The impedance of the inductor and capacitor depends on the frequency of the sinusoidal signal applied to a network. This resonance is shown by the relationships:

\[ X_L = j \omega L \]

\[ X_C = -\frac{j}{\omega C} \]

This is illustrated in the following example.

**Example 13.1**

Consider the circuit shown in Figure 13.1.

![Circuit Diagram](image)

The 100 V is the source voltage.

\[ L = 0.5 \text{ H} \]

\[ C = 100 \mu \text{F} \]

We first put the circuit in phasor form.

\[ L = 0.5 \text{ H} \Rightarrow j \omega L = j 100 \times 0.5 = j 50 \Omega \]
\[ C = 100 \mu F \Rightarrow \frac{-j}{\omega C} = \frac{-j}{100 \times 100 \times 10^6} = -j100 \Omega \]

\[ 80 \cos (100t - 30^\circ) \Rightarrow 80 \angle -30^\circ \text{ V} \]

The phasor circuit is shown in Figure 13.2.

![Diagram](image.png)

**Figure 13.2: Circuit in "Phasor" form.**

We can apply any of the techniques we learned in DC circuits to solving the above circuit. The major difference, technique wise, is that we are dealing with complex number rather than all real numbers.

Suppose the solutions for \( I_1 \) and \( I_2 \) are desired. We can use mesh analysis to find these.

**Mesh Analysis for A.C. Circuits:**

We start at point A in Figure 13.2 and go clockwise use \( \sum \text{drops} = 0 \)
\[20I_1 + 30I_1 + 40I_1 + (I_1 - I_2)(20-j100) = 0\]

or
\[
(80-j30)I_1 + (-20+j100)I_2 = 0
\]

(13.1)

Next we start at "B" and go clockwise around mesh 2 using \(\sum\text{drops} = 0\). This gives the following equation:

\[
(I_2 - I_1)(20-j100) + j50I_2 + 80I_1 - 30 = 0
\]

or
\[
(-20+j100)I_1 + (20-j50)I_2 = -80I_1 - 30
\]

(13.2)

Express (13.1) and (13.2) in matrix form:

\[
\begin{bmatrix}
80-j30 & (-20+j100) \\
(-20+j100) & (20-j50)
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
-80\end{bmatrix}
\]

Express the solutions for \(I_1\) and \(I_2\) in both rectangular forms:

\[I_1 = -0.817 + j0.647 \text{ A} = 1.042/141.6^\circ \text{ A}\]

\[I_2 = -1.09 + j0.0167 \text{ A} = 1.09/199.1^\circ \text{ A}\]

You can also solve the above as a matrix with MATLAB as:

\[
\begin{bmatrix}
1 & -1 \\
2 & -1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
-80\end{bmatrix}
\]
I do not know anyway in MATLAB to express polar for so one would need to write \(-80/\angle 0^\circ\) as \(-69.28 - j40\).

The above equation was solved with MATLAB as shown below.

```
% Solving the matrix equation that results
% from analysis of an AC circuit.
% (80-j20)I1 + (-20+j100)I2 = 0
% (-20+j100)I1 + (20-j50)I2 = -69.28 - j40
% wg: office computer: Oct 19,'06: program: AC_solution.m

Z = [80-j*70 -20+j*100; -20+j*100 20-j*50];
V = [0; -69.28-j*40];
I = inv(Z)*V

>>
>>
>> AC_solution
I =
-0.8171 + 0.6473i
-1.0865 + 0.0167i
```

The solution for \(V\) (see Figure 13.2) can be obtained by

\[
\hat{V} = (20-j100)(\hat{I} - \hat{I}_2)
\]

We find, using

\[
\hat{V} = (20-j100)*((1.042\angle 141.9) - (1.07\angle 120.1))
\]

\[
\hat{V} = 68.48-j14.72 \quad V = 70.04 \angle -121.3 \quad V
\]

We will use nodal analysis to check the answer.
Using Eq. leaving node \( V \) in Figure 13.2 gives,

\[
\frac{V}{60+j30} + \frac{V}{20-j100} + \frac{V - 80}{j50} = 0
\]

This is a tedious problem to solve. There is no "easy" way. We use

\[
(0.0133-j0.0067)V + (0.0019+j0.0096)\ V
- j0.02V - 0.8 + j1.386 = 0
\]

\[
(0.0152-j0.0171)V = 1.8 - j1.386
\]

\[
V = 68.51 - j14.11 \ \text{V}
\]

Close to previous answer but there is a difference in the decimal part. Probably because I did not take it out to as enough decimal places.

Another problem will be presented to be sure we get the hang of things.

