wkg

Introduction To Operational Amplifiers

Lesson 19
Lesson 20

The term operational amplifier itself induces one to ask, "what is this"? Most people in science and engineering know, or have a good idea, what amplifier is and basically what it does. The "operational" part may be puzzling.

First, to shorten the writing perhaps, operational Amplifier becomes op amp. These devices were more or less introduced in the 1930's — with George Philbrick being recognized for pioneering work with the devices.

Second, we shall shortly see how op Amps can be used to

- add signals
- subtract signals
- integrate signals
- differentiate signals (not recommended)
- multiply signals (not directly but in conjunction with other electronics)

It is because the device can perform those mathematical operations, that it was referred to (still is) as an operational Amplifier.

In the text;

"Fundamentals of Electric Circuit";
Alexander & Sadiku; 3rd Edition,
copyright 2007, ISBN-13 978-0-07-297718-9,
McGraw-Hill Book company

the statements are made

- measurements are a tool for understanding the physical world,
- instruments are tools for measurement,
- the operational amplifier is a building block for modern electronic instrumentation (one might add signal processing and control)

It behooves a person of science and engineering to have a basic understanding (at least from the standpoint of using as a device) how an operational amplifier functions.

Physical Structure & History

In the 1940-1960 time frame the operational amplifier was constructed with vacuum tubes. The circuitry was encapulated in the base that held the vacuum tubes (usually 2 dual 12AU7's).

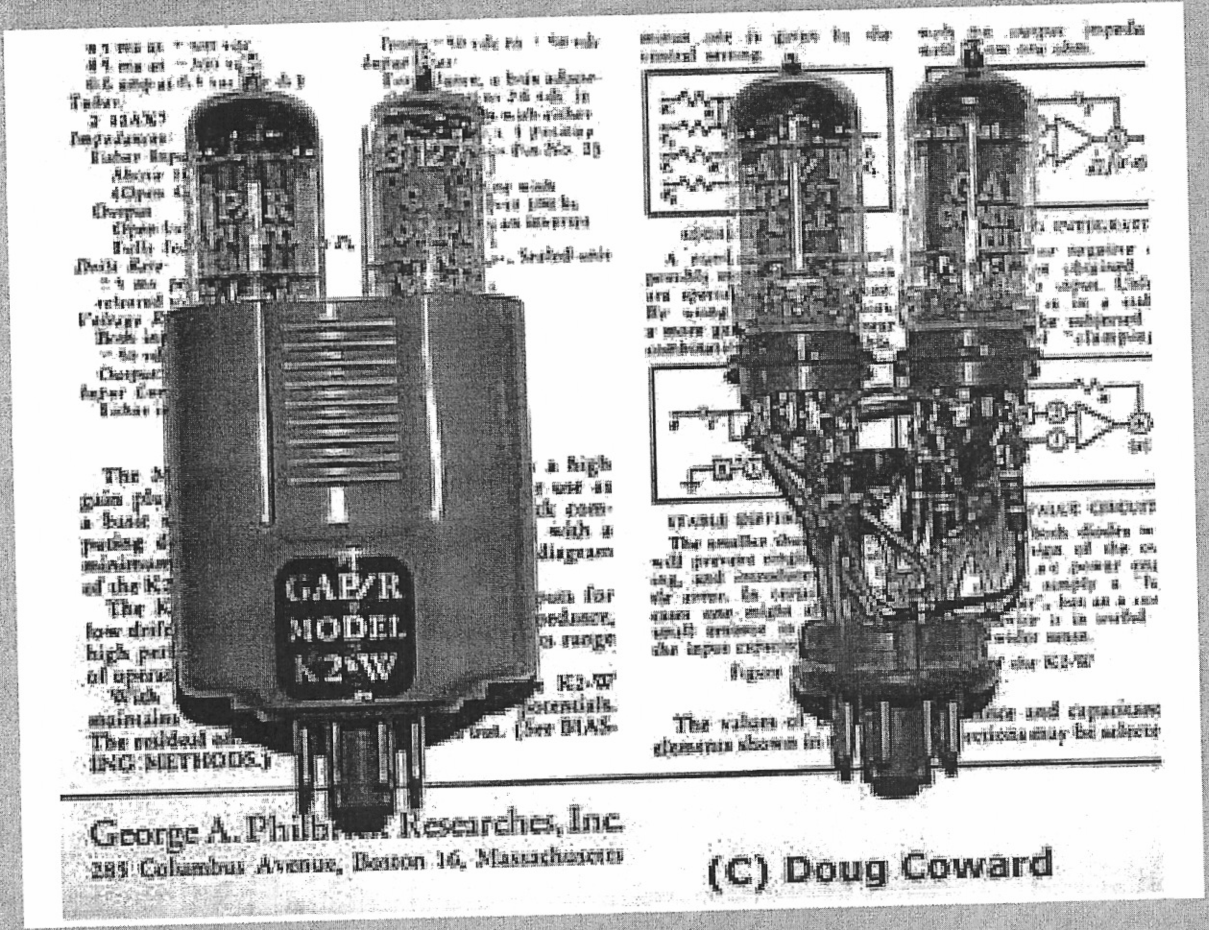
A picture of the GAP/R Model K2-W Op Amp can be seen on my web site

http://www.ece.utk.edu/ngreen/

Look under ECE 300 Sp'06; Operational Amplifier (power point). These devices were expensive; ranging from $50-$90 per unit. They were the backbone of the analog computer. Basically the analog computer was used to solve differential equations that represented the models of physical systems. The vacuum tube op amp has been replaced by solid state semi-conductor (chip) op amps. The price for a garden variety op amp (μA741) is less than one dollar. At the same time, analog computers (which can be readily constructed from low-cost solid state op-amps) have all but faded into the past. Most simulation today is performed using digital computers with software packages such as MATLAB/Simulink. We continue now on looking at the physical properties of the modern day op amps.

Basic Electric Circuits

Operational Amplifiers



The Philbrick Operational Amplifier.

From "Operational Amplifier", by Tony van Roon: http://www.uoguelph.ca/~antoon/gadgets/741/741.html

Perhaps the work hourse of op amps is the low cost LM741. One can purchase these with from one to four op-amps on a DIP (dual in-line pack). The "half chip" op amp is very popular, selling for under 50¢ per unit. A pin connection for the device is shown in Figure 18.1

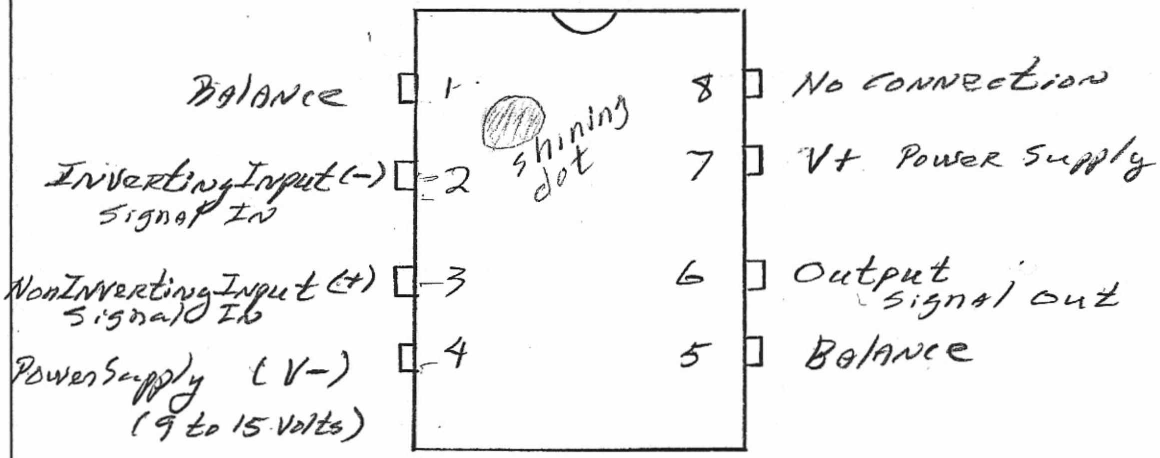


Figure 18.1: Pin layout for the LM 741 op amp.

The physical size is on the order of 0.25" by (3/8)", top view, by (1/8)" deep.

In simple connections it is most often not necessary to use the "Balance" input. The no connection pin is there simple because of manufacturing format of an 8 pin chip.

The presentation of the op amp as shown in Figure 18.1 is non-descript as a device. The conventional display (drawing) is shown in Figure 18.2.

