B.

Wiley

ECE 301
Test 2B
Fall 2006

(1) You are given the circuit of Figure 1A. The switch has been open for a long time and is closed at t = 0. The voltage, v(t), as shown in Figure 1A has a waveform as shown in Figure 1B. Use the plot of Figure 1B and your knowledge of RL circuits to answer the following.

(a) Find the value of the voltage source, E.
(b) Determine the time constant of the circuit for $t \geq 0$.
(c) Determine the value of the inductor, L.

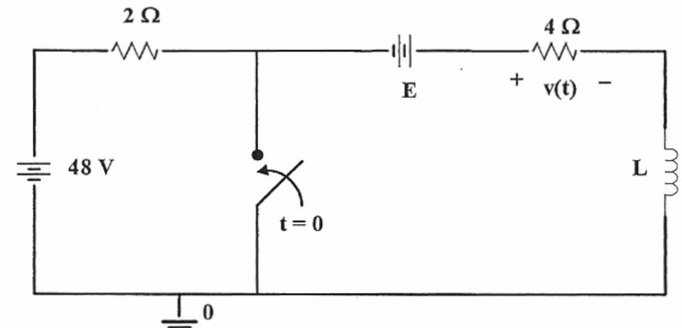
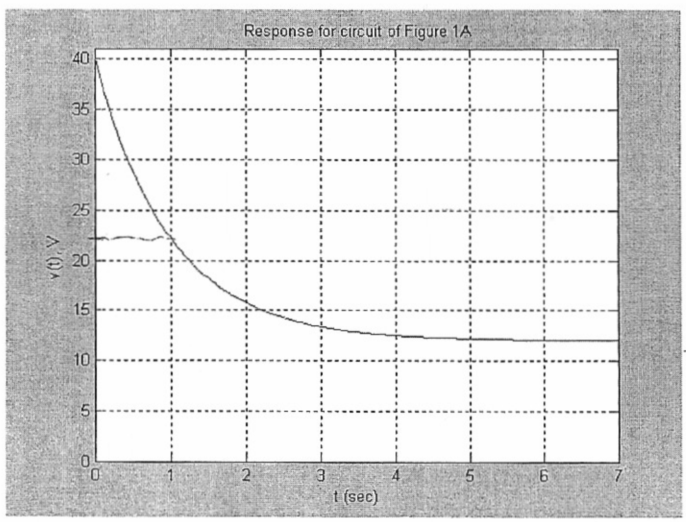



Figure 1A: Circuit for problem 1.




(a) $V(0^-) = V(0^+) = \frac{(48+E)\times 4}{4+2} = 40$

$\boxed{E = 12\text{ V}}$

(b) Look for $40 - (40-12)\cdot .632 = 22.3\text{ V}$

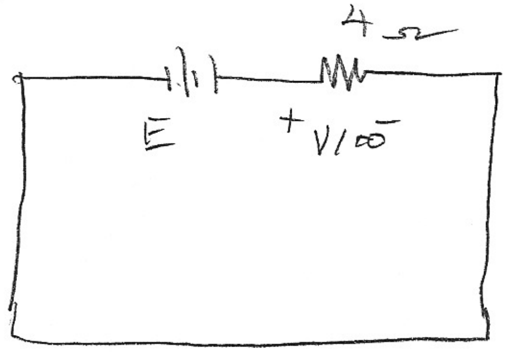
Project horizontally: then down: $\boxed{\tau = 1\text{ sec}}$

(c) $\tau = \frac{L}{R}$ $\quad L = \tau \times R = 1 \times 4 = 4\text{ H}$

$\boxed{L = 4\text{ H}}$

#1 continued. (easier way)

$t \to \infty$



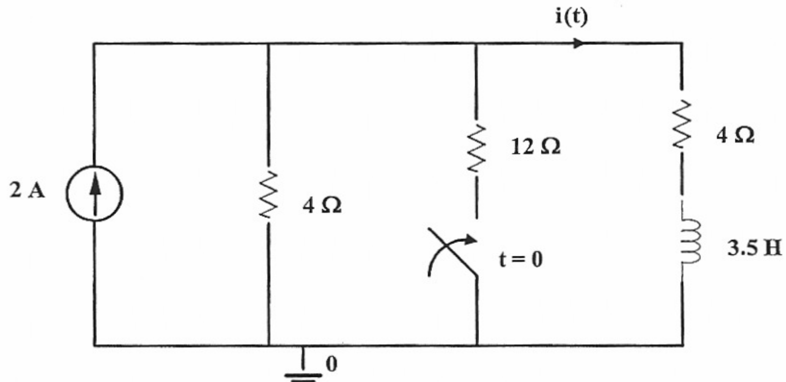

From the plot $V(\infty) = 12\,V$

so $E = 12\,V$

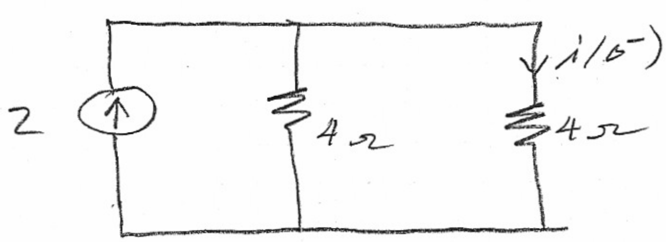
B

(2) You are given the circuit of Figure 2. The switch has been open for a very long time and is closed at t = 0.