It might be added that generally it is easier to solve AC circuits using mesh analysis. There can be exceptions.
Example 13.2

Use mesh analysis to find $I_1$ and $I_2$ in the following circuit. Also find $A_1(t)$.

\[ V = 50 \sin(40t + 30^\circ) \]

\[ A \to B \]

\[ 50 \sin(40t + 30^\circ) = 50 \cos(40t - 60^\circ) \]

\[ \Rightarrow 50L - 60^\circ \text{ V} \]

\[ \text{Figure 13.4: Phasor circuit for example 13.2.} \]
Start at "A" go CW using $\sum\Delta\!P_{\text{sys}}=0$.

\[-50 \angle 60^\circ + j50 I_1^2 + (30+j20)(I_1 - I_2) = 0\]

or

\[(30 + j20) I_1^2 + (-30 - j20) I_2^2 = 50 \angle 60^\circ\]

Start at "B" go CW use $\sum\Delta\!P_{\text{sys}}=0$

\[(30 + j20)(I_2 - I_1) + (20 - j10) I_2^2 + 50 \angle 10^\circ = 0\]

or

\[(-30 - j20) I_1^2 + (50 + j10) I_2 = -50 \angle 10^\circ\]

or

\[
\begin{bmatrix}
(30 + j20) & (-30 - j20) \\
(-30 - j20) & (50 + j10)
\end{bmatrix}
\begin{bmatrix}
I_1^2 \\
I_2^2
\end{bmatrix} =
\begin{bmatrix}
60 \angle 60^\circ \\
-50 \angle 10^\circ
\end{bmatrix}
\]

$I_1^2 = -1 - j0.126 A = 1.008 \angle -172.7^\circ A$

$I_2^2 = -1.533 + j0.0037 A = 1.53 \angle 179.89^\circ A$

To get $I_1(t), I_2(t)$ we go back to the cosine reference. Thus

$I_1(t) = 1.008 \cos(40t - 172.7^\circ) A$

$I_2(t) = 1.533 \cos(40t + 179.89^\circ) A$
When we have current sources we follow the same rules (techniques) we used with DC circuits. We illustrate this with an example.

**Example 13.3**

Compute $V_o$ in the circuit of Figure 13.5, using mesh analysis.

![Circuit Diagram]

**Figure 13.5: Circuit for Example 13.3.**

We assign mesh currents as shown. We open the current sources, redraw the circuit.

![Redrawn Circuit Diagram]
We have only one "closed" mesh to write KVL. It gives
\[ 2(I_2 - I_1) - 3I_2 + 12 + 2(I_2 - I_3) = 0 \]
or
\[ -2 I_1 + (4 + j3)I_2 - 2I_3 = -120 \]
We have a constraint equation
\[ I_1 = 4\angle 90^\circ \]
or
\[ I_1 + 0I_2 + 0I_3 = 4\angle 90^\circ \]
We have another current constraint equation,
\[ I_3 = -210^\circ \]
or
\[ 0I_1 + 0I_2 + I_3 = -210 \]
In matrix form
\[
\begin{bmatrix}
-2 & (4 + j3) & -2 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
= \begin{bmatrix}
-12 \\
0.4 \\
-2
\end{bmatrix}
\]
\[ I_1 = j4A, \quad I_2 = (-3.52 - j0.64)A \]
\[ I_3 = -2A \]
\[ V_0 = 2 \left( I_1 - I_2 \right) = 2 (4 + 3.52 + 0.64) \]
\[ V_0 = 2 \left( 3.52 + j 4.64 \right) = 7.04 + j 9.28 = 11.65 \angle 52.8^\circ \]

We handle dependent sources with the same techniques as in DC circuits.

**Example 13.4**

Apply nodal analysis to find \( I_0 \) in the following circuit (From Alexander, 3rd Ed. Problem 10.11 p. 444)

\[ \begin{align*}
  V_0 &= 4 + j 4.64 \\
  V_2 &= 2j \pi \\
  \end{align*} \]

At \( V_1 \),
\[ \frac{V_1 - 4}{2} + \frac{V_1 - V_2}{2} - 2I_0 = 0 \]

But
\[ I_0 = \frac{4 - V_2}{-j5} = -j0.2V_2 + j0.8 \]

Substitute into the previous equation
\[ V_1 - 0.5 V_2 - 2 I_0 = 2 \]
\[ V_1 - 0.5 V_2 - 2(-j0.2 V_2 + j0.8) = 2 \]
\[ \sqrt{V_1 + (-0.5 + j0.4) V_2} = 2 + j1.6 \]

At \( V_2 \)

\[ \frac{V_2 - V_1}{2} + \frac{V_2}{j8} + \frac{V_2 - 4}{-j5} = 0 \]
\[ -0.5 V_2 - 0.125 V_1 - j0.2 V_2 + j0.8 = 0 \]
\[ -0.5 V_1 + (0.5 + j0.075) V_2 = j0.8 \]

\[
\begin{bmatrix}
1 & (-0.5 + j0.4) \\
(-0.5 & (0.5 + j0.075))
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= \begin{bmatrix}
2 + j1.6 \\
j0.8
\end{bmatrix}
\]

\[ V_1 = 4.86 + j0.054 \text{ V} \]
\[ V_2 = 4.9955 + j0.905 \text{ V} \]

\[ I_0 = \frac{4 - V_2}{-j5} = \frac{4 - 4.9955 - j0.905}{-j5} \]

\[ I_0 = 0.181 - j0.1991 \text{ A} \]