This lesson will consist entirely of example problems. Most of the problems will involve power calculations. Three reference sources, in addition to Rizzoni, were used as reference. They are:

- *Basic Engineering Circuit Analysis; Irwin & Nelms* Eight Editions 2005; Publisher: John Wiley.

**Maximum Power Transfer**

**Example 16.1**

Given the circuit of Figure 16.1, find Z_L for max power transfer and the power delivered.

![Circuit Diagram]

**Figure 16.1: Circuit for Example 16.1**

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We remove the load $Z_L$, deactivate all independent sources to the left of a-b and determine $Z_{TH}$. The circuit of Fig 16.2 is used to determine $Z_{TH}$.

![Circuit diagram](image)

**Figure 16.2: Circuit for determining $Z_{TH}$, Example 16.1.**

\[
\frac{5 \Omega \parallel 30 \Omega}{5 \times 30} = 4.29 \Omega
\]

\[
Z_{TH} = \frac{(-j5) \parallel (4.29 + j10)}{4.29 + j10 - j5}
\]

\[
Z_{TH} = 2.471 - j7.880 \Omega
\]

\[
Z_L = Z_{TH} = 2.471 + j7.880 \Omega \text{ for maximum power transfer.}
\]

Next find $V_{TH}$. Refer back to Fig 16.1 with $Z_L$ removed to find the open circuit voltage.

![Circuit diagram](image)

**Figure 16.3: Circuit for finding $V_{TH}$.**
There are several ways to find $V_{th}$. Here I will find the Thévenin to the left of cd and connect back to the original circuit to find $V_{th}$. Reconnecting gives the circuit of Figure 16.4.

![Diagram of circuit](image)

Figure 16.4: Modified circuit for finding $V_{th}$.

Applying voltage division gives,

$$V_{th} = \frac{(17.14)(-0.5)}{4.29 \times 10 - j5} = 13.01 \angle -139.37^\circ V$$

The circuit for finding maximum power transfer is shown in Figure 16.5.

![Diagram of circuit](image)

Figure 16.5: Circuit for finding maximum power transfer, Example 16.1.
By direct calculation,

\[ I = \frac{12.01 - 139.37}{2 \times 2.471} = 2.63 \angle -139.37^\circ \ A \]

\[ P_L = \frac{12.01^2}{2} \frac{2.471}{2} = 2.63 \times 2.471 \approx 8.55 \text{ W} \]

Using the expression

\[ P_L = \frac{1 V_0^2}{8 R_L} = \frac{(13.01)^2}{8 \times 2.47} = 8.56 \text{ W} \]

(close enough)

**Example 16.2**

Given the circuit of Figure 16.6.

(a) Find \( R_L \) for maximum power to be delivered to the load.

(b) Find the power delivered to \( R_L \) as found in (a).

![Figure 16.6: Circuit for maximum power transfer, Example 16.2.](image)
It has been stated in class that for maximum power transfer, when the load is constrained to be a resistor, is

\[ R_L = \frac{1}{2\pi f L} \]

or

\[ R_L = \sqrt{8^2 + 2^2} = 8.246 \text{ ohms} \]

The circuit becomes as shown in Figure 16.7. Note RMS source voltage

![Circuit Diagram]

**Figure 16.7: Final Circuit for Determining \( P_{\text{max}} \) for Example 16.2.**

From Figure 16.2:

\[ \frac{2}{Z} = \frac{110\sqrt{2}}{8+j2+8.246} = 6.72/\angle7.07^\circ \text{ A} \]

\[ P_L = \frac{1}{2} \frac{2}{R_L} = 6.72^2 \times 8.246 = 372.38 \text{ W} \]

Note: You cannot use

\[ P_L = \frac{1}{8R_L} \]
Example 16.3

The purpose of this example is to illustrate how to calculate complex power, real power, reactive power in an AC circuit.

Given the circuit of Figure 16.8

\[ \begin{align*}
& I_1 \\
& \downarrow \\
& 220 \angle 0^\circ \text{ V (rms)} \\
& \uparrow \\
& I_2 \\
& \downarrow \\
& \text{Load 1} \\
& \uparrow \\
& \text{Load 2} \\
& \downarrow \\
& 8 \text{ kW} \\
& 0.8 \text{ PF lagging} \\
& \uparrow \\
& V_L \\
& \downarrow \\
& 20 \text{ kVA} \\
& 0.6 \text{ PF lagging} \\
& \uparrow \\
& I_3 \\
& \downarrow \\
& 3 \\
& \uparrow \\
& I_2 \\
& \downarrow \\
& 3_2 \\
& \uparrow \\
& \end{align*} \]

Figure 16.8: Circuit for Example 16.3.

(a) Determine \( 3_1 \) and \( 3_2 \).
(b) Determine \( I_2 \).
(c) Determine \( \dot{2} \).
(d) Show that \( 3 = 3_1 + 3_2 \), \( P = \dot{P} + \dot{P}_2 \), \( Q = \dot{Q} + \dot{Q}_2 \).

