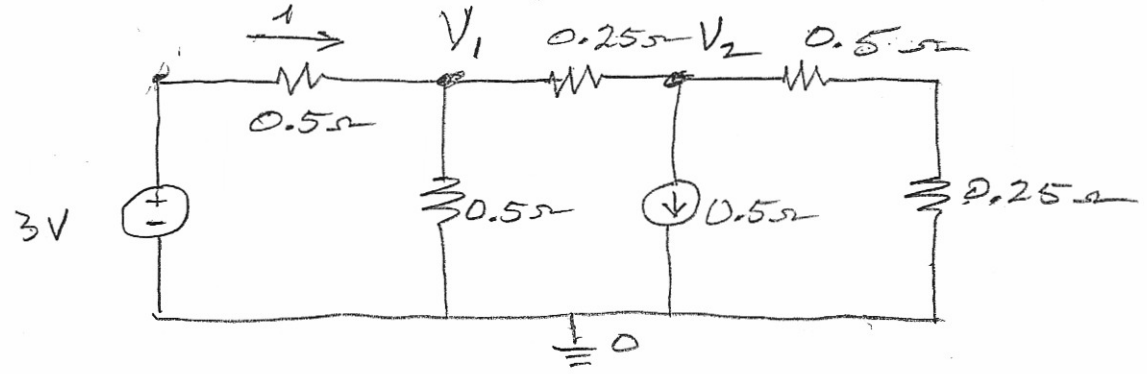
$$\begin{bmatrix} 5 & -3 & 0 & 0 \\ -3 & 4 & 0 & 1 \\ 0 & 0 & 5 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_x \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

$I_1 = -0.33\ A$, $I_2 = -1.2\ A$, $I_3 = 0.778\ A$

$V_x = 3.88\ V$ check

(3) 3.11

Use nodal analysis to find V_1, V_2 & i in the following



At V_1

$$\times .5 \quad \frac{V_1 - 3}{.5} + \frac{V_1}{.5} + \frac{V_1 - V_2}{0.25} = 0$$

$$V_1 - 3 + V_1 + 2V_1 - 2V_2 = 0$$

$$\boxed{4V_1 - 2V_2 = 3} \qquad (1)$$

At V_2

$$\times .75 \quad \frac{V_2 - V_1}{0.25} + \frac{V_2}{.75} + 0.5 = 0$$

$$3V_2 - 3V_1 + V_2 = -0.375$$

$$\boxed{-3V_1 + 4V_2 = -0.375} \qquad (2)$$

From (1) & (2)

$$\begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -.375 \end{bmatrix}$$

(3) prob 3.17

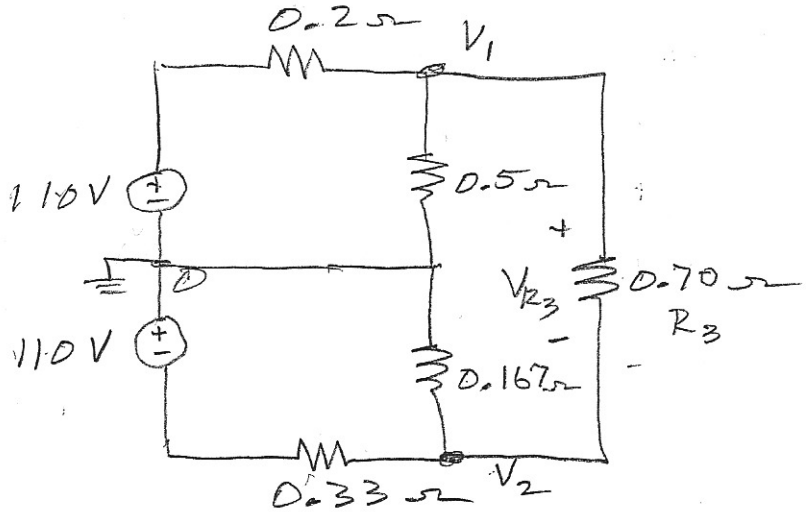
$V_1 = 1.125\ V$ $\qquad$ $V_2 = 0.75\ V$

$$i = \frac{3 - V_1}{.5} = 2(3 - V_1)$$

$$i = 3.75\ A$$

(5) Prob 3.22

Given the following circuit.