(a) Find i(t) for t < 0.
(b) Find i(t) for $t \geq 0$.
(c) Find the numerical value of the circuit time constant for $t \geq 0$.
(d) Sketch the current waveform for $t \geq 0$. Show the starting value of i(t), the ending value of i(t) and the approximate location of the time constant on the time axis.

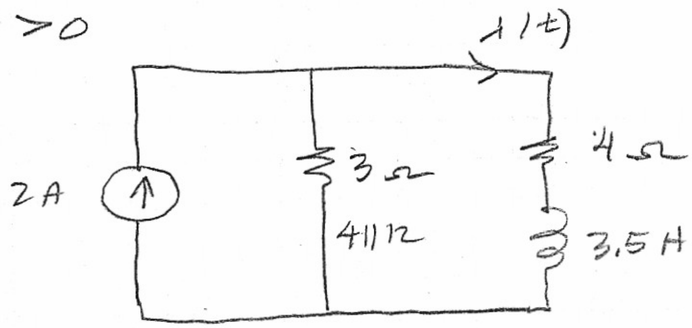



(a) t < 0




$i(0^-) = 1A = i(0^+)$

(b) t > 0




$R_{eq} = 4 + 3 = 7\Omega \qquad \tau = \frac{L}{R} = 0.5 \text{ sec}$

$i(\infty) = \frac{2 \times 3}{7} = \frac{6}{7}$

$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-\frac{t}{\tau}}$

B

2.1

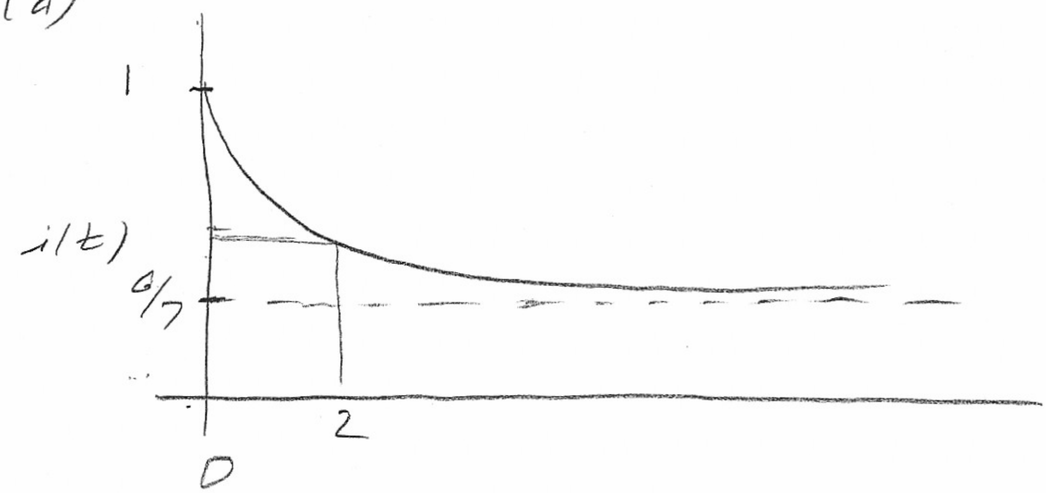
(b)

$$i(t) = \frac{6}{7} + \left[1 - \frac{6}{7}\right] e^{-2t}$$

$$\boxed{i(t) = \frac{6}{7} + \frac{1}{7} e^{-2t} \quad A}$$

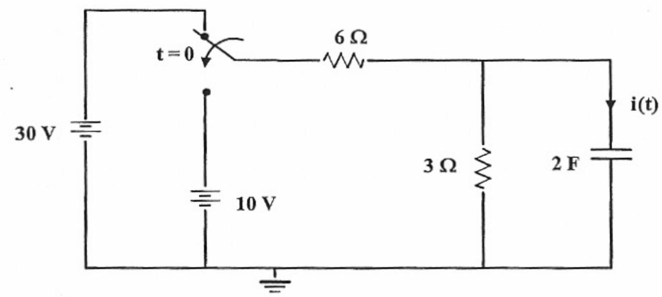
(c) $\tau = \frac{L}{R} = \frac{3.5}{7} = 0.5$ sec