Like in most circuit problems, there is more than one way to find the solution.

(e) The complex power triangle for load 1 is as shown in the following sketch.
\[ P_f = 0.8 \quad \text{loss in} \]
\[ \cos \theta = 0.8 \]
\[ \theta = \cos^{-1} 0.8 = 36.7^\circ \]

\[ \cos 36.7 = 0.8 = \frac{8 \text{ kVA}}{13.1} \]
\[ \text{Load 1} \]

Knowing the angle from the triangle gives

\[ S_1 = 10 \angle 36.7^\circ \text{ kVA} = (8 + j6) \text{ kVA} \]

\[ \text{Load 2} \]

\[ P_f = 0.6 \quad \text{loss in} \]
\[ \theta = \cos^{-1} 0.6 = 53.13^\circ \]

\[ S_2 = 20 \angle 53.13^\circ \text{ kVA} = (12 + j16) \text{ kVA} \]

16) Referring to Figure 16.8

\[ S_1 = V_L I_1^* \quad \left( V_L = 220 \text{ Volts RMS} \right) \]
\[ I_1^* = \frac{S_1}{V_L} = \frac{10 \angle 36.7^\circ \text{ kVA}}{220 \text{ Volts}} = 45.45 / 35.9 \text{ A} \]
\[ I_1 = 45.45 \angle -35.9^\circ \text{ A} \quad \text{(RMS)} \]
(b) \[ I_2^* = \frac{Z_2}{V_L} = \frac{20 \angle 53.12}{220 \Omega} \]
\[ I_2^* = 90.71 / 53.12 \text{ A}_{\text{rms}} \]
\[ I_L = I_i + I_2^* = (45.45 / -36.9) + (90.71 / 53.12) \]
\[ I_L = 101.62 / 26.6^\circ \text{ A}_{\text{rms}} \]

(c) Determine S
\[ S = V_L I_L^* = (220 \Omega) (101.62 / -26.6) \]
\[ S = (20 - j10) \text{ kVA} = 22.356 / -26.6 \text{ kVA} \]

Check
\[ S_1 = S_i + S_2 \]
\[ S_1 = (8 + j4) \text{ kVA} + (12 + j16) \text{ kVA} \]
\[ S_1 = (20 + j22) \text{ kVA} ?? \]

What is wrong? Where has the error been made? You find it. I made an error in the answer. That is often made.

The correct answer is
\[ S = (20 + j22) \text{ kVA} \]
Example 16.4

In the circuit shown in Figure 16.9, a load having an impedance of 40 + j20 \( \Omega \) is fed from a voltage source through a line having an impedance of 0.5 + j20 \( \Omega \). The rms value of the source voltage is 220 V.

(a) Calculate the load current \( I_L \) and voltage \( V_L \).

(b) Calculate the average and reactive power delivered to the load.

(c) Calculate the average and reactive power delivered to the line.

(d) Calculate the average and reactive power supplied by the source.

Figure 16.9: Circuit for Example 16.4.
(a) \[ I_L = \frac{220\Omega}{0.5 + j2 + 40 + j20} = 4.77 \angle -28.5^\circ \text{A} \]

\[ V_L = (40 + j20)I_L = 213.47 \angle -1.75^\circ \text{V} \]

(b) \[ S_L = V_L \times I_L^* = (213.47 \angle -1.75^\circ) (4.77 \angle 28.5^\circ) \]

\[ S_L = (910.87 + j455.14) = 1018.26 \text{ kVA} \]

\[ P_L = 910.87 \text{ W} \quad Q_L = 455.14 \text{ VAr} \]

(c) \[ S_L' = |I_L|^2 Z_L = (4.77)^2 (0.5 + j2) \]

\[ S_L' = (11.38 + j45.5) \text{ VA} = 46.9 \angle 76^\circ \text{ VA} \]

\[ P_L' = 11.38 \text{ W} \quad Q_L' = 45.5 \text{ VAr} \]

(d) \[ S_5 = V_5 \times I_5^* = (220\Omega) (4.77 \angle 28.5^\circ) \]

\[ S_5 = (922.23 + j500.73) \text{ VA} \]

For a check

\[ S_5 = S_{25} + S_L = (1018.26 \text{ kVA} + 46.9 \angle 76^\circ) \text{ VA} \]

\[ S_5 = (921.6 + j501.3) \text{ VA} \quad \text{Close} \]
Example 16.5

This is problem 16.18 at the end of the chapter of Rizzoni.

A single phase motor draws 220 W at a power factor of 80% (lagging) when connected across a 200 V, 60 Hz source. A capacitor is connected in parallel with the load to give unity power factor. Find the required capacitance.

What we have is shown in Figure 16.10.