(a) solve by nodal analysis

At V_1

$$\frac{V_1 - 110}{.2} + \frac{V_1}{.5} + \frac{V_1 - V_2}{.7} = 0$$

$$5V_1 - 550 + 2V_1 + 1.43V_1 - 1.43V_2 = 0$$

$$\boxed{8.43V_1 - 1.43V_2 = 550} \quad (1)$$

At V_2

$$\frac{V_2}{.167} + \frac{V_2 - V_1}{.7} + \frac{V_2 + 110}{.33} = 0$$

$$5.99V_2 + 1.43V_2 - 1.43V_1 + 3V_2 + 330 = 0$$

$$\boxed{-1.43V_1 + 10.42V_2 = -330} \quad (2)$$

(5) p 3.22 nodal

$$\begin{bmatrix} 8.43 & -1.43 \\ -1.43 & 10.42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 550 \\ -330 \end{bmatrix}$$

$V_1 = 61.3\,V, \quad V_2 = -23.26\,V$

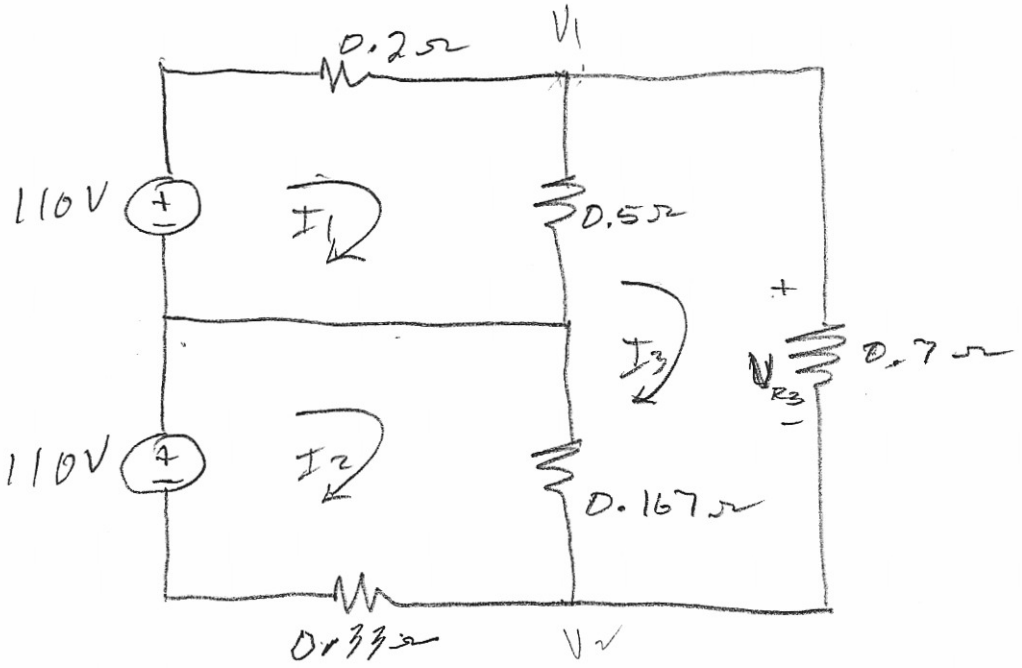
From my circuit;

$V_1 - V_{R3} - V_2 = 0$

$V_{R3} = V_1 - V_2 = 84.56\,V$

$\boxed{V_{R3} = 84.56\,V}$ within rounding

Mesh Analysis



(5) P 3.22

Mesh 1

$$-110 + .2I_1 + .5(I_1 - I_3) = 0$$

$$\boxed{0.7I_1 + 0 \cdot I_2 - 0.5I_3 = 110} \quad (3)$$

Mesh 2

$$.167(I_2 - I_3) + .333I_2 - 110 = 0$$

$$\boxed{0I_1 + 0.5I_2 - 0.167I_3 = 110} \quad (4)$$

Mesh 3

$$-.5(I_1 - I_3) + 0.7I_3 - 0.167(I_2 - I_3) = 0$$

$$\boxed{-0.5I_1 - 0.167I_2 + 1.367I_3 = 0} \quad (5)$$

From (3), (4), (5)

$$\begin{bmatrix} 0.7 & 0 & -.5 \\ 0 & 0.5 & -.167 \\ -.5 & -.167 & 1.367 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 110 \\ 110 \\ 0 \end{bmatrix}$$