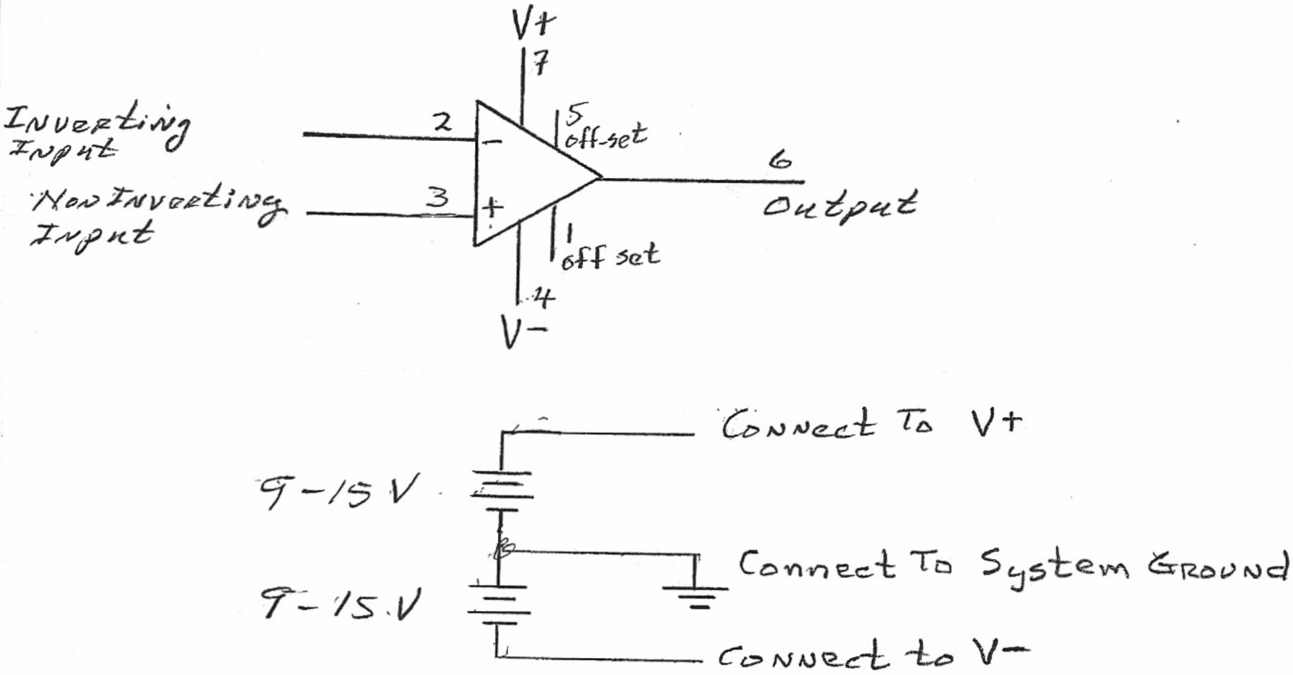


Figure 18.2: Showing Hook-Up of The LM741 Op Amp.

It is not uncommon for the first-time student to forget (or not realize) the the supply voltage must externally be connected to the device. Most op amps have an upper/lower voltage rating (this case ± 15 V), often called the rail-to-rail voltage, but will operate quite successfully on lower voltages (± 9V is shown above).

Basic Electric Circuits

Operational Amplifiers

The op amp is built using VLSI techniques. The circuit diagram of an LM 741 from National Semiconductor is shown below.

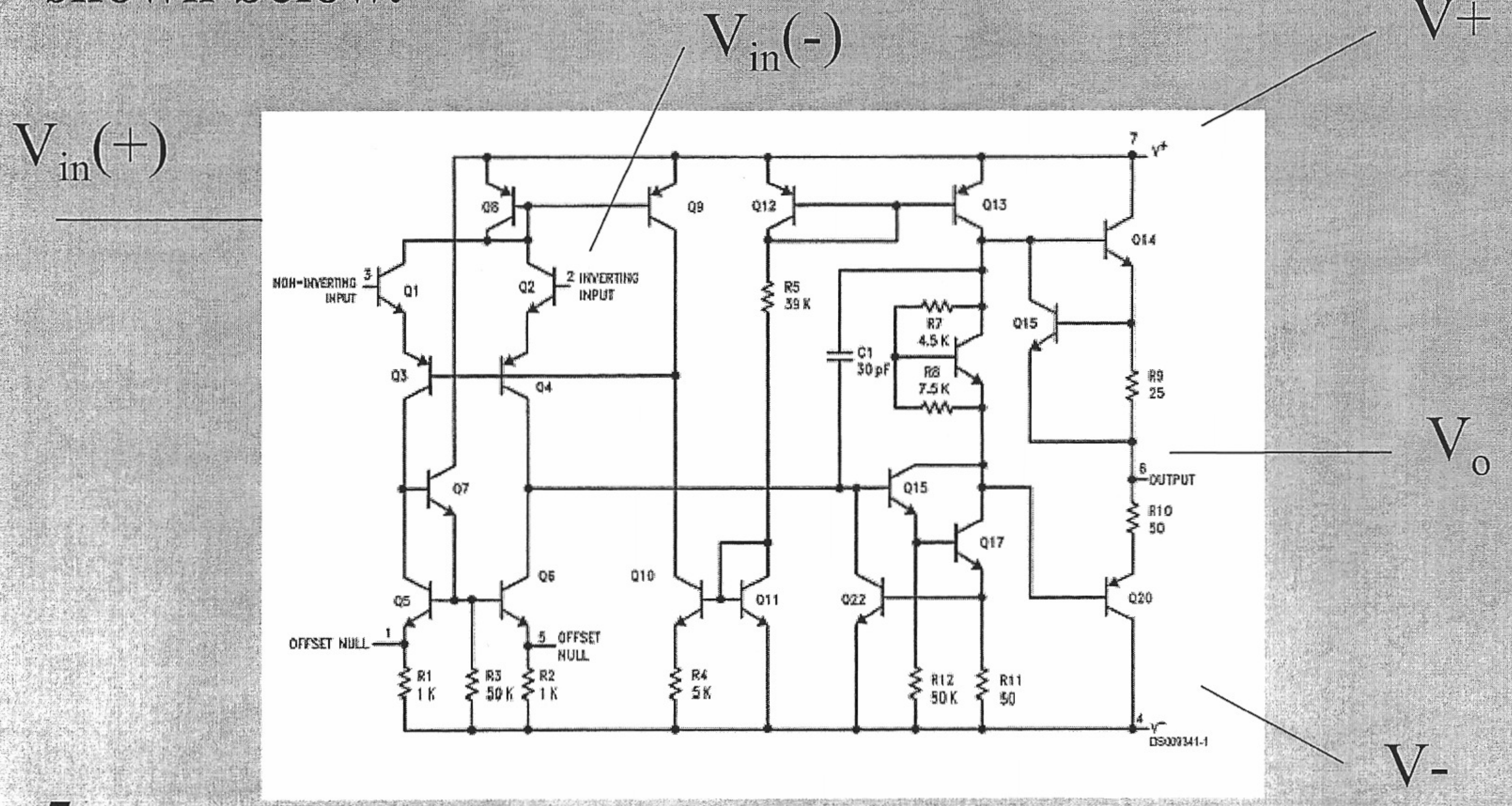


Figure 8.1: Internal circuitry of LM741.

Taken from National Semiconductor data sheet as shown on the web.

We will see later that an op amp does not include pin numbers and voltage supply pins identification when the op amp is connected to external circuitry. When this happens, one must keep in mind that the current for the device comes from the external power supply and input signals as shown in Figure 18.3.

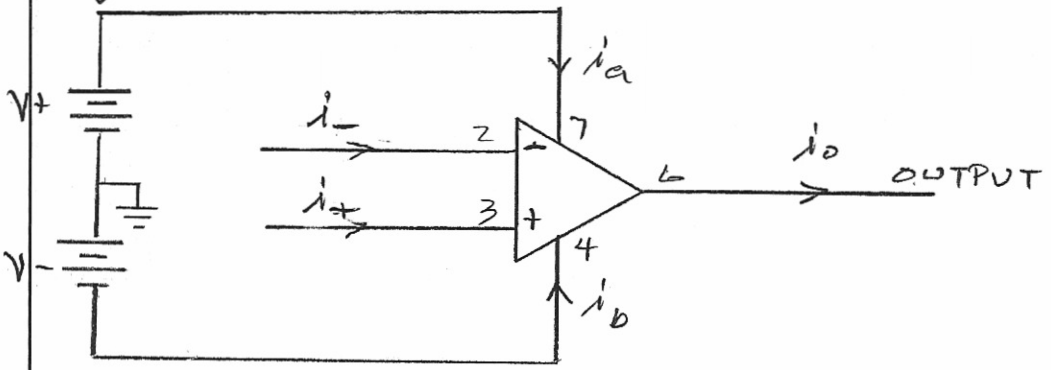


Figure 18.3: Reminder. Circuit showing external currents for an op amp.

obviously, from this diagram;

$$i_o = i_a + i_b + i_+ + i_- \qquad (18.1)$$

In our earlier study of circuits, we have see dependent and independent sources. The model (typical model) of the op amp relys in dependent sources.

