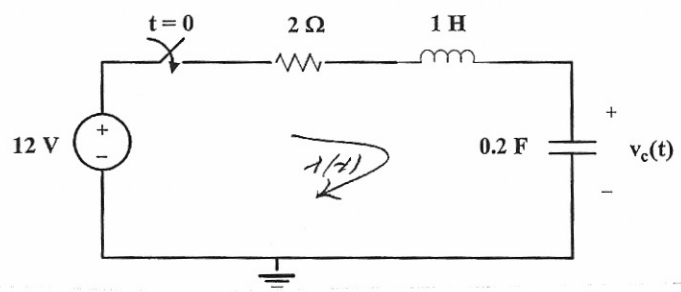
wlg

ECE 301
H.W. #7
Fall 2006

(1) You are given the circuit of Figure 7.1. $v_c(0^-) = 8$ V. Find $v_c(t)$ for $t>0$.

Ans: $12 - e^{-t}[4\cos 2t + 2\sin 2t]$ V



We write the differential equation for the circuit (use R, L, C and V_s to avoid errors).

$$Ri(t) + L\frac{di}{dt} + V_c(t) = V_s \qquad (1.1)$$

but

$$i(t) = C\frac{dV_c}{dt} \qquad (1.2$$

Place (1.2) into (1.1)

$$RC\frac{dV_c}{dt} + LC\frac{d^2V_c}{dt^2} + V_c(t) = V_s$$

And

$$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c}{LC} = \frac{V_s}{LC}$$

Characteristic equation is

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

(1) cont.

Putting in numerical values,

$$s^2 + 2s + 5 = 0$$

$$(s+1+j2)(s+1-j2) = 0$$

The solution will be of the form

$$V_c(t) = V_{c_t}(t) + V_{c_{ss}}$$

$V_{c_{ss}} = V_s = 12$ (by inspection)

so

$$V_c(t) = 12 + e^{-t}\left[A\cos 2t + B\sin 2t\right] \qquad (1.3)$$

Now we know

$$V_c(0^-) = 8 = V_c(0^+)$$

$$i(0^-) = 0 = C\frac{dV_c}{dt} = i(0^+)$$

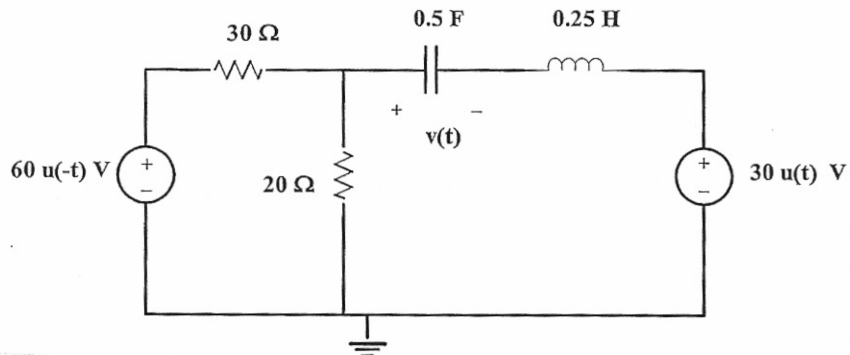
so $\dfrac{dV_c(0^+)}{dt} = 0$

Evaluate (1.3) @ $V_c(0^+)$ and $\dot{V}_c(0^+)$ to get two equations and solve for A & B as follows.

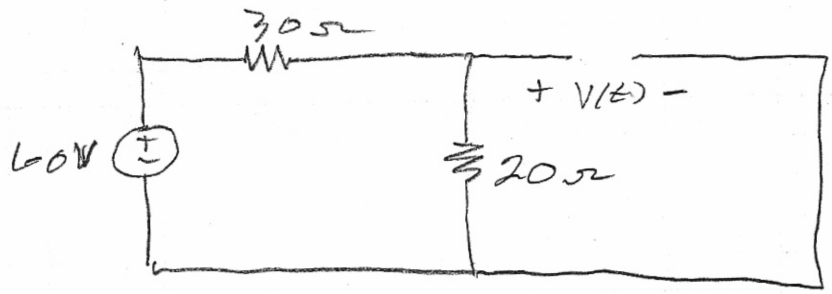
$$V_c(0^+) = 8 = 12 + A \Rightarrow \boxed{A = -4}$$

#1

(2) You are given the circuit of Figure 7.2. Find v(t) for t > 0. Ans $-30 - 0.19e^{-47.83t} + 54.19e^{-0.167t}$ V