$I_1 = 243.47\ A$, $I_2 = 260.37A$, $I_3 = 120.86\ A$

$$V_{R3} = I_3 \times 0.7 = 120.86 \times 0.7$$

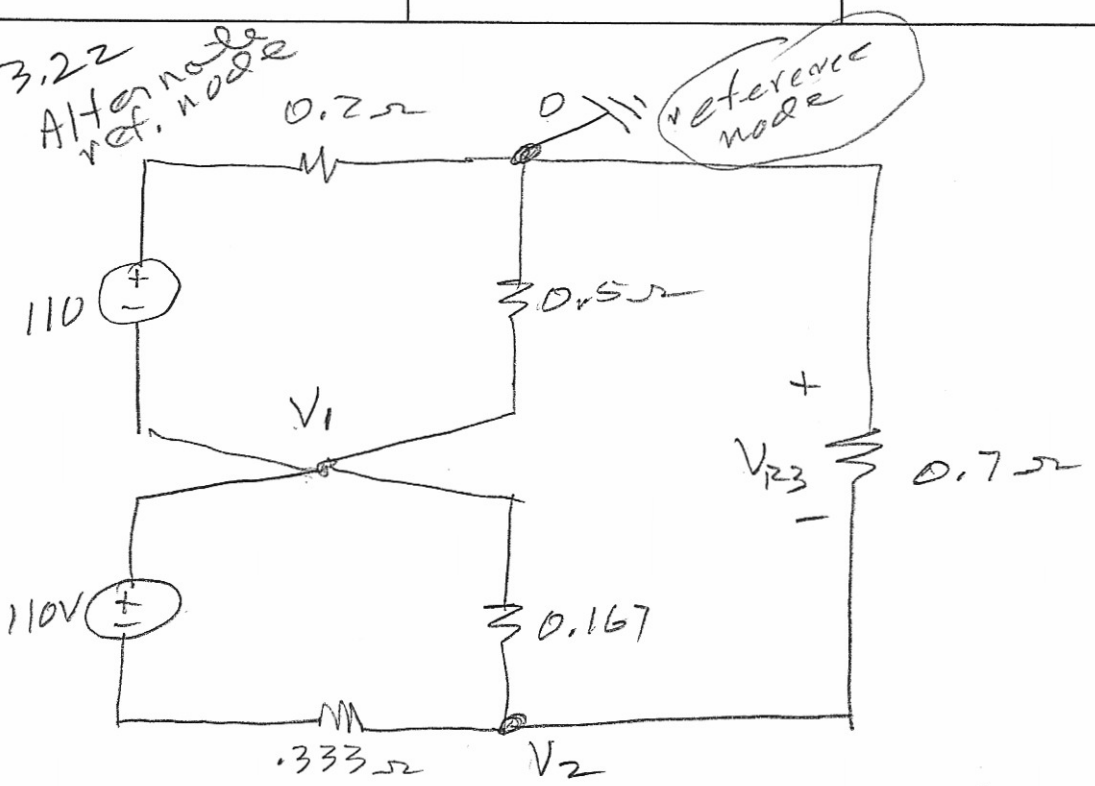
$$\boxed{V_{R3} = 84.6 \quad V}$$

check

Probably node is less work here.

(b)

3.22 Alternate ref. node



At V_1

$$\frac{V_1 + 110}{0.2} + \frac{V_1 - 110 - V_2}{.333} + \frac{V_1}{.5} + \frac{V_1 - V_2}{.167} = 0$$

so,

$$5V_1 + 550 + 3V_1 - 330 - 3V_2 + 2V_1 + 5.99V_1 - 5.99V_2 = 0$$

$$\boxed{15.99V_1 - 8.99V_2 = -220}$$

At V_2

$$\frac{V_2 + 110 - V_1}{.333} + \frac{V_2 - V_1}{.167} + \frac{V_2}{.7} = 0$$

so

$$3V_2 + 330 - 3V_1 + 5.99V_2 - 5.99V_1 + 1.43V_2 = 0$$

$$\boxed{-8.99V_1 + 10.42V_2 = -330}$$

(b) 3.22 cont
Alternate node

$$\begin{bmatrix} 15.99 & -8.99 \\ -8.99 & 10.42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -220 \\ -330 \end{bmatrix}$$