A model of the op-amp that gives reasonably good results is shown in Figure 18.4

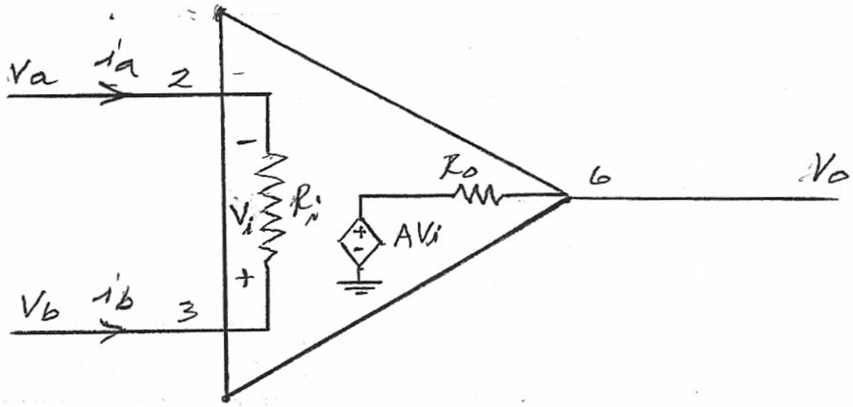


Figure 18.4: A basic model of the op amp.

We note that a dependent voltage controlled voltage source is present at the output. The output voltage is given by

$$V_o = A\left(V_b - V_a\right) \qquad (18.2)$$

A is the open-loop gain (no external feedback) of the op-amp.

Table 18.1 gives some typical values of op amp parameters

Parameter	Range	Ideal Values
Open Loop Gain, A	$10^5 - 10^8$	∞
Input Resistance, R_i	$10^4 - 10^{12}\ \Omega$	$\infty\ \Omega$
Output Resistance, R_o	$10 - 100\ \Omega$	$0\ \Omega$
Supply Voltage	$5 - 20$ V	

Table 18.1: Typical Op Amp Parameter Values.

There are limitations on how much current the op amp can output and how much the voltage can swing.

The typical output swing is judged by the approximate sketch shown in Figure 18.5.

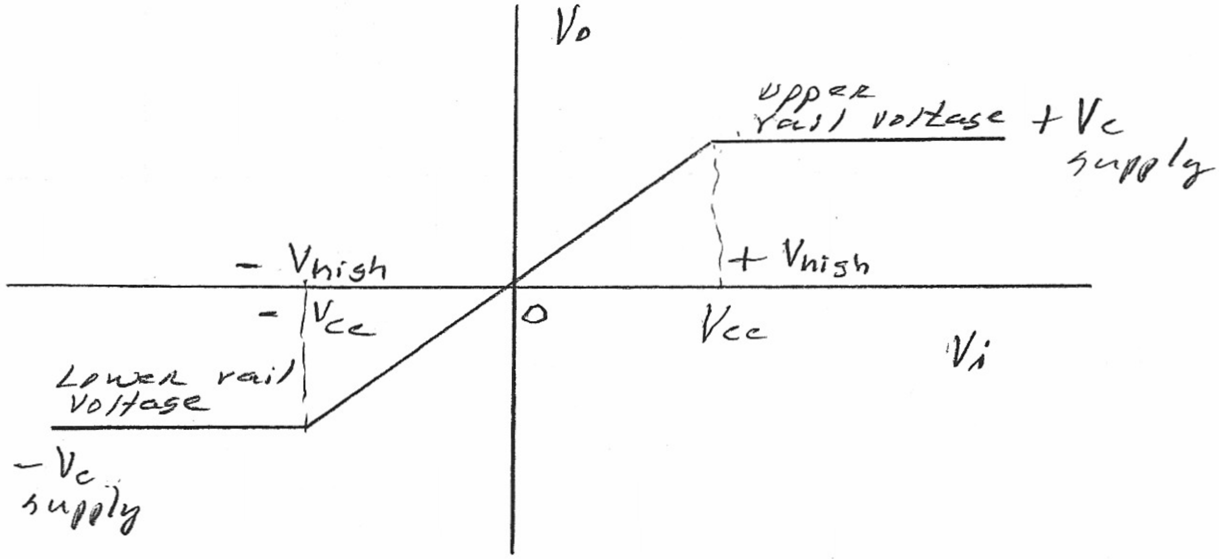


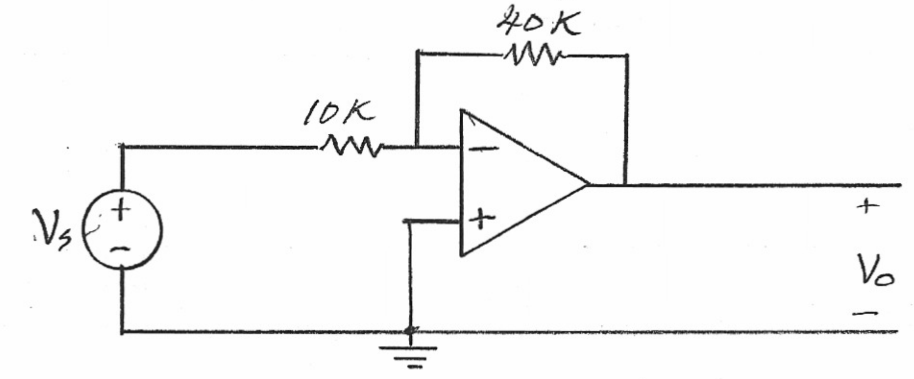
Figure 18.5: Sketch showing linear range of operation for an op amp

The device can successfully operate over ± V_{high}, which is the range of the supply voltage applied at pins 7 and 4 on the 741. This range, $-V_{cc}$ to V_{cc} is sometimes called the rail to rail voltage.

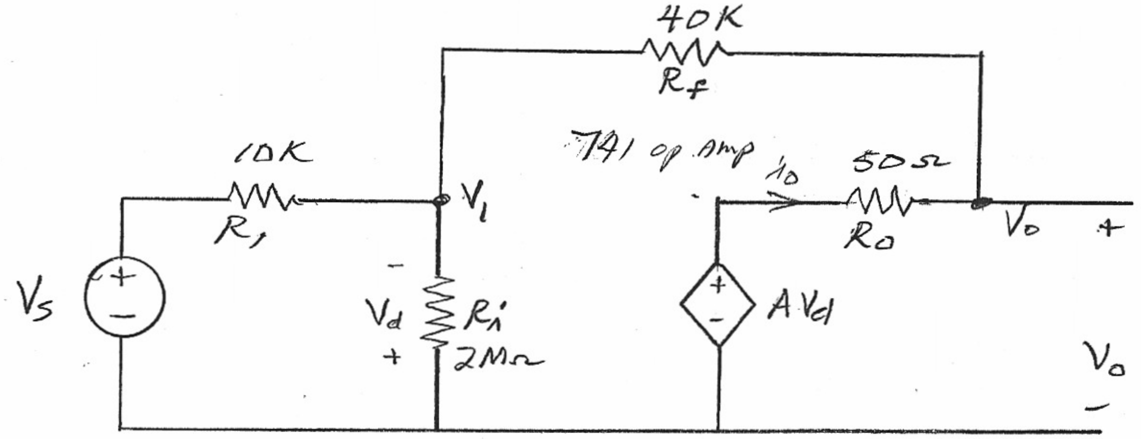
Analysis

We now consider a typical application of the op amp.

A normal way of showing external circuitry connected to an op amp is shown in Fig 18.5a. Using the model of Figure 18.4, produces the circuit of Figure 18.6b.



(a) A popular op amp configuration.



(b) Circuit of Figure 18.6a but using full model of the op amp.

Figure 18.6

We desire to find the transfer function Vo/Vs. An important note: We carry out the analysis of op amp circuits use nodal analysis. A great deal of this has to do with us adopting a simplier model later in which case we would have difficulty using, say mesh analysis.

