wrg

ECE 301
Test 2A
FALL 2006

(1) You are given the circuit of Figure 1A. The switch has been closed for a very long time and is opened at t = 0. The voltage response for $t \geq 0$ is shown in Figure 1B. Use the plot of Figure 1B and your knowledge of RC circuits to answer the following questions.

(a) What is the value of the source voltage, E?
(b) What is the numerical value of the circuit time constant for $t \geq 0$?
(c) What is the numerical value of the circuit capacitor, C?

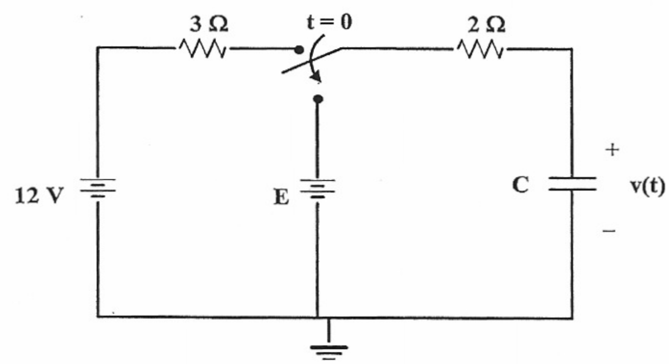
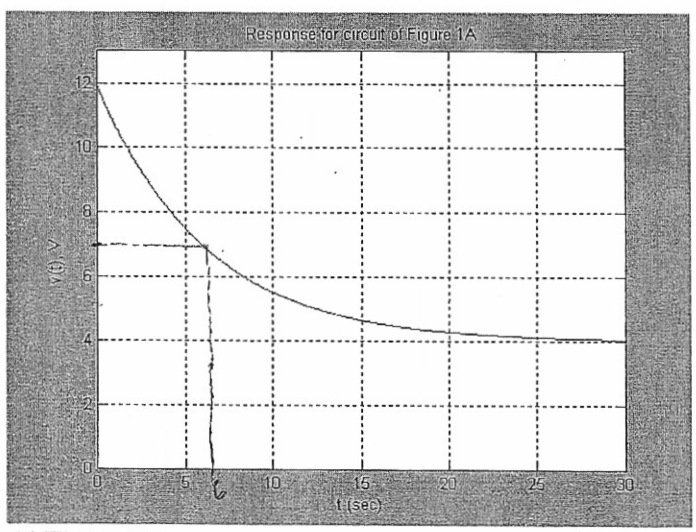


Figure 1A: Circuit for problem 1A.



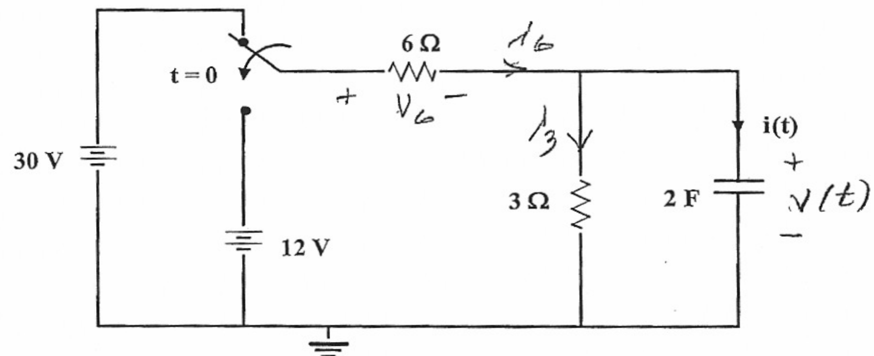
(a) From the graph, since $v(t) = 12\,V$ @ $t = 0^+$;
~~E = 12 V~~ $E = 4V$

(b) Look for $12 - .632 \times (12 - 4) = 6.94$
At 6.94 project horizontally, then vertical:
Find $\boxed{\tau \doteq 6 \text{ sec.}}$

(c) $RC = 6$, since $R = 2$, $\boxed{C = \frac{6}{2} = 3F}$

(2) You are given the circuit of Figure 2. The switch has been in the position shown for a very long time and is switched to the 12 V source as shown at t = 0.

(a) Find i(t) for t < 0.
(b) Find i(t) for $t \geq 0$.
(c) Give the numerical value of the circuit time constant for $t \geq 0$.
(d) Sketch the waveform of the current i(t) showing the initial value of the current, the final value of the current and the time where one time constant occurs.