$V_1 = -61.3\,V$ $\quad$ $V_2 = -84.56\,V$

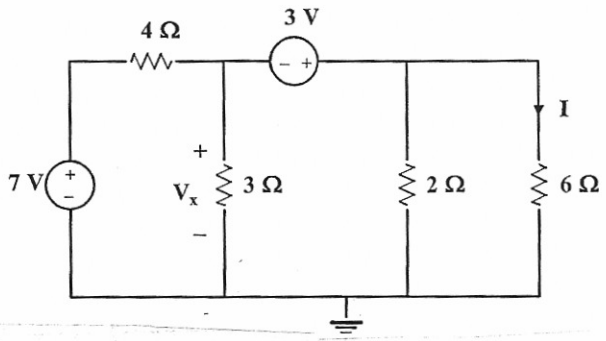
From the circuit:

$$V_{R3} = -V_2 = 84.56\,V$$

$V_{R3} = 84.56V$ (same as using other ref. point)

This V_1 is the negative of the previous V_1, as it should be.

(6) You are given the circuit in the following diagram. Use nodal analysis to find V_x and I. Treat the 3 V source as as a super node. Ans: $V_x = -0.2$ V, $I = 0.467$ A..



At V_x

$$\frac{V_x - 7}{4} + \frac{V_x}{3} + \frac{V_y}{2} + \frac{V_y}{6} = 0$$

$$3V_x - 21 + 4V_x + 6V_y + 2V_y = 0$$

$$\boxed{7V_x + 8V_y = 21}$$

Constraint Equation

$$V_x + 3 - V_y = 0$$

$$\boxed{V_x - V_y = -3}$$

$$\begin{bmatrix} 7 & 8 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} 21 \\ -3 \end{bmatrix}$$

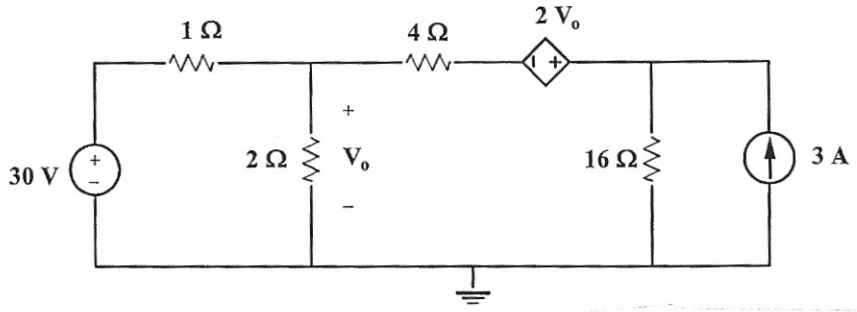
$$V_x = -0.2\,V, \quad V_y = 2.8\,V$$

FOR I:

$$I = \frac{V_y}{6} = \frac{2.8}{6}$$

$$I = 0.467\,A$$

(7) You are given the circuit in the following diagram. Use nodal analysis to find V_o. Ans: 19.64 V