At node V_1:

$$\frac{V_1 - V_s}{10K} + \frac{V_1}{2M} + \frac{V_1 - V_o}{40K} = 0$$

OR

$$200V_1 - 200V_s + V_1 + 50V_1 - 50V_o = 0$$

$$200V_s = 251V_1 - 50V_o$$

OR

$$V_1 = 0.7968V_s + 0.1992V_o \qquad (18.3)$$

At V_o:

$$\frac{V_o - V_1}{40K} + \frac{V_o - AV_d}{50} = 0$$

OR

$$\frac{V_o - V_1}{40K} + \frac{V_o - AV_1}{50} = 0$$

Using $A \simeq 200{,}000$ (741 typical)

$$V_o - V_1 + 800V_o - 160\times10^6 V_1 = 0 \qquad (18.4)$$

Using (18.3) in (18.4)

$$V_o - 0.7968V_s - .1992V_o + 800V_o - 160\times10^6(.7968V_s + .1992V_o) = 0$$

OR

$$V_o - 0.7968V_s - .1992V_o + 800V_o - 127.488\times10^6 V_s - 31.872\times10^6 V_o = 0$$

essentially

$$V_o = -\frac{127.488\times10^6 V_s}{31.872800} = -3.9998996\,V_s$$

OR

$$\frac{V_o}{V_s} = -3.9998996$$

(really close to 4)

A Simplior Model

We have seen that

$$|V_o| < V_c \quad (\text{supply})$$

which means

$$A(V_b - V_a) < V_c$$

OR

$$(V_b - V_a) < \frac{V_c}{A}$$

but V_c is very finite, say 15V;
"A" runs on the order of 1×10^6 or more
OR

$$(V_b - V_a) \doteq 15 \text{ micro volts}$$

so we make the approximation

$$\boxed{V_b = V_a} \tag{18.5}$$

An implication of this is that

$$i_b = -i_a = \frac{V_b - V_a}{R_{in}} \tag{18.6}$$

$V_b - V_a$ is very, very small; R_{in} is very large, so we say

$$i_b = -i_a = 0 \tag{18.7}$$

This sets the stage for our simplier model. We make the following assumptions.

(1) A, gain, is ∞

(2) R_i, input resistance, is ∞

(3) R_o, output resistance, is 0

(4) $i_a = i_b = 0$

(5) $V_a = V_b$

Some of this is portaid in Figure 18.7

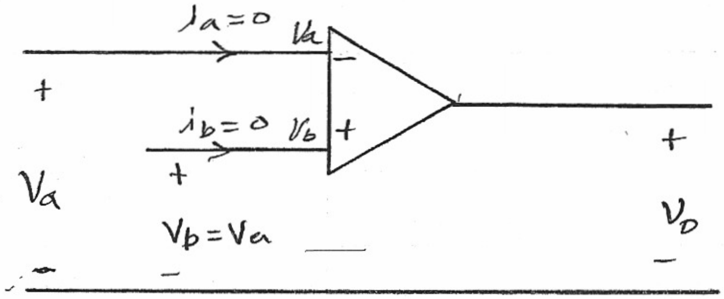


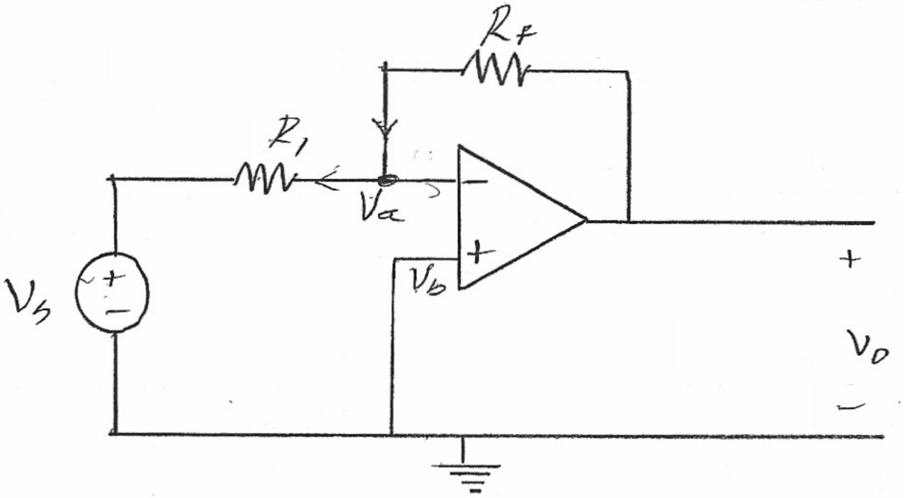
Figure 18.7: Symplified Op Amp Model.

Two important things to remember:

- $i_a = i_b = 0$ (no current enters the non-inverting and inverting terminals)
- V_a follows the voltage of V_b.

This greatly simplifies the analysis of op amps. These approximations work very good in most all cases.

Return and Analyze the earlier example



At V_a:

$$\frac{V_a - V_s}{R_1} = \frac{V_o - V_a}{R_f} \qquad (18.8)$$

Since $V_a = V_b$ and V_b is tied to ground, (0 Volts) then $V_a = 0$. Then (18.8) becomes

$$\frac{-V_s}{R_1} = \frac{V_o}{R_f}$$

OR

$$\boxed{\frac{V_o}{V_s} = -\frac{R_f}{R_1}} \qquad (18.9)$$

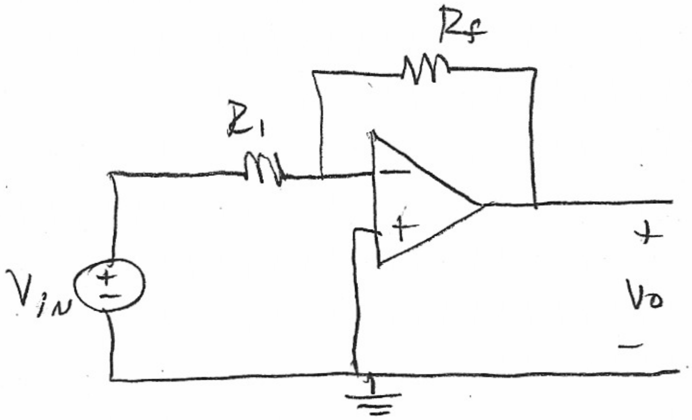
which is sort of a famous equation for an op amp. Using 40k and 10k we get

$$\frac{V_o}{V_s} = -4$$

As compared to

$$\frac{V_o}{V_s} = -3.9998996$$

Gain Configuration



$$V_O = -\frac{R_f}{R_1} V_{IN}$$
(18.10)

Figure 18.8: A voltage controlled voltage

Summing Amplifier

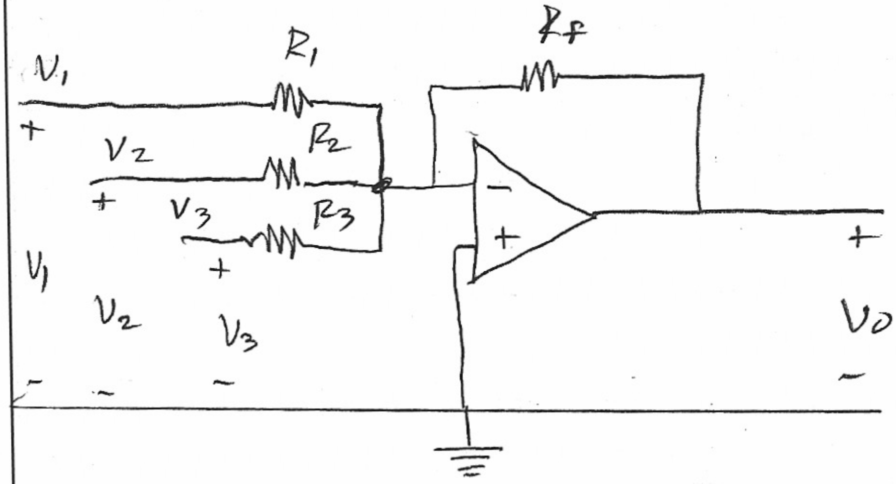


Figure 18.9: A summing op amp configuration.

It is easy to show

$$V_o = -\left[\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3\right]$$

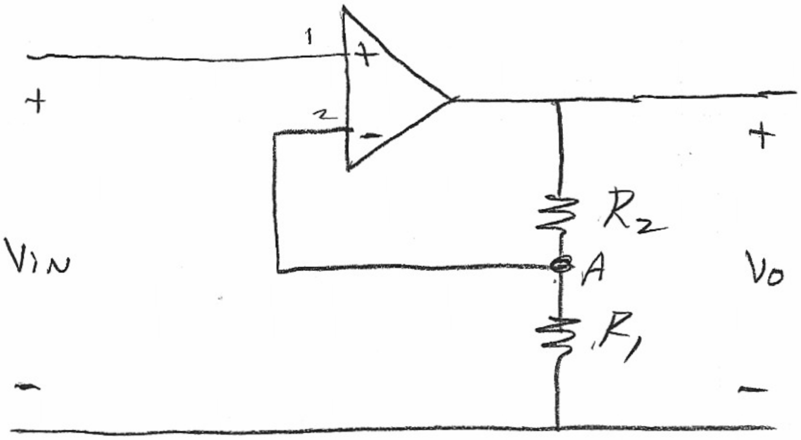
making $R_1 = R_2 = R_3$ gives

$$V_o = -\left[V_1 + V_2 + V_3\right] \frac{R_f}{R_1} \qquad (18.11)$$

Non-Inverting Configuration.

In all cases up to now the output polarity is the negative of the input signal.

Consider the configuration shown in Figure 18.10.