(a) From the physics of the circuit and the nature of a capacitor;

$i(0^-) = 0$;

$v(0^-) = V(0^+) = \frac{30 \times 3}{3+6} = 10\,V$

(b) The voltage across the 3 Ω resistor is 10 V @ $t = 0^+$. This means

$i_3(0^+) = \frac{10}{3}$ A

By KVL; $V_6(0^+) = 2\,V$

$i_6(0^+) = \frac{2}{6} = \frac{1}{3}$ A.

so $\frac{1}{3} = \frac{10}{3} + i(0^+)$

$i(0^+) = \frac{1}{3} - \frac{10}{3} = -3\,A$

$R_{eq} = \frac{3 \times 6}{3+6} = 2\,\Omega$

$\tau = R_{eq} C = 2 \times 2 = 4$ sec

(2) cont,

For $t \to \infty$, $i(\infty) = 0$

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$$

$$\boxed{i(t) = -3e^{-\frac{t}{4}} \text{ A}}$$

#

OR

$$V(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-\frac{t}{4}}$$

$$V(\infty) = \frac{12 \times 3}{3+6} = 4V$$

$$V(0^+) = 10V$$

$$V(t) = 4 + (10-4)e^{-.25t}$$

$$V(t) = 4 + 6e^{-.25t}$$

$$i(t) = C\frac{dV}{dt} = 2[-.25 \times 6e^{-.25t}]$$

$$\boxed{i(t) = -3e^{-0.25t}}$$

A

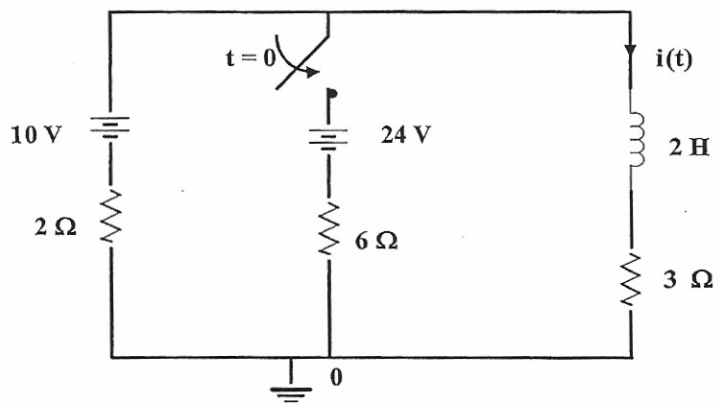
(3) You are given the circuit of Figure 3.

(a) Find i(t) for t<0.

(b) Find i(t) for $t \geq 0$.

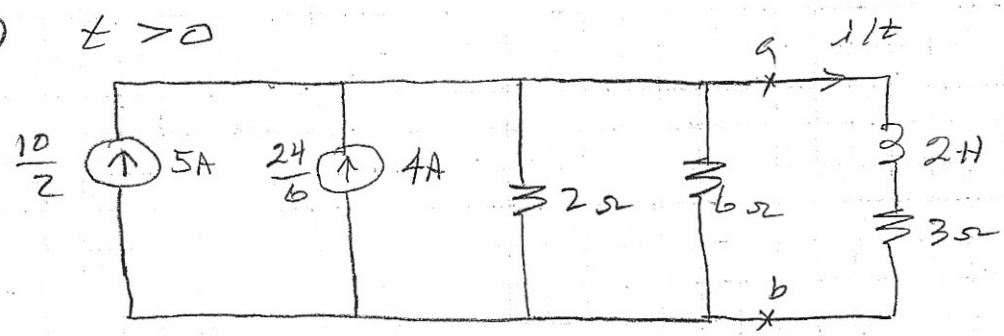
(c) Give the numerical value of the circuit time constant for $t \geq 0$.

(d) Sketch the current waveform for $t \geq 0$ showing initial current, final current, time constant on the time axis.



(a) $i(0^-) = \frac{10}{5} = 2\,A$

(b) $t > 0$



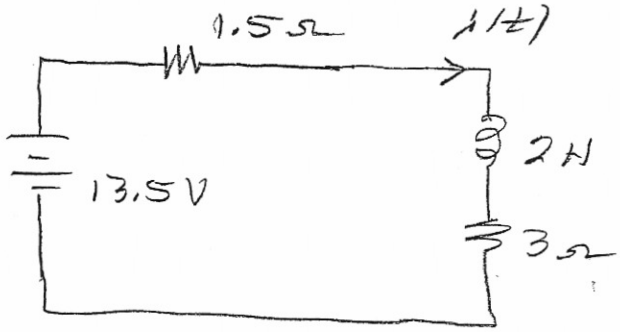
Make a Thevenin equivalent to the left of a-b;

$$R_{TH} = \frac{2 \times 6}{2+6} = \frac{12}{8} = \frac{3}{2} = 1.5\,\Omega$$

$$V_{TH} = 9 \times 1.5 = 13.5\,V$$

Now connect the Thevenin ckt to the load.