For $t<0$, the circuit is

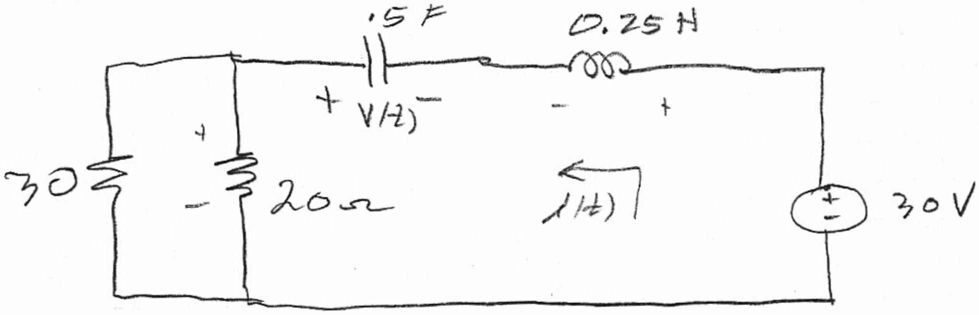


$$V(0) = \frac{60 \times 20}{20+30} = 24V$$

We see that

$$V(0^-) = 24V = V(0^+)$$

For $t>0$, the circuit is



$$-30 + L\frac{di}{dt} - V(t) + R_q\, i(t) = 0$$

$$i = -C\frac{dv}{dt} = -0.5\frac{dv}{dt}$$

$$R_q = 30 \| 20 = 12\Omega$$

(1) cont.

$$\frac{dv_c}{dt} = e^{-t}\left[-2(-4)\sin 2t + 2B\cos 2t\right] - e^{-t}\left[-4\cos 2t + B\sin 2t\right]\Big|_{t=0} = 0$$

$$0 = 2B + 4$$

$$B = -2$$

$\therefore$

$$v_c(t) = 12 - e^{-t}\left[4\cos 2t + 2\sin 2t\right]$$

(2) cont.

substituting for $i(t)$ gives

$$LC\frac{d^2V}{dt^2} + RC\frac{dV}{dt} + V = -30$$

$$\frac{dV}{dt} + \frac{R}{L}\frac{dV}{dt} + \frac{V}{LC} = -\frac{30}{LC}$$

$$s^2 + 48s + 8 = 0$$

$$(s+.167)(s+47.83)=0$$

$$V(t) = -30 + K_1 e^{-0.167t} + K_2 e^{-47.83t}, V$$

$$\boxed{V(0^+) = 24} \quad i = -C\frac{dV}{dt} = 0 \quad i(0^+) = 0$$

$$24 = -30 + K_1 + K_2$$

$$\boxed{K_1 + K_2 = 54}$$

$$\frac{dV}{dt} = -.167K_1 e^{-.167t} - 47.83e^{-47.83t}$$

$$0 = 0.167K_1 + 47.83K_2$$

$$\begin{bmatrix} 1 & 1 \\ .167 & 47.83 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 54 \\ 0 \end{bmatrix}$$

(2) cont.

$K_1 = 54.19$

$K_2 = -0.19$

$\therefore$

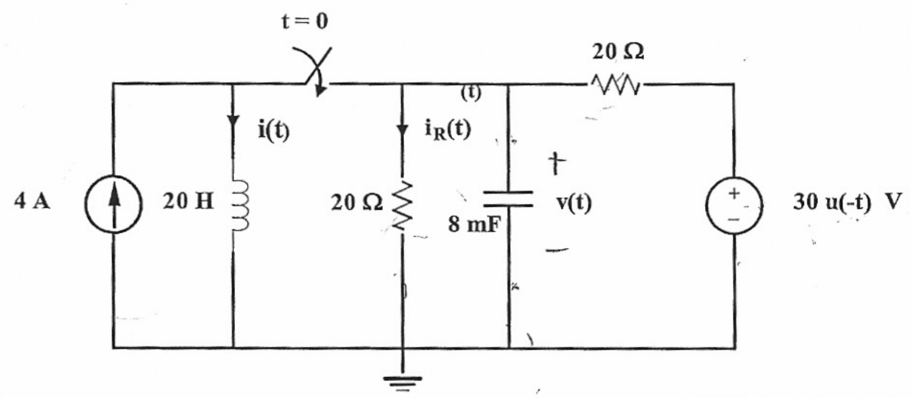
$$v(t) = \left(-30 + 54e^{-.167t} - 0.19e^{-47.83t}\right)V$$

