(1) For the circuit below, the RMS phasor voltages (as indicated) are $V_R = 35.4 \angle 45^\circ \text{Vrms}$, $V_L = 35.4 \angle 135^\circ \text{V rms}$, $V_C = 70.7 \angle -45^\circ \text{V rms}$. Draw the phaser diagram (approx. to scale) showing these voltages and the source voltage $V_S$ all on the same diagram.
(2) Find the power factor for the circuit of Figure 2. Indicate which case of leading/lagging.

\[ Z = 20 - j12 + 4j \left( 10 + j10 \right) \]
\[ = (20 - j12) + \frac{4 (10 + j10)}{14 + j10} = 23.2 - j11.46 \]
\[ Z = 25.9 \angle -26.2 \degree \]

P.f. = \cos (-26.2) = 0.897 \text{ leading}
(3) You are given the circuit of Figure 3. Find the current $I_1$ as an rms phasor current.

\[ 40 \sin(100t + 30) \rightarrow 40 \cos(100t - 60) \rightarrow 40 \angle -60^\circ \text{ V} \]

\[ \frac{1}{j \omega C} = \frac{-j}{1 \times 10^{-2} \times 5 \times 10^{-4}} = -j20 \cdot \frac{1}{j} \]

The correct analysis gives:

\[
\begin{bmatrix}
25 + j20 & -15 - j30 \\
-15 - j30 & 35 + j10
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
= \begin{bmatrix}
40 \angle -60^\circ \\
-50
\end{bmatrix}
\]

\[ I_1 = 0.1504 - j1.867 = 1.87 \angle -85.4^\circ \text{ A} \]

\[ I_2 = 0.0041 - j0.683 \text{ A} \]

\[ I_{1,\text{ rms}} = \frac{I_1}{\sqrt{2}} = 1.32 \angle -85.4^\circ \text{ A rms} \]

\[ I_{2,\text{ rms}} = 0.075 - j0.93 \text{ A rms} \]
The circuit shown in Figure 4 is used for experimental work in an aerospace project. It is required that the current in the circuit be in phase with the source voltage. The frequency of the source voltage is 100 Hz. The inductor at the output of the circuit is variable. What must be the value of \( L \) so that the requirements concerning the phase of the source voltage and current are met?

For the voltage and current to be in phase, the phase angle \( \theta \) is zero and the impedance looking into the circuit must have only a real part. At the present

\[
\begin{align*}
Z &= 20 \angle 4 \text{ } \Omega \\
Z &= 30 \angle 0 \text{ } \Omega \\
\text{Want} \quad Z &= 30 \\
\text{So,} \quad j \omega L &= 10
\end{align*}
\]

\[
L = \frac{10}{200 \pi} = 0.0159 \text{ } \mu \text{F}
\]
(5) You are given the periodic voltage waveform of Fig 5. Find the RMS value of the voltage.

\[ V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T V(t)^2 \, dt} = \sqrt{\frac{1}{T} \int_0^T \left( \frac{2V_m t}{T} \right)^2 \, dt + \int_0^T V_m^2 \, dt} \]

\[ V_{\text{RMS}} = \sqrt{\int_0^T \left[ \frac{4V_m^2 t^2}{T^2} \frac{T^3}{8} + \frac{V_m^2 T}{2} \right] \, dt} \]

\[ = V_m \sqrt{\frac{1}{6} + \frac{1}{2}} \]

\[ V_{\text{RMS}} = V_m \sqrt{\frac{2}{3}} = 0.816 \, V_m \]
(6) You are given the circuit of Figure 6.

![Circuit Diagram](image)

Figure 6: Circuit for problem 6.

(a) Determine the average complex power supplied by the source.
(b) Determine the average real power delivered (absorbed) by the resistor.
(c) Determine the average reactive power supplied to the inductor.
(d) Determine the average reactive power supplied to the capacitor.
(e) Draw (sketch) the complex power triangle for the circuit.
(f) Determine the power factor of the circuit, including leading/lagging.

\[
S = \overline{V_i} \times \overline{I} = \frac{|V|}{2} = \frac{220^2}{40-j15}
\]

\[
S = 1061 + j398 = 1133\sqrt{20.56^\circ} \text{ VA}
\]

\[
\begin{align*}
P_e &= P = 1061 \text{ W} \\
Q_L &= 796 \text{ VArS} \\
Q_C &= 1133\sqrt{20.56^\circ} \text{ VArS}
\end{align*}
\]

(e) continued

\[ b = 1133 \]
\[ 20.56 \]
\[ 1061 \]

(f) \[ P_f = 105 \text{ (20.56)} \]
\[ P_f = 0.936 \text{ lagging} \]