![Circuit Diagram]

Figure 16.10: Circuit for problem 16.5

The complex power triangle is as shown below.

\[ \cos \theta = 0.8 \]

\[ \theta = 36.87^\circ \]
The existing load \( Q \) is

\[
\frac{Q}{220} = \tan 36.87
\]

\[
Q = 165 \text{ VARs}
\]

\[
3 = 220 + j 165 \text{ VA}
\]

We want to eliminate the \( j165 \text{ VARs} \) with

\[
\frac{1}{C} = \frac{\left| V_c \right|^2}{Z_c} = \frac{\omega C \left| V_c \right|^2}{-j} \]

\[
377 \left( \frac{200}{2} \right) C = 165
\]

\[
C = 10.74 \text{ \mu F}
\]

**Example 16.6**

This problem is 7.21 from Rizzoni at the end of the chapter.

The motor inside a blender can be modeled as a resistance in series with an inductor as shown in Figure 16.11.

(a) What is the average power, \( P_{av} \), dissipated in the load?

(b) What is the motor's power factor?

(c) What \( C \) placed in parallel with the motor will change the p.f. to 0.9 lagging?
Figure 16.11: Circuit for Example 16.6.

(a) First find $X_L$.

$$X_L = \omega L = 2\pi \times 60 \times 20 \times 10^{-3} = 7.54$$

$$Z_L = 10 + j7.54$$

$$P_{AV} = \left| I_L \right|^2 R_L$$

$$I_L = \frac{120}{12 + j7.54} = 8.47/ -32.1° \text{ A rns}$$

$$P_{AV} = (8.47)^2 \times 10 = 717.41 \text{ W}$$

$$S_L = \left| I_L \right|^2 Z_L = (8.47)^2 (10 + j7.54) \text{ VA}$$

$$S_L = (717.41 + j540.93) \text{ VA}$$

$$S_L = 898.49 / 37° \text{ VA}$$
(b) Motor p.f.
Check at the angle of $\theta_c$. It is the same as the angle of $\theta_L$.
The angle of $\theta_c$ is the p.f. angle.

$$\text{p.f.} = \cos(37^\circ) = 0.7786 \text{ lagging}$$

(c) Now find C for p.f. = 0.9 lagging

![Diagram]

This is what you have:

$$\cos \theta = 0.9$$

$$\alpha = 717.41$$

$$37^\circ \Rightarrow 25.84$$

$$\alpha = 717.41 \text{ cos } 25.84 = 347.43 \text{ VARs}$$

The capacitor must provide reactive power of:

$$Q_c = 540.93 - 347.43 = 193.5 \text{ VAr}$$

$$\text{but}$$

$$Q_c = \frac{\text{W V}_{L}^2}{C}$$

$$\sqrt{V_{L}} = |(10 \angle 7.5^\circ)| = 106.08 \text{ V rms}$$

$$C = \frac{193.5}{3.77 \times (106.08)^2} = 45.6 \mu\text{F}$$
Example 16.7

You are given the circuit shown in Figure 16.12.

Figure 16.12: Circuit for Example 16.7.

(a) Find the complex power supplied to the circuit \( P^* \) by the source.

(b) Determine the power factor of the source.

(c) Determine \( S_1, S_2, S_3, S_4, S_5 \) and \( S_6 \) show that

\[
S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6
\]

agrees with \( S \) determined in (a).

Probably the easiest way to approach this problem is to assign mesh currents \( I_1 \) and \( I_2 \) as indicated. Solve for \( I_1, I_2 \).
By inspiration

\[
\begin{bmatrix}
20+j20 & -15-j30 \\
-15-j30 & 35+j12
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
= \begin{bmatrix}
120+j0 \\
0
\end{bmatrix}
\]

Solve with calculator

\[
I_1 = 3.94+j1.137 = 3.91/16.91° \text{ A rms}
\]

\[
I_2 = 1.676+j3.112 = 3.544/61.42° \text{ A rms}
\]

\[
Z = \sqrt{3} I_1 = 120 (3.91/16.91) \]

\[
Z = 469.21/16.91 \text{ VA} = (478.91-j136.48) \text{ VA}
\]

(1) Power factor

\[
PF = \cos(-16.91) = 0.9568 \text{ leading}
\]

(2) \[
Z_1 = I_1 I_1^* = 76.44 \text{ VA (or watt here)}
\]

\[
Z_2 = I_2 I_2^* = -j152.88 \text{ VA}
\]

\[
Z_3 = I_3 I_3^* = 251.2 \text{ VA}
\]

\[
Z_4 = \sqrt{(3.91/16.91)^2 - (3.544/61.42)^2} = 2.84 \text{ VA}
\]

\[
Z_5 = (2.84)^2 \times 15 = 120.68
\]

\[
Z_6 = I_3 I_3^* (-j18) = 3.544(-j18) = -j226.08
\]
\[ \sum_{j=1}^{6} S_j = 76.44 - j152.88 + j251.2 + j242 + 120.98 - j226.08 \]

\[ S_f = (448.62 - j156.96) \text{ VA} \]

\[ Z = (448.91 - j136.48) \]

in part (b) good enough