At V_o

$$\frac{V_o - 30}{1} + \frac{V_o}{2} + \frac{V_o + 2V_o - V_1}{4} = 0$$

$$4V_o - 120 + 2V_o + 3V_o - V_1 = 0$$

$$9V_o - V_1 = 120$$

At V_1

$$\frac{V_1 - 2V_o - V_o}{4} + \frac{V_1}{10} = 3$$

$$4V_1 - 8V_o - 4V_o + V_1 = 48$$

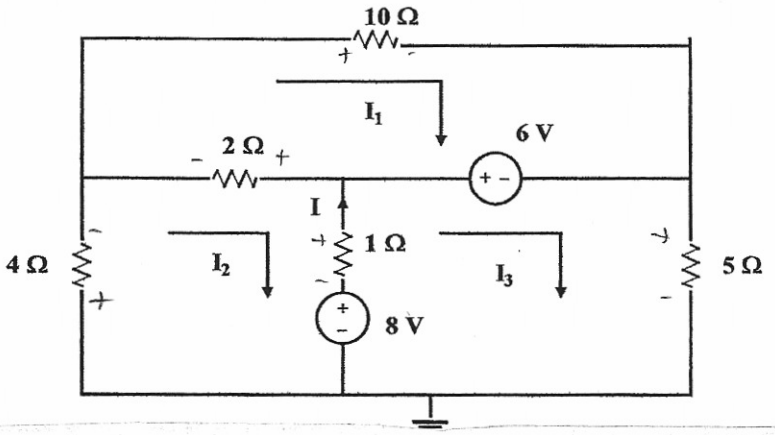
$$-12V_o + 5V_1 = 48$$

$$\begin{bmatrix} 9 & -1 \\ -12 & 5 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \end{bmatrix} = \begin{bmatrix} 120 \\ 48 \end{bmatrix}$$

$$V_o = 19.64 \text{ V}$$

$$V_1 = 56.73 \text{ V}$$

(8) You are given the circuit in the following diagram. Apply mesh analysis to find I_1, I_2, I_3 and I. Ans: I = 1.186 A: Other currents on your own.



Mesh 1

$$10I_1 - 6 + 2(I_1 - I_2) = 0$$

$$12I_1 - 2I_2 + 0I_3 = 6 \quad (1)$$

Mesh 2

$$4I_2 - 2(I_1 - I_2) + 1(I_2 - I_3) + 8 = 0$$

$$-2I_1 + 7I_2 - I_3 = -8 \quad (2)$$

mesh 3

$$-8 - (I_2 - I_3) + 6 + 5I_3 = 0$$

$$0I_1 - I_2 + 6I_3 = 2 \quad (3)$$

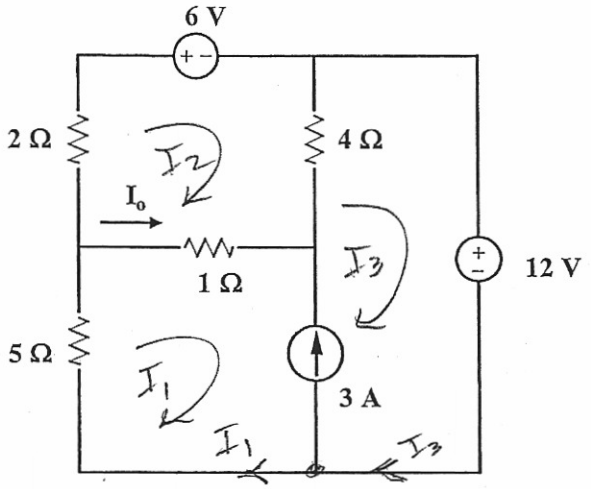
From (1), (2), (3)

$$\begin{bmatrix} 12 & -2 & 0 \\ -2 & 7 & -1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 2 \end{bmatrix}$$

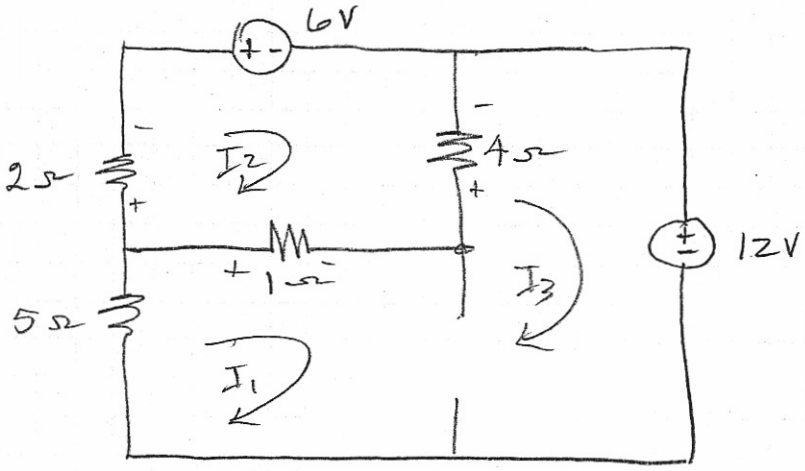
$I_1 = 0.329$ A $I_2 = -1.026$ A $I_3 = 0.162$ A

$I = I_3 - I_2 = 1.188$ A

(9) You are given the circuit in the following diagram. Use mesh analysis to find the current I_o.
Ans: $I_o = -1.733$ A



Omitting the current source but leave currents assigned.