$V_o = \left[1 + \frac{R_2}{R_1}\right] V_{IN}$

Figure 18.10: A non-inverting op amp.

Now the voltage at 1 = voltage at 2 = voltage at A. So we can write the following nodal equation at A.

$$\frac{V_{IN} - V_o}{R_2} + \frac{V_{IN}}{R_1} = 0$$

OR $R_1 V_{IN} - R_1 V_O + R_2 V_{IN} = 0$

$R_1 V_o = (R_1 + R_2) V_{IN}$

OR

$$V_o = \frac{[R_1 + R_2]\,V_{IN}}{R_1}$$

OR

$$\boxed{V_o = \left[1 + \frac{R_2}{R_1}\right] V_{IN}} \qquad (18.12)$$

famous equation

Voltage Controlled Current Source

Considered the op amp configuration shown in Figure 18-11.

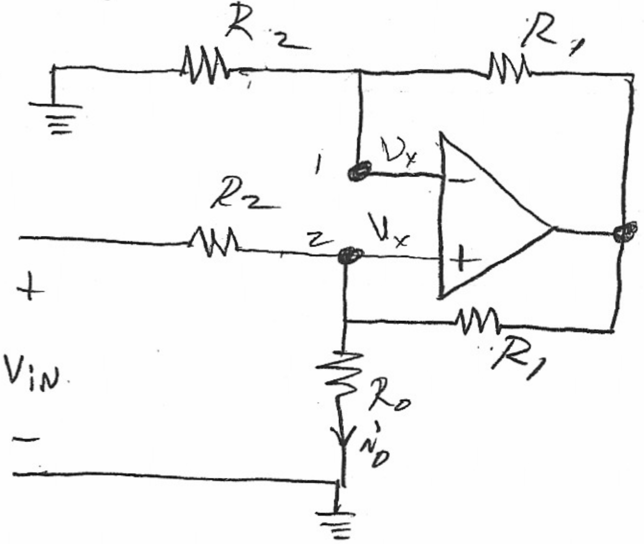


Figure 18.11; A voltage controlled current source configuration.

We write nodal equations at ① and ②.

At ①

$$\frac{V_x}{R_2} = \frac{V_o - V_x}{R_1} \qquad (18.13)$$

At ②

$$\frac{V_x - V_{IN}}{R_2} + \frac{V_x - V_o}{R_1} + i_o = 0 \qquad (18.14)$$

From (18.13)

$$R_1 V_x = R_2 V_o - R_2 V_x$$

or

$$V_o = \frac{(R_1 + R_2) V_x}{R_2} \qquad (18.15)$$

From (18.14), getting a common denominator,

$$R_1 V_x - R_1 V_{IN} + R_2 V_x - R_2 V_o + R_1 R_2 i_o = 0$$

$$(R_1 + R_2) V_x - R_1 V_{IN} - R_2 V_o + R_1 R_2 i_o = 0 \quad (18.16)$$

Substitute (18.15) into (18.16)

$$(R_1 + R_2) V_x - R_1 V_{IN} - R_2 \left(\frac{R_1 + R_2}{R_2}\right) V_x + R_1 R_2 i_o = 0$$

or

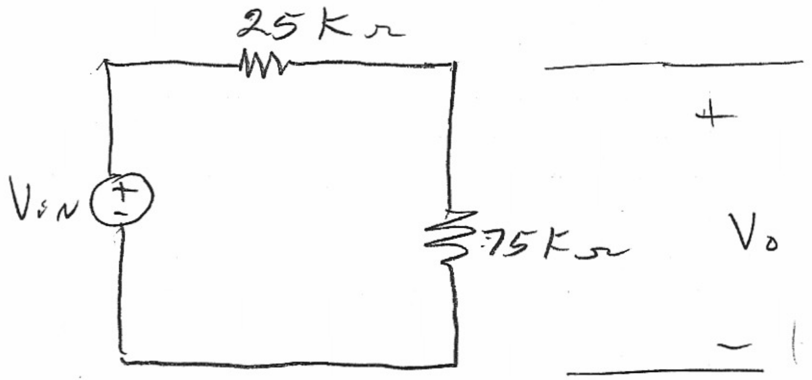
$$R_1 R_2 i_o = R_1 V_{IN}$$

$$\boxed{i_o = \frac{V_{IN}}{R_2}} \qquad (18.17)$$

So the output current is controlled by the input voltage.

Buffer Amplifier

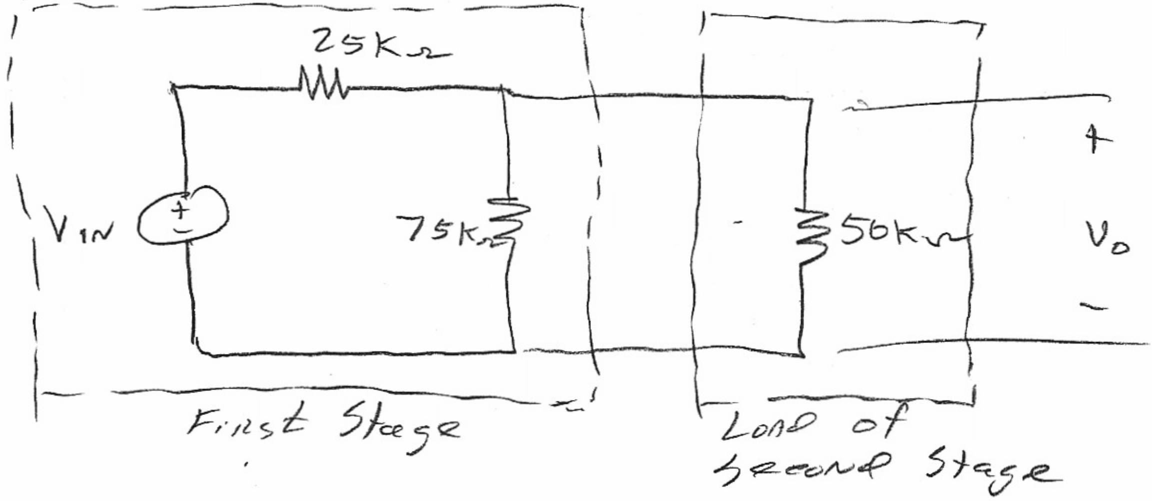
We often need a "buffer" between two stages of a process so that the final stage, when connected to the initial stage, does not load-down the initial stage. Consider the oversimplified following case.



We see that

$$\frac{V_o}{V_{IN}} = \frac{75k}{25k+75k} = 0.75$$

Suppose we now have the case,

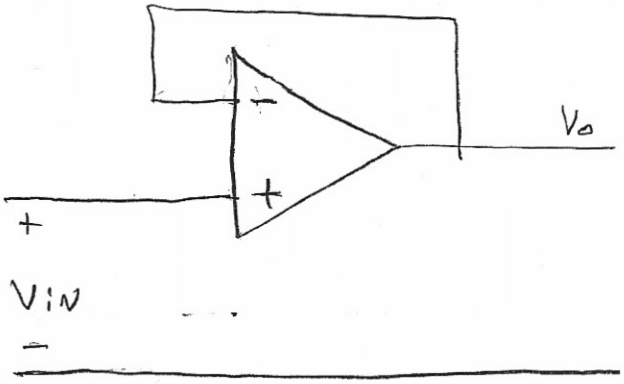


With $75K \parallel 50K = \frac{75 \times 50}{125} = 30K$

Therefore;

$$\frac{V_o}{V} = \frac{30K}{30K + 25K} = 0.545$$

We see the 50K load has loaded down the first stage and changed V_o. We can avoid this by placing a buffer between the two stages. The buffer, as an operational amplifier is shown in Figure 18.12



$V_o = V_{IN}$

Figure 18.12: Buffer amplifier.

The input impedance is very high (> 1 meg Ω), the output impedance (looking back in the op-amp) is low (normally < 50 Ω).

with the buffer amplifier inserted

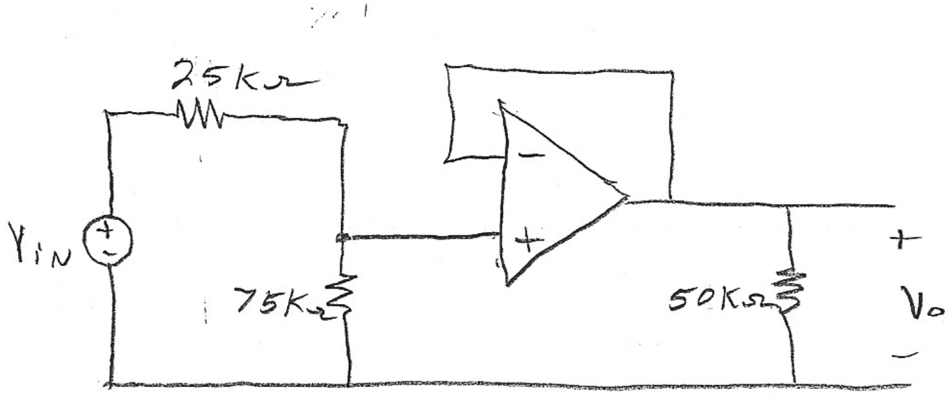


Figure 18.13: Two stages separated by a buffer amplifier.