(d)

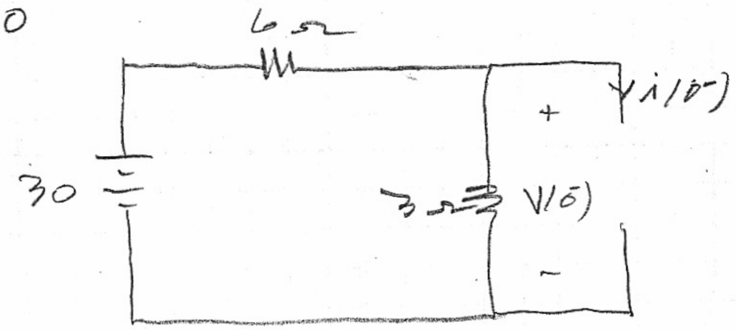


(3) You are given the circuit of Figure 3. The switch has been in the position shown for a very long time and is switched to the 12 V source as shown at t = 0.

(a) Find i(t) for t < 0.
(b) Find i(t) for $t \geq 0$.
(c) Give the numerical value of the circuit time constant for $t \geq 0$.
(d) Sketch the waveform of the current i(t) showing the initial value of the current, the final value of the current and the time where one time constant occurs.

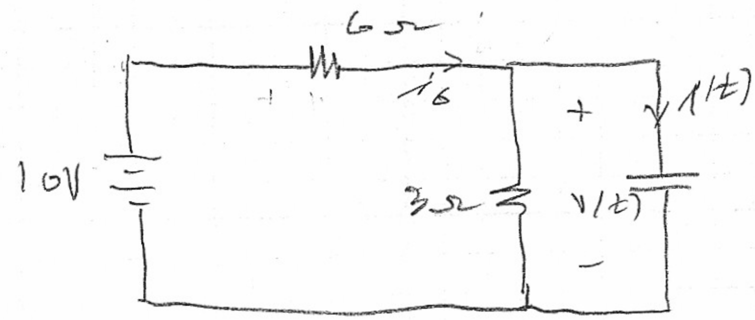



(a) $t < 0$




$i(0^-) = 0 \qquad V(0^-) = \frac{30 \times 3}{3+6} = 10V = V(0^+)$

(b) $t > 0$




Solve for $v(t)$; use $i = C\frac{dv}{dt}$ to find $i(t)$

B

(3) cont 

$$V(0^+) = 10\,V$$

$$V(\infty) = \frac{10 \times 3}{6+3} = \frac{30}{9} = \frac{10}{3}$$

$$R_{eq} = 3 \parallel 6 = 2\,\Omega$$

$$\tau = R_{eq}C = 2 \times 2 = 4\ \text{sec}$$

$$V(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-\frac{t}{\tau}}$$

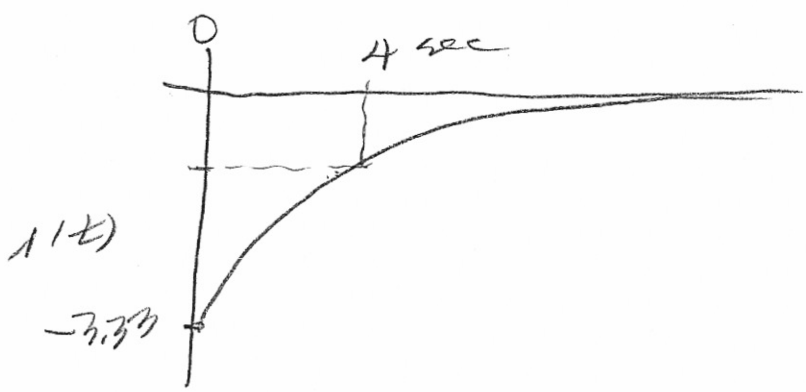
$$V(t) = \frac{10}{3} + \left[10 - \frac{10}{3}\right]e^{-.25t}$$

$$V(t) = \frac{10}{3} + \frac{20}{3}e^{-.25t}$$

$$i(t) = C\frac{dv}{dt}$$

$$i = 2\left[-\frac{1}{4} \times \frac{20}{3}e^{-.25t}\right]$$

$$\boxed{i(t) = -\frac{10}{3}e^{-.25t}}$$



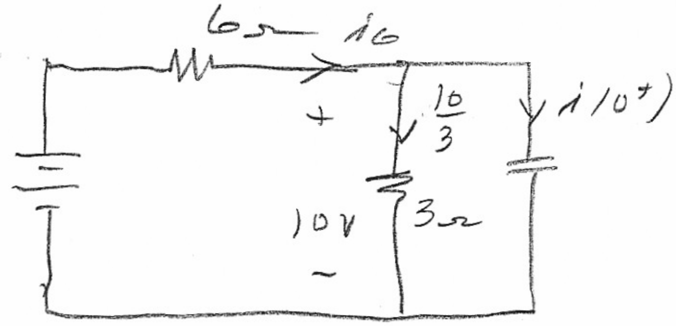

See next page for easier solution

(3) cont.