Supermesh

$$5I_1 + 1\times(I_1 - I_2) + 4(I_3 - I_2) + 12 = 0$$

$$6I_1 - 5I_2 + 4I_3 = -12 \quad (1)$$

Mesh 2

$$2I_2 + 6 - 4(I_3 - I_2) - (I_1 - I_2) = 0$$

$$-I_1 + 7I_2 - 4I_3 = -6 \quad (2)$$

(9) Current constraint:

$$I_3 = I_1 + 3$$

$$\boxed{-I_1 + 0I_2 + I_3 = +3} \quad (3)$$

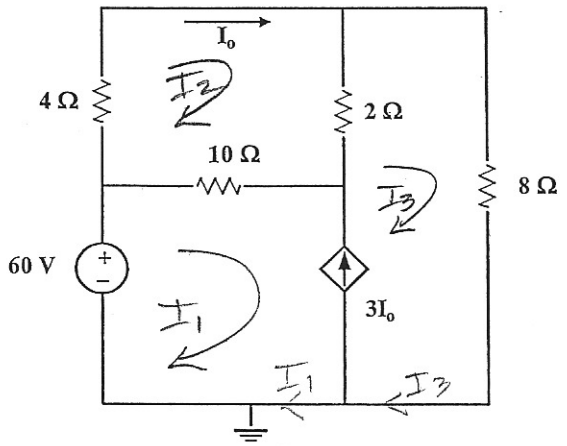
$$\begin{bmatrix} 6 & -5 & 4 \\ -1 & 7 & -4 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ -6 \\ +3 \end{bmatrix}$$

$I_1 = -3.067\,A \quad I_2 = -1.333\,A \quad I_3 = -0.0667\,A$

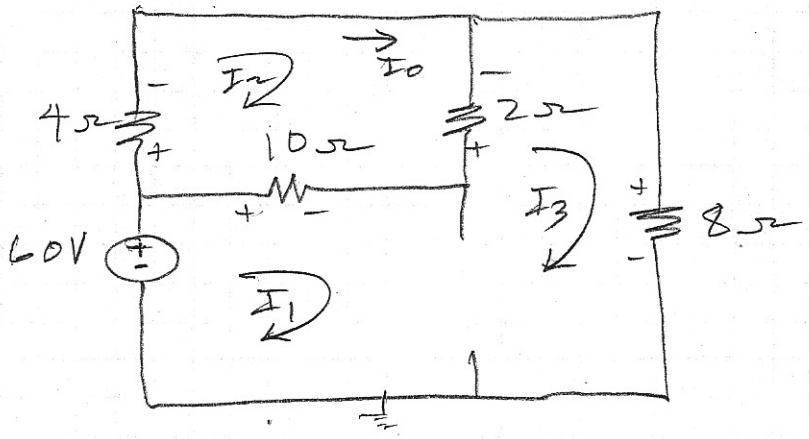
$I_o = I_1 - I_2 = -3.067 + 1.333$

$$\boxed{I_o = -1.734\,A}$$

(10) You are given the circuit in the following diagram. Use mesh analysis to find the current I_o.
Ans: $I_o = 1.731$ A



Super mesh circuit



Around the super mesh

$$-60 + 10(I_1 - I_2) + 2(I_3 - I_2) + 8I_3 = 0$$

$$\boxed{10I_1 - 12I_2 + 10I_3 = 60} \quad (1)$$

Around Mesh 2

$$4I_2 - 2(I_3 - I_2) - 10(I_1 - I_2) = 0$$

$$\boxed{-10I_1 + 16I_2 - 2I_3 = 0} \quad (2)$$

(10) cont

Current constraint

$$I_3 = 3I_0 + I_1$$

but $I_0 = I_2$

$$\boxed{-I_1 - 3I_2 + I_3 = 0} \quad (3)$$

From (1), (2), (3)

$$\begin{bmatrix} 10 & -12 & 10 \\ -10 & 16 & -2 \\ -1 & -3 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 1.44\,A$ $I_2 = 1.73\,A$, $I_3 = 6.635\,A$

$I_0 = I_2 = 1.73\,A$

$$\boxed{I_0 = 1.73\,A}$$