wlg

Examples Solving Differential Equations Using The Symbolic Tool Kit of Matlab.

Example 1

Given:

$$\frac{dv}{dt} + 5v(t) = 10 \qquad v(0) = 0$$

```
>>    dsolve('Dv + 5*v = 10', 'v(0) = 0')

ans =

2-2*exp(-5*t)
```

Example 2

Given:

$$\frac{d^2v}{dt^2} + 8\frac{dv}{dt} + 12v = 5$$

```
>>
>>    dsolve('D2v + 8*Dv + 12*v = 5', 'v(0) = 0', 'Dv(0) = 0')

ans =

5/12-5/8*exp(-2*t)+5/24*exp(-6*t)
```

Example 3

```
>>
>>    % The following DE has complex roots
>>
>>    dsolve('D2v + 4*Dv + 16*v = 0', 'v(0) = 2', 'Dv(0) = 0')

ans =

2/3*3^(1/2)*exp(-2*t)*sin(2*3^(1/2)*t)+2*exp(-2*t)*cos(2*3^(1/2)*t)
```

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 16v = 0;$$

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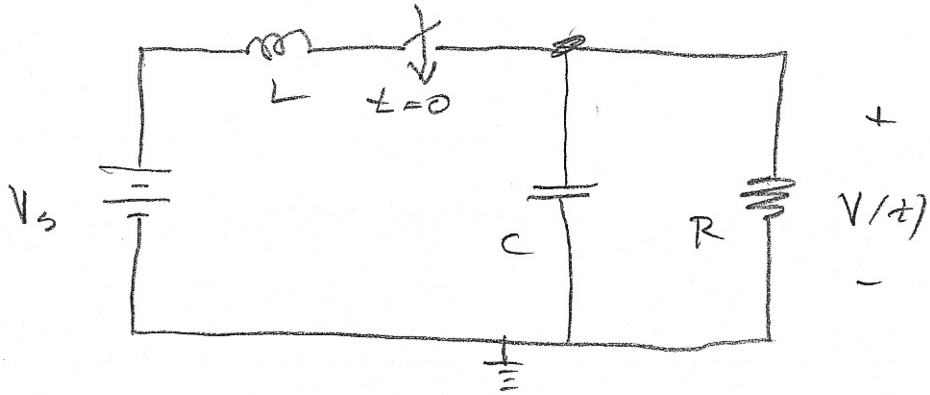
Solution for Take Home Exam Problem.

Start with the circuit.



(a)

Use nodal analysis to develop the following equation.

$$C\frac{dV(t)}{dt} + \frac{V(t)}{R} + \frac{1}{L}\int_0^t (V(t) - V_s)\,dt = 0$$

Take the derivative of the above equation.

$$C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{(V(t) - V_s)}{L} = 0$$

clearing:

$$\boxed{\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V(t)}{LC} = \frac{V_s}{LC}} \quad (1)$$

We know the solution will be of the form

$$v(t) = V_{ss} + V_t$$

where $V_{ss} = V_s$

so

$$v(t) = V_s + e^{-\zeta \omega_n t}\left[A\cos\omega_d t + B\sin\omega_d t\right]$$

where we use the characteristic equation as

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

and roots of

$$s_1, s_2 = -\zeta\omega_n \pm j\omega_d$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

We know $v(t)$ can be expressed as

$$v(t) = V_s + e^{-\zeta\omega_n t}\left[\sqrt{A^2+B^2}\cos(\omega_d t + \phi)\right]$$

We don't know, prior to selecting ζ & ω_n, the values of A & B.

As a ball park approximation we say that

$$\sqrt{A^2+B^2} \doteq V_0 = 10 \quad \text{(for here)}$$

We know we want

$$V(t_{ss}) = 10 + e^{-\zeta\omega_n t_{ss}}\left[\sqrt{A^2+B^2}\cos(\omega_d t_{ss}+\theta)\right]$$

$$\doteq 10 + e^{-\zeta\omega_n t_{ss}}\sqrt{A^2+B^2}$$

$$V(t_{ss}) \doteq 10 + e^{-\zeta\omega_n t_{ss}}\,10 = 10 \pm .01$$

OR we want

$$10e^{-\zeta\omega_n t_{ss}} \leq .01$$

$$t_{ss} = 0.1, \quad \zeta = 0.4$$

$$e^{-.04\omega_n} \leq .001$$

$$-.04\omega_n \ln e \leq \ln .001$$

OR $.04\,\omega_n > +6.91$

$$\omega_n \geq 173$$

TRY $\omega_n = 175$

This gives

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad (\zeta = .4,\ \omega_n = 175)$$

$$s^2 + 140s + 30625 = 0$$

The differential equation will be

$$\frac{d^2 v}{dt^2} + 140\frac{dv}{dt} + 30625\,v(t) = 306250$$

Characteristic equation factors as

$$(s + 70 + j160.4)(s + 70 - j160.4) = 0$$

$$v(t) = 10 + e^{-70t}\left[A\cos 160t + B\sin 160t\right]$$

$v(0^+) = 0;$

$\dot{v}(0^+) = 0$

$$0 = 10 + A \Rightarrow A = -10$$

$$\frac{dv}{dt} = e^{-70t}\left[-160A\sin 160t + 160B\cos 160t\right]$$
$$- 70e^{-70t}\left[A\cos 160t + B\sin 160t\right]$$

$$0 = 160.4B - 70A$$

$$B = \frac{-700}{160.4} = -4.364$$

$$v(t) = 10 - e^{-70t}\left[10\times\cos 160.4t + 4.364\sin 160.4t\right]$$

Now check with symbolic:

```
>>
>>
>> dsolve('D2v + 140*Dv + 30625*v = 306250' ,  'v(0) = 0', 'Dv(0) = 0')

ans =

10-10*exp(-70*t)*cos(35*21^(1/2)*t)-20/21*21^(1/2)*exp(-70*t)*sin(35*21^(1/2)*t)

>>
```

$$10 - 10e^{-70t}\cos 160.4t - 4.364e^{-70t}\sin 160.4t$$

checks.

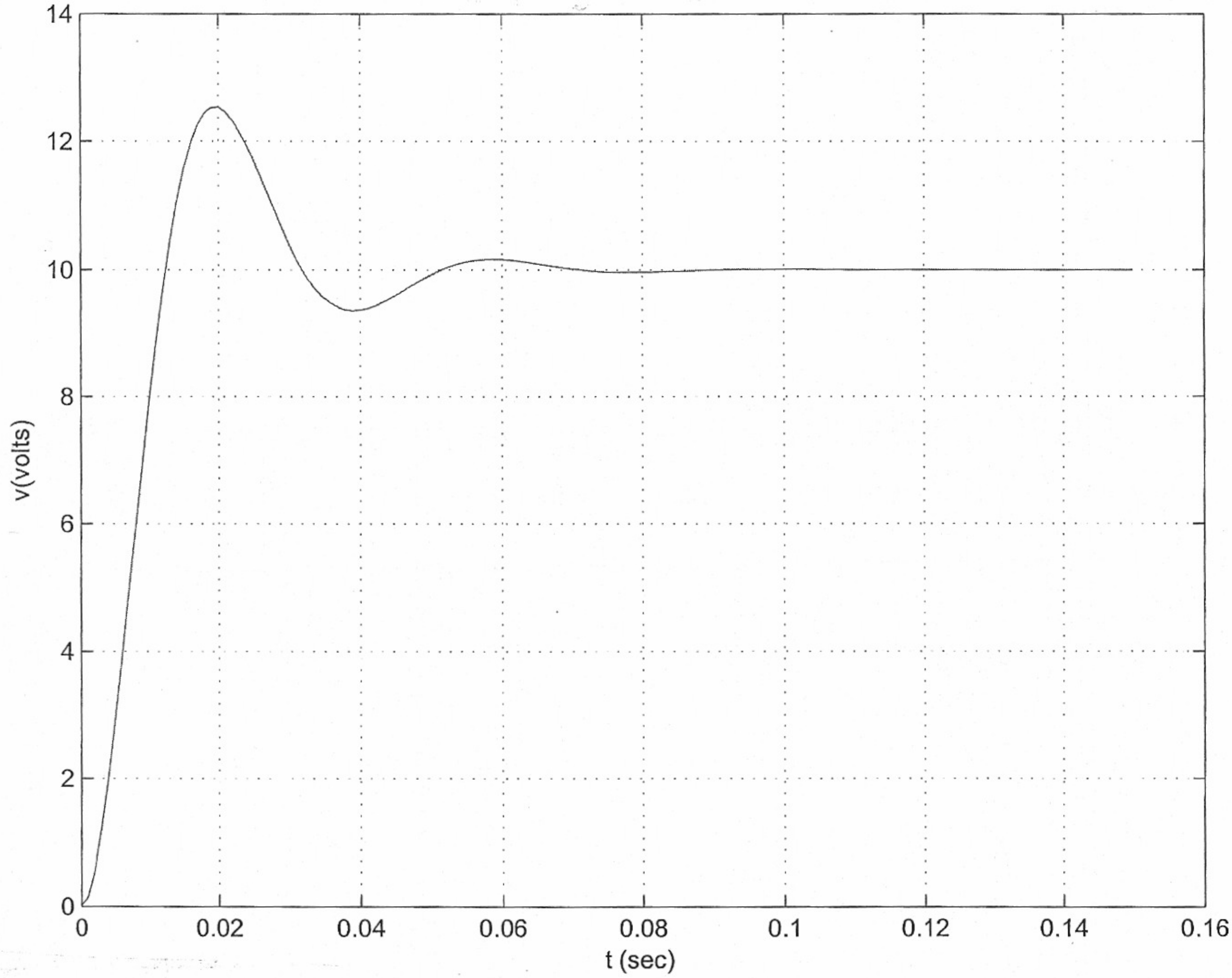
Now run MATLAB to check response and steady state.

```
% program to veryify take home problem
% for ECE 301.  Fall Semester, 2006.
%  save as:  take_home_RLCm.m

t = 0:.001:.15;

v = [10-10*exp(-70*t).*cos(35*21^(1/2)*t)-20/21*21^(1/2)*exp(-70*t).*sin(35*21^(1/2)*t)];
[t;v]'
plot(t,v)
grid
ylabel('v(volts)')
xlabel('t (sec) ')
```

(6)



0.0950	10.0090
0.0960	10.0099
0.0970	10.0104
0.0980	10.0105
0.0990	10.0104
0.1000	10.0099
0.1010	10.0092
0.1020	10.0084
0.1030	10.0074
0.1040	10.0064
0.1050	10.0053
0.1060	10.0042

Now set R, L, C.

use $\boxed{C = 100\,\mu F}$

$$\frac{1}{LC} = 30625$$

$$L = \frac{1}{30625C} = \frac{10{,}000}{30625}$$

$$\boxed{L = 0.327\,H}$$

$$\frac{1}{RC} = 140$$

$$R = \frac{1}{140C} = \frac{10{,}000}{140}$$

$$\boxed{R = 71.4\,\Omega}$$

$0.3H < L < 4H$

$$\frac{100000}{30625}$$

3.26