(3) cont



Now $i(0^-) = i(0^+) = 2A$

$$i(\infty) = \frac{13.5}{3+1.5} = 3A$$

$$\tau = \frac{L}{R_q} = \frac{2}{4.5}$$

$$\frac{1}{\tau} = 2.25 \text{ sec}^{-1}$$

so

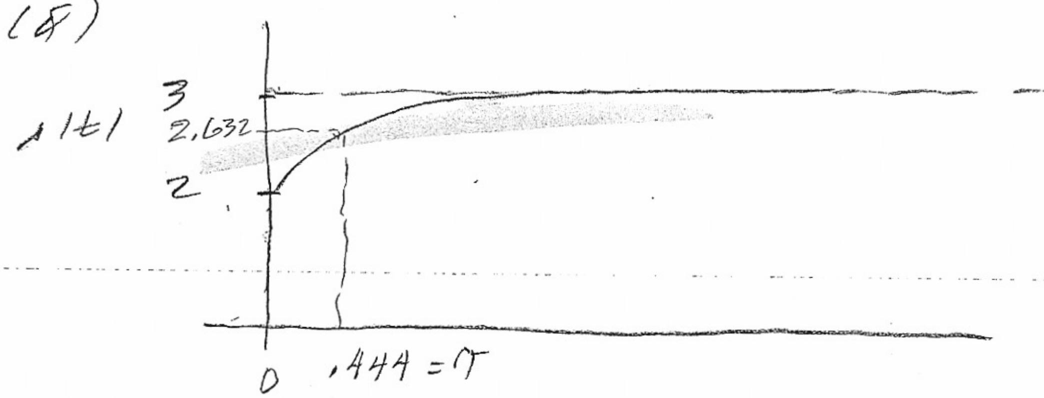
$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$$

$$i(t) = 3 + (2-3)e^{-2.25t}$$

$$\boxed{i(t) = [3 - e^{-2.25t}]\ A}$$

(c) $\tau = \dfrac{2}{4.5} = 0.444$ sec

(d)



(4) You are given the circuit shown in Figure 4.

(a) Develop the differential equation that can be used to solve for v(t). Give this equation In terms of V_S, R, L, and C but do not use numerical values.

(b) If $V_S = 12$ V, $R = 400\ \Omega$, $L = 1.25$ H and $C = 1.25\ \mu F$, give the characteristic equation in numerical form. Give the roots of the characteristic equation.

(c) Consider that the characteristic equation is expressed as

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

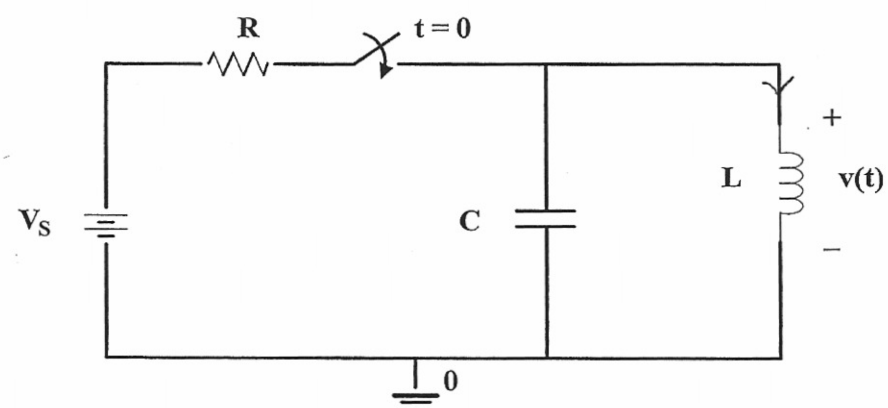
give the numerical values of ξ and ω_n.

(d) Which damping case applies here; (i) overdamped, (ii) critically damped, (iii) underdamped damped?

(e) Give the values of $v(0^+)$ and $\frac{dv(0^+)}{dt}$.

(f) Solve the differential equation for v(t).

(g) Sketch the response for v(t). Give the approximate time (for all practical purposes) that it takes for the response to settle.



(a) Using nodal analysis;

$$\frac{v(t) - V_s}{R} + C\frac{dV}{dt} + \frac{1}{L}\int_0^t v(t)\,dt = 0$$

Take the derivative with respect to t.

(4) cont

$$\frac{1}{R}\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{V(t)}{L} = 0$$

$$C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{V(t)}{L} = 0$$

$$\boxed{\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V(t)}{LC} = 0}$$

(b) using numerical values.

$$\frac{d^2V}{dt^2} + \frac{1\times10^6}{400\times1.25}\frac{dV}{dt} + \frac{V(t)\times10^6}{1.25\times1.25} = 0$$

OR

$$\frac{d^2V}{dt^2} + 2000\frac{dV}{dt} + 640000 = 0$$

$$s^2 + 2000s + 640{,}000 = 0$$

$$(s+400)(s+1600) = 0$$

(c) $s^2 + 2000s + 640{,}000$

compare with

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = 640{,}000$$

$$\boxed{\omega_n = 800}$$

$$\zeta = \frac{2000}{2\times800}$$

$$\boxed{\zeta = 1.25}$$

(d) overdamped

(e) FOR $V(0^+)$

Look at the circuit, Voltage across the capacitor cannot change inst.

so $\boxed{V(0^+) = V(0^-) = 0}$

$$i(0^+) = C\frac{dV(0^+)}{dt}$$

$$i(0^+) = \frac{V_s}{R} = \frac{12}{400} = 0.03\ A$$

$$\frac{dV(0^+)}{dt} = \frac{.03}{C} = \frac{.03 \times 1 \times 10^{6}}{1.25}$$

$$\boxed{\frac{dV(0^+)}{dt} = 24{,}000}$$

(f)
$$V(t) = V_{ss} + k_1 e^{-400t} + k_2 e^{-1600t}$$

$$V_{ss} = 0$$

$$V(t) = k_1 e^{-400t} + k_2 e^{-1600t}$$

$$\boxed{V(0^+) = 0 = k_1 + k_2}$$

$$\frac{dV}{dt} = -400 k_1 e^{-400t} - 1600 k_2 e^{-1600t}$$

@ $t = 0^+$

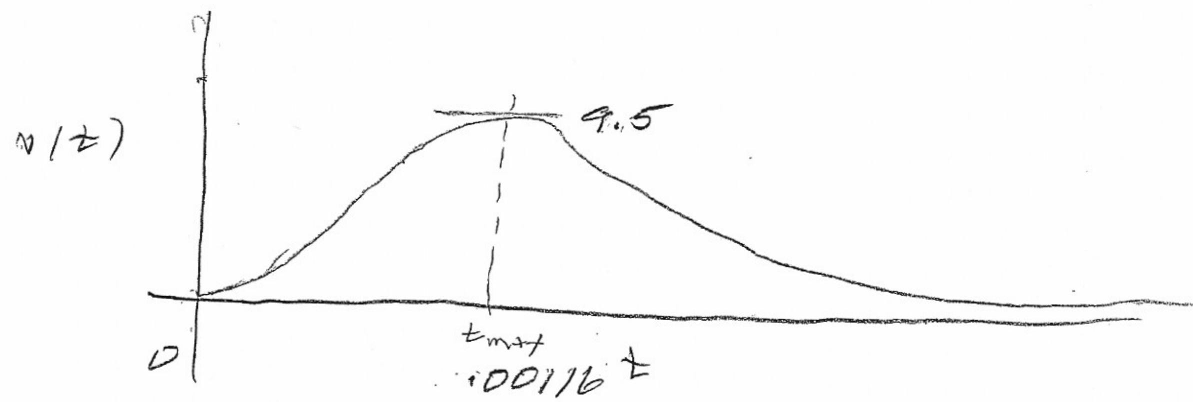
$$24{,}000 = -400 k_1 - 1600 k_2$$

$$\begin{bmatrix} 1 & 1 \\ -400 & -1600 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 24000 \end{bmatrix}$$

$k_1 = 20, \quad k_2 = -20$

$$\boxed{v(t) = 20e^{-400t} - 20e^{-1600t}}$$

(e) governed by $e^{-400t} \rightarrow \tau = \frac{1}{400}$



$$\frac{dv}{dt} = -8000e^{-400t} + 32000e^{-1600t} = 0$$

$$e^{-400t} - 4e^{-1600t} = 0$$

$$1 - 4e^{-1200t} = 0$$

$$e^{-1200t} = .25$$

$$-1200t \ln e = \ln .25$$

$$-1200t = -1.386$$

$$t_{MAX} = 0.000116 \text{ sec}$$

Peak_T2A.m

MATLAB gives $t_{max} \doteq .0012$ sec

4.5

$$V_{MAX} = 20e^{-400\times 1.16\times 10^{-3}} - 20e^{-1.6\times 10^{3}\times 1.16\times 10^{-3}}$$

$$V_{MAX} = 20\,[.629 - .156] = \underline{\underline{9.46\ V}}$$

RUN MATLAB; checks; see below.

```
% Verifying tmax and vmax for problem 4 on test 2A
% of ECE 301, October 10, 2006.  Call program peak_T2A

t = 0:0.000001:0.01;


v = 20*(exp(-400*t)-exp(-1600*t));


plot(t,v)
grid
ylabel('v(t)')
xlabel('t (sec)')
```