The ratio of V_o/V_{IN} should now return to 0.75. You are encouraged to try this on your own to see if it works.

A Difference Amplifier

Suppose you have two signals V_1 and V_2 and you would like to subtract V_2 from V_1. Consider the configuration of Fig 18.14

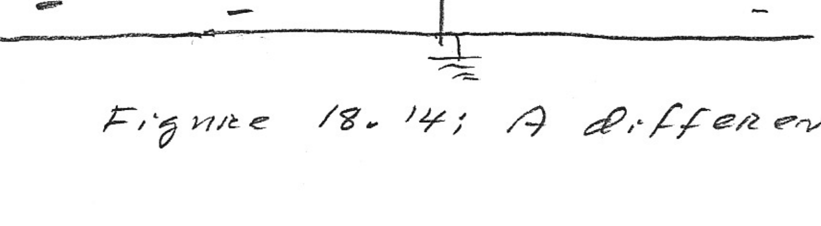


Figure 18.14: A difference amplifier.

At the upper V_x in Fig 18.14

$$\frac{V_x - V_2}{R_1} + \frac{V_x - V_o}{R_2} = 0$$

OR

$$V_x\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{V_2}{R_1} + \frac{V_o}{R_2} \qquad (18.18)$$

At the lower V_x of Figure 18.14

$$\frac{V_x - V_1}{R_1} + \frac{V_x}{R_2} = 0$$

OR

$$V_x\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{V_1}{R_1} \qquad (18.19)$$

Equate (18.18) to (18.19)

$$\frac{V_1}{R_1} = \frac{V_2}{R_1} + \frac{V_o}{R_2}$$

$$\frac{V_o}{R_2} = \frac{V_1}{R_1} - \frac{V_2}{R_1}$$

OR

$$\boxed{V_o = \frac{R_2}{R_1}\left[V_1 - V_2\right]} \qquad (18.20)$$

One can control gain by a selection of the resistors R_1 & R_2

Example 1

You are given the following op amp configuration. Find the voltage V_o

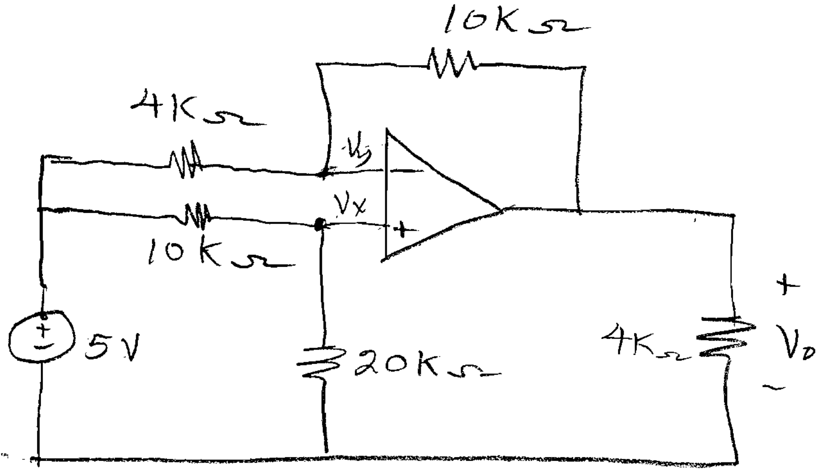


Figure 18.15: Op Amp circuit for Example 1.

At V_x

$$\frac{V_x - 5}{10K} + \frac{V_x}{20K} = 0$$

or

$$2V_x - 10 + V_x = 0$$

$$3V_x = 10 \Rightarrow V_x = \frac{10}{3} \qquad (18.21)$$

At V_y

$$\frac{V_y - 5}{4K} + \frac{V_y - V_o}{10K} = 0$$

$$5V_y - 25 + 2V_y - 2V_o = 0$$

$$7V_y = 2V_o + 25$$

$$V_y = \frac{2V_o + 25}{7} \qquad (18.22)$$

but $V_x = V_y$ so set $(18.21) = \overset{23}{(18.22)}$

$$\frac{10}{3} = \frac{2V_o + 25}{7}$$

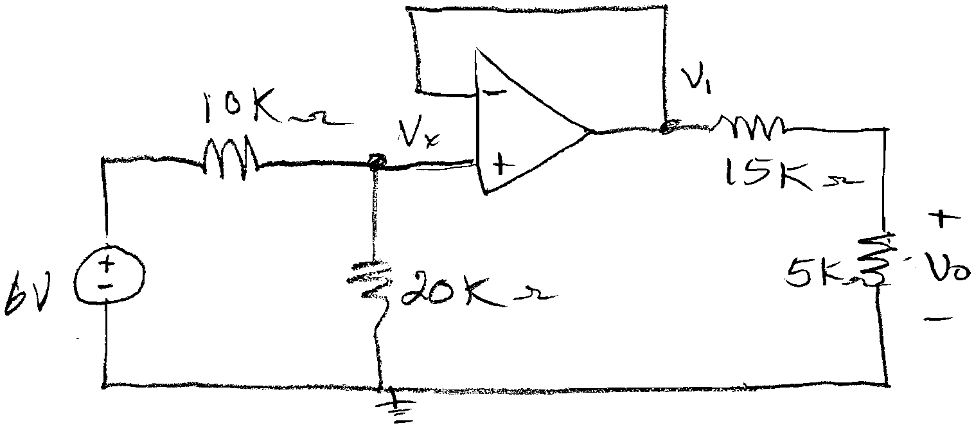
$$70 = 6V_o + 75$$

$$6V_o = 70 - 75$$

$$\boxed{V_o = \frac{-5}{6}}$$

Example 2

Find Vo for the following op amp circuit.



$$\frac{V_x - 6}{10K} + \frac{V_x}{20K} = 0$$

$$2V_x - 12 + V_x = 0$$

$$3V_x = 12 \Rightarrow V_x = 4V = V_1$$

$$V_o = \frac{V_1 \times 5K}{5K + 15K} = \frac{V_1}{4} = \frac{4}{4} = 1\ V$$

$$\boxed{V_o = 1V}$$

Example 3

You are given the op amp circuit as shown in Figure 18.16. What value of R_f will give an output of

$$V_o = 5 - 4V_a \quad ?$$

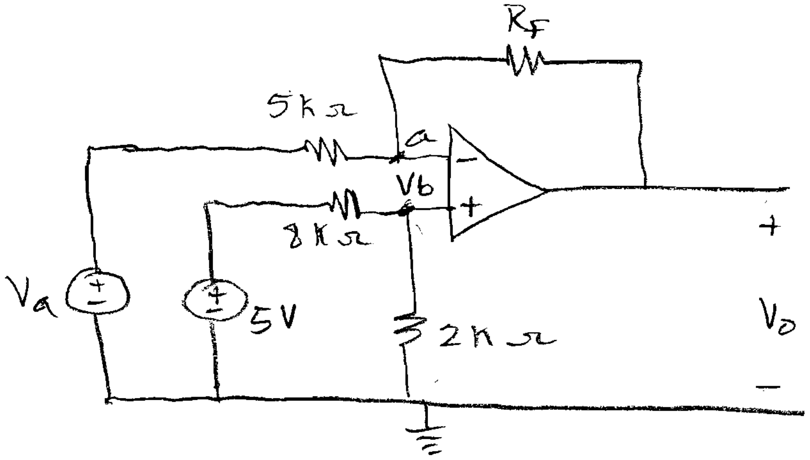


Figure 18.16: Circuit for Example 3.