Another solution:

Since $v(0^+) = 10$




$i_6(0^+) = 0$

so. $\frac{10}{3} + i(0^+) = 0$

$i(0^+) = -\frac{10}{3}$

use

$i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-\frac{t}{4}}$

$i(\infty) = 0$

$$\boxed{i(t) = -\frac{10}{3} e^{-0.25t}}$$

B

(4) You are given the circuit of Figure 4.

(a) Develop the differential equation that can be used to solve for v(t). Give this equation in terms of $V_S$, R, L, and C but do not use numerical values.

(b) If $V_S = 10$ V, $R = 11\ \Omega$, $L = 1$ H, and $C = 0.1$ F, give the characteristic equation in numerical form. Give the roots of the characteristic equation.

(c) Consider that the characteristic equation is expressed as

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

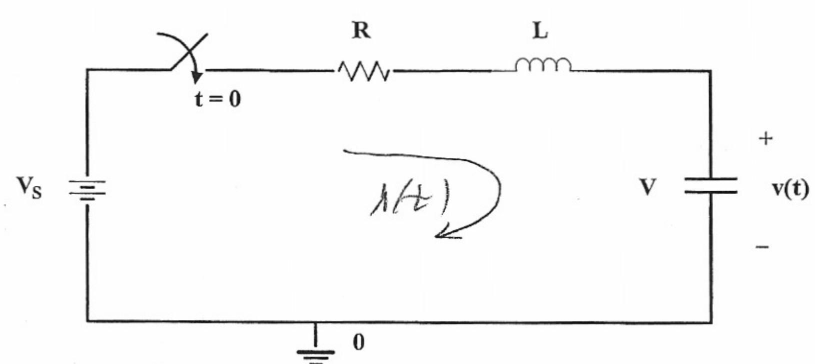
give the numerical values of $\xi$ and $\omega_n$.

(d) Which damping case applies here; (i) overdamped, (ii) critically damped, (iii) underdamped damped?

(e) Give the values of $v(0^+)$ and $\frac{dv(0^+)}{dt}$.

(f) Solve the differential equation for v(t).

(g) Sketch the response for v(t). Give the approximate time (for all practical purposes) that it takes for the response to settle.




1a) $$R i(t) + L\frac{di}{dt} + V(t) = V_s$$

but $$i(t) = C\frac{dV}{dt}$$

$$RC\frac{dV}{dt} + LC\frac{d^2V}{dt^2} + V(t) = V_s$$

$$\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{V(t)}{LC} = \frac{V_s}{LC}$$

B

(4) (b)

$$\frac{d^2V}{dt} + \frac{11}{1}\frac{dV}{dt} + \frac{1}{1\times .1} = \frac{10}{.1}$$

$$\frac{d^2V}{dt^2} + 11\frac{dV}{dt} + 10V = 100$$

$$s^2 + 11s + 10 = 0$$

$$(s+1)(s+10) = 0$$

(c) $s^2 + 2\zeta\omega_n s + \omega_n^2$

$$s^2 + 11s + 10 = 0$$

$$\omega_n^2 = 10$$

$$\omega_n = \sqrt{10}$$

$$\zeta = \frac{11}{2\sqrt{10}} \doteq 1.74$$

(d) overdamped

(e)

From the circuit

$$\boxed{V(0^+) = 0}$$

$$i(0^+) = C\frac{dV(0^+)}{dt}$$

current thru inductor cannot change inst. i.e. $i(0^-) = 0 = i(0^+)$

This leave

$$\boxed{\frac{dV(0^+)}{dt} = 0}$$

B

4.2

(4)

$$V(t) = V_{ss} + K_1 e^{-t} + K_2 e^{-10t}$$

$$V_{ss} = V_s = 10$$

$$V(t) = 10 + K_1 e^{-t} + K_2 e^{-10t}$$

$$V(0^+) = 0 = 10 + K_1 + K_2$$

$$\boxed{K_1 + K_2 = -10}$$

$$\frac{dV}{dt} = -K_1 e^{-t} - 10 K_2 e^{-10t}$$

$$\frac{dV(0^+)}{dt} = 0 = -K_1 - 10 K_2$$

$$\boxed{K_1 + 10 K_2 = 0}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 10 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

$$K_1 = -11.1 \quad ; \quad K_2 = 1.11$$

slowest time constant

$$V(t) = 10 - 11.1 e^{-t} + 1.11 e^{-10t}$$