#1

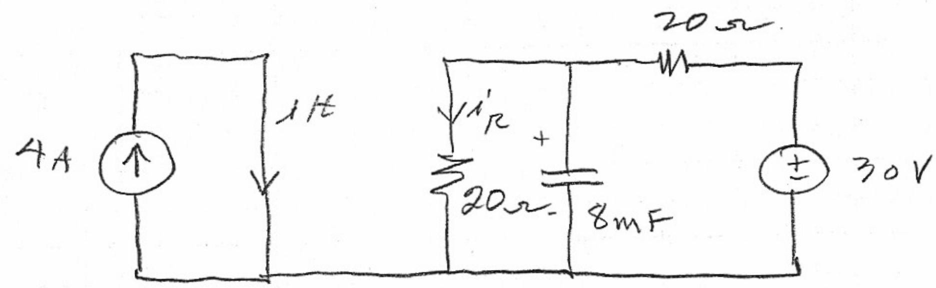
(3) You are given the circuit of Figure 7.3.

(a) Find i(t) for t > 0. Ans: $4 + 0.0655(e^{-0.522t} - e^{-11.98t})$ A

(b) Find $i_R(t)$ for t > 0. Ans: $0.785e^{-11.98t} - 0.0342e^{-0.522t}$ A



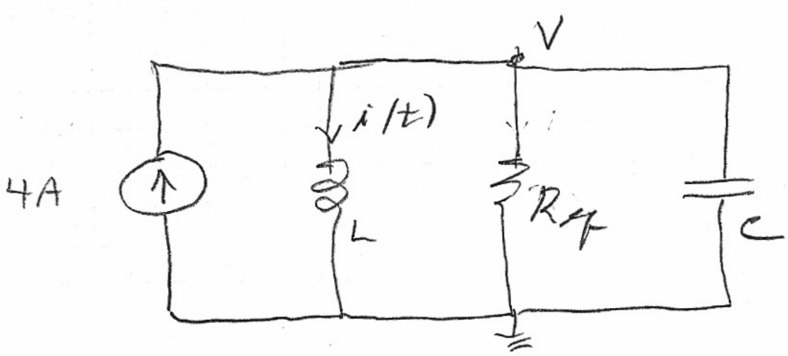
$t < 0$



$i(0^-) = 4A = i(0^+)$

$V(0^-) = V(0^+) = 15V$

$t > 0$



Write a general equation

(b)

$$\frac{V}{R_q} + C\frac{dV}{dt} + i = I_s$$

$$V = L\frac{di}{dt}$$

$$\frac{L}{R_q}\frac{di}{dt} + LC\frac{d^2i}{dt^2} + i = I_s$$

$$\frac{d^2i}{dt^2} + \frac{1}{R_qC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

$R_q = 10\,\Omega$, $C = 8\,mF$, $L = 20\,H$

$$s^2 + \frac{1\times10^3}{10\times8}s + \frac{1\times10^3}{20\times8} = 0$$

$$(s^2 + 12.5s + 6.25) = 0$$

$$(s + .522)(s + 11.98) = 0$$

so

$$i(t) = \underline{4} + k_1e^{-0.522t} + k_2e^{-11.98t}$$

$$i(0^+) = 4 = 4 + k_1 + k_2$$

$$\boxed{k_1 + k_2 = 0}$$

$V_L = L\frac{di}{dt}$

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{15}{L} = \frac{15}{20} = 0.75$$

$$\boxed{.75 = -.522k_1 - 11.98k_2}$$

$$\begin{bmatrix} 1 & 1 \\ -.522 & -11.48 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ .75 \end{bmatrix}$$

$K_1 = 0.0655$

$K_2 = -0.0655$

$i(t) = 4 + 0.0655\left[e^{-0.522t} - e^{-11.98t}\right] A$

FOR $i_R(t)$

$V = L\frac{di}{dt}$; $i_R = \frac{V}{20} = \frac{L}{20}\frac{di}{dt}$ $\quad L = 20\,H$

$i_R(t) = \frac{20}{20}\left[-.522 \times .0655 e^{-.522t} + .0655 \times 11.98 e^{-11.98t}\right]$

$= \left(-.034 e^{-.522t} + 0.785 e^{-11.98t}\right) A$

OR (turn around)

$$\boxed{i_R(t) = \left(0.785 e^{-11.98t} - 0.034 e^{-0.522t}\right) A}$$