At Vb

$$V_b = \frac{5 \times 2K}{2K + 8K} = 1\,V$$

At "a"

$$\frac{1 - V_a}{5K} = \frac{V_o - 1}{R_f}$$

$$V_o - 1 = \frac{R_f(1 - V_a)}{5K}$$

$$V_o = \frac{R_f(1 - V_a)}{5K} + 1$$

So

$$\frac{R_f(1-V_a)}{5K} + 1 = 5 - 4V_a$$

or

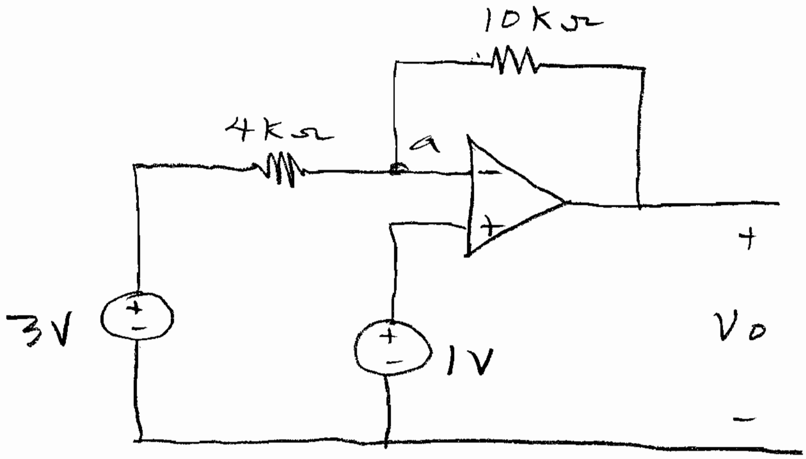
$$R_f \frac{(1-V_a)}{5K} = 4(1-V_a)$$

$$R_f(1-V_a) = 4 \times 5k(1-V_a)$$

$$\therefore \boxed{R_f = 20k\,\Omega}$$

Example 4:

You are given the op amp circuit of Figure 18.17. Determine V_o.



At "a"

$$\frac{1-3}{4K} = \frac{V_o - 1}{10K}$$

$$(-0.5)10 + 1 = V_o$$

$$\boxed{V_o = -4\text{ V}}$$

15   $\frac{12}{10}$

Example 5

Determine the output current i_x for the following op amp circuit.

$$\frac{.5 \times 10k}{13k}$$

$$5/13$$

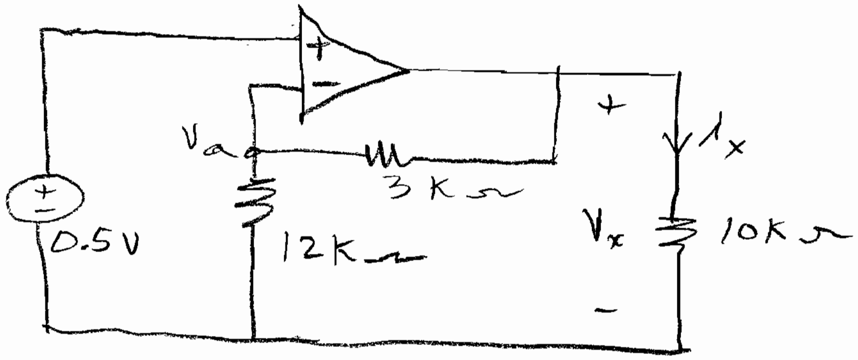


Figure 18.17: Circuit for example 5.

$V_a = 0.5\ V$

but

$$V_a = 0.5 = \frac{V_x \times 12K}{(12+3)K} = \frac{4}{5} V_x$$

$$V_x = \frac{2.5}{4}$$

$$i_x = \frac{V_x}{10K} = 0.0625\ mA$$

Example 6

Voltage Follower.

You are given the circuit of Figure 18.18.

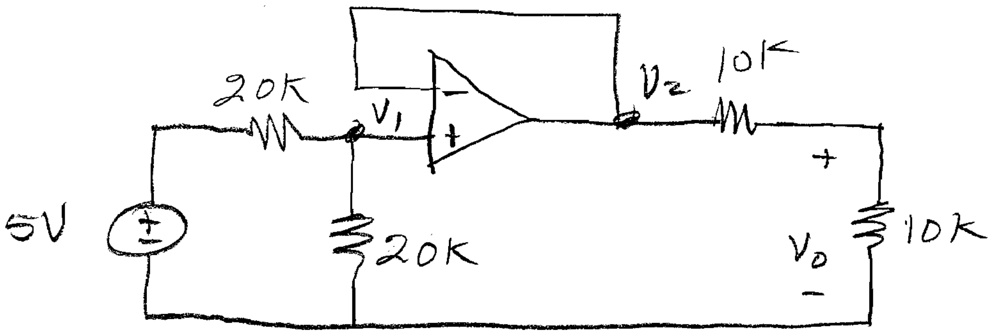


Figure 18.18: Circuit for Example 6.

The voltage V_2 follows V_1. Since

$$V_1 = \frac{5 \times 20k}{20k + 20k} = 2.5\,V$$

$$V_2 = V_1 = 2.5\,V$$

$$V_o = \frac{V_2 \times 10k}{10k + 10k} = \frac{V_2}{2}$$

$$\boxed{V_o = 1.25\,V}$$

Example 7

For the op amp circuit of Figure 18.19 find i_x and i_y.

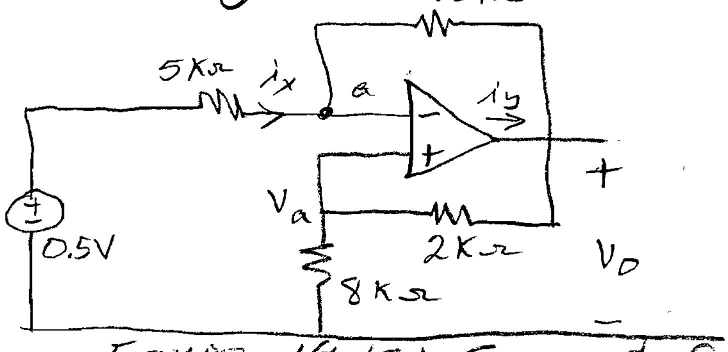


Figure 18.19: Circuit for Example 7.

$$V_a = \frac{V_o \times 8K}{8K + 2K} = 0.8 V_o \qquad (1)$$

At "a"

$$i_x = \frac{0.8 V_o - V_o}{10K} = -0.02 V_o \text{ mA}$$

At "a"

$$\frac{.8 V_o - .5}{5\cancel{K}} = \frac{V_o - .8 V_o}{10\cancel{K}}$$

$$1.6 V_o - 1 = + .2 V_o$$

$$1.4 V_o = 1$$

$$V_o = \frac{1}{1.4} \text{ V}$$

$$\boxed{i_x = -\frac{.02}{1.4} \text{ mA} = -0.01428 \text{ mA}}$$

$$i_y = \frac{V_o - .8 V_o}{10K} + \frac{V_o}{10K}$$

$$= \frac{.2 V_o + V_o}{10K} = \frac{1.2 V_o}{10K} = \frac{(1.2)\left(\frac{1}{1.4}\right)}{10K}$$

$$\boxed{i_y = 0.0857 \text{ mA}}$$

Example 8

Find the voltage gain for the following op amp circuit.

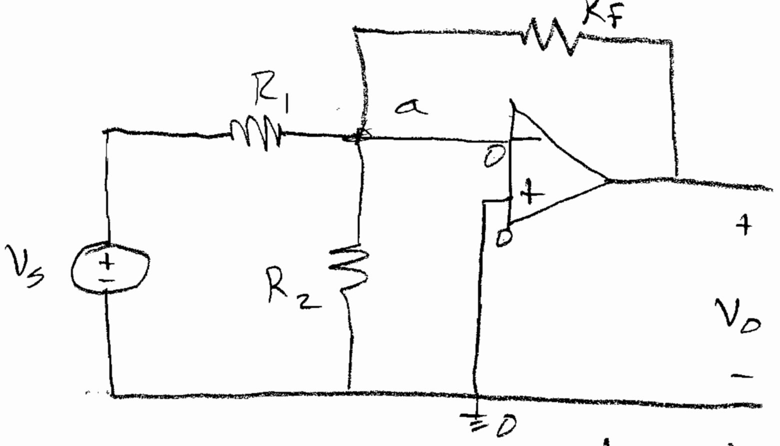


Figure 18.20: Circuit for Example 8.

At "a"

$$\frac{0 - V_s}{R_1} = \frac{V_o - 0}{R_f}$$ (the current thru $R_2 = 0$)

$$\boxed{\frac{V_o}{V_f} = -\frac{R_f}{R_1}}$$

In earlier times, one of the most important roles of the op amp was its use as an integrator, that is, the ability to electrically perform the function of mathematically integrating a signal. This was the back bone of the analog computer. Whereas analog computers have expired, the operation of integration is still important in signal processing. We now look at the process of integration with the op amp.

Example 9

Consider the op amp circuit shown in Figure 18.21

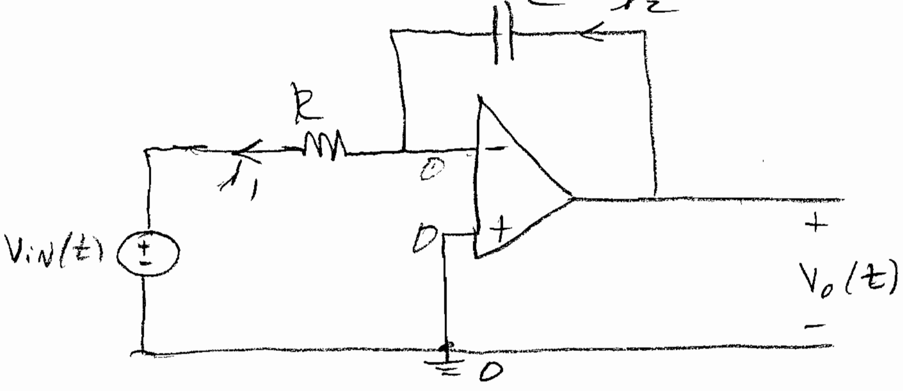


Figure 18.21: Op Amp circuit used as an integrator.

From the diagram we see that

$$i_1 = i_2$$ (no current enters the inverting terminal)

or

$$0 - \frac{V_{in}(t)}{R} = C\frac{dV_o(t)}{dt}$$

and

$$\frac{dV_o(t)}{dt} = -\frac{V_{in}(t)}{RC} \qquad (18.23)$$

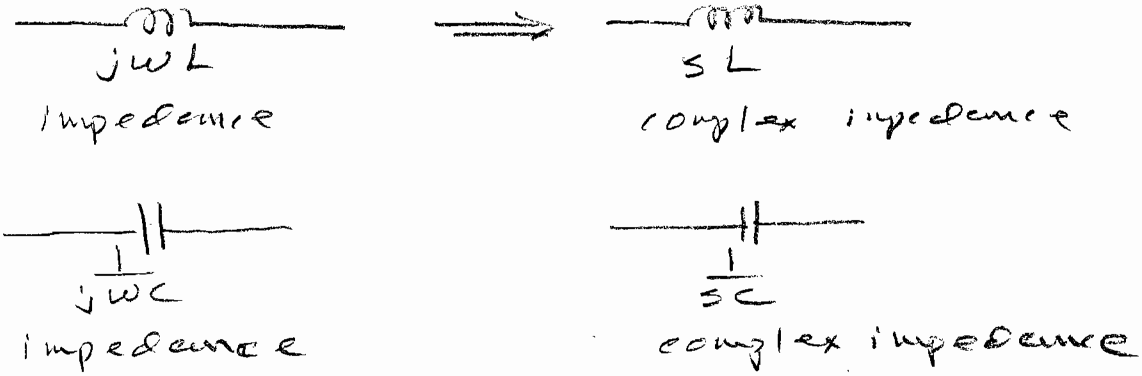
We separate variables and integrate;

$$\int_{\tau=0}^{\tau=t} dV_o(\tau) = -\frac{1}{RC}\int_{\tau=0}^{\tau=t} V_{in}(\tau)\,d\tau$$

$$\boxed{V_o(t) - V_o(0) = -\frac{1}{RC}\int_{\tau=0}^{\tau=t} V_{in}(\tau)\,d\tau} \qquad (18.24)$$

Initial condition, $V_o(0)$, can be applied to the op amp by applying an initial voltage to the capacitor.

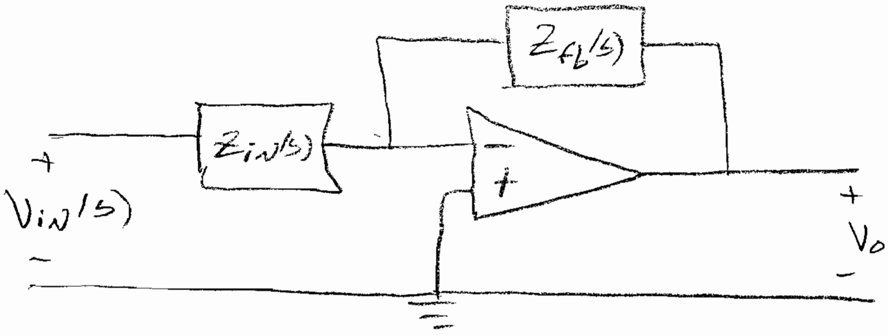
Most often when we use op amps we use complex impedance rather than actually carrying out integration as above. For example



And mathematically

$$\mathcal{L}\left[\int f(t)\,dt\right] = \frac{F(s)}{s}$$ with 0 initial condition

We recall



$$\frac{V_o(s)}{V_{in}(s)} = - \frac{Z_{fb}(s)}{Z_{in}(s)}$$

If $Z_{fb}(s)$ corresponds to the complex impedance of a capacitor

$$Z_{fb}(s) = \frac{1}{sC}$$

and $Z_{in}(s)$ is the complex impedance of a resistor;

$$Z_{in}(s) = R$$

So

$$V_o(s) = - \frac{V_{in}(s)}{RC\, s}$$

in the time domain

$$V_o(t) = -\frac{1}{RC}\int V_{in}(t)\,dt$$

which is the result we got earlier.

Example 10

We saw earlier on page 14 how an op amp can be set-up to add input signals. This example illustrates how to set-up an op amp so it averages the input signals. Consider the circuit of Figure 18.22.

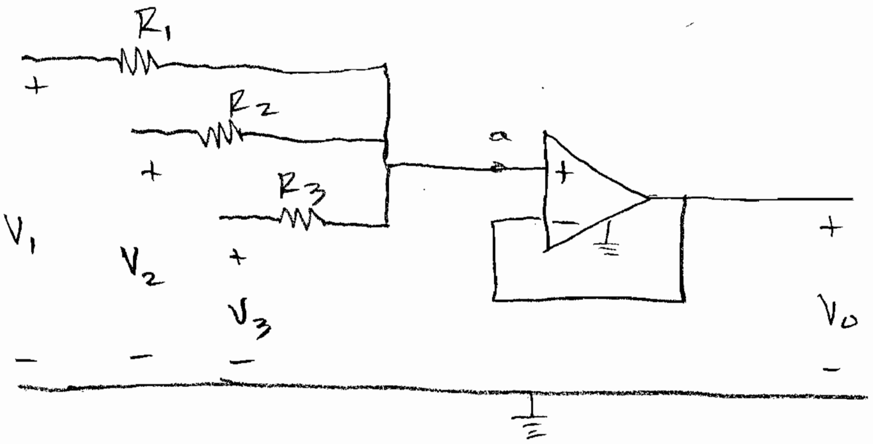


Figure 18.22: Op Amp used for averaging.

The voltage at "a" is V_0 so we can write

$$\frac{V_1 - V_0}{R_1} + \frac{V_2 - V_0}{R_2} + \frac{V_3 - V_0}{R_3} = 0$$

or

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = V_0\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

or

$$R_2R_3V_1 + R_1R_3V_2 + R_1R_2V_3 = V_0\left[R_2R_3 + R_1R_3 + R_1R_2\right]$$

OR

$$V_0 = \frac{R_2 R_3 V_1 + R_1 R_3 V_2 + R_1 R_2 V_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

If $R_1 = R_2 = R_3 = R$

$$V_0 = \frac{(V_1 + V_2 + V_3)\cancel{R^2}}{3\cancel{R^2}}$$

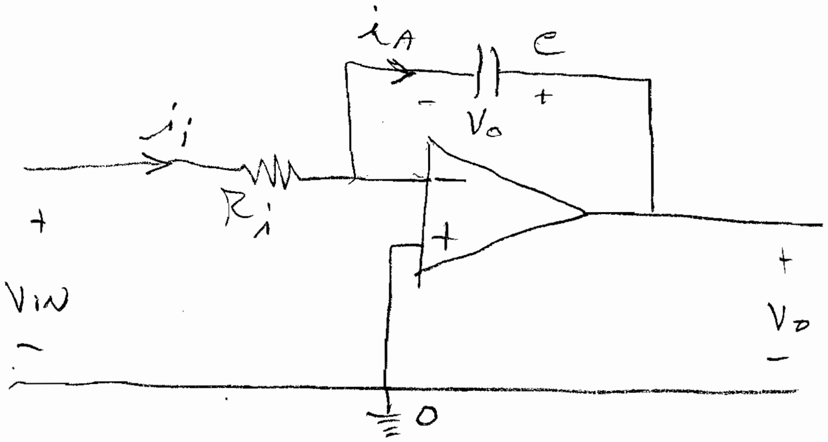
OR

$$\boxed{V_0 = \frac{(V_1 + V_2 + V_3)}{3}}$$

Closure on Op-Amps

Integrating with an op-amp.

Consider the following:



$$i_i = i_A$$

$$i_i = \frac{V_{IN}}{R_i} \qquad i_A = -C\frac{\partial V_o}{\partial t}$$

$$C\frac{\partial V_o}{\partial t} = -\frac{V_{IN}}{R_i}$$

$$\partial V_o = -\frac{1}{R_iC} V_{in}\,\partial t$$

Assume $V_o(0) = 0$

$$V_o = \left[-\frac{1}{R_iC}\right]\int V_{in}(t)\,\partial t$$

The output is equal to the input multiplied by the scaling factor $\left[-\frac{1}{R_iC}\right]$

Suppose you have

$$\frac{dx(t)}{dt} + 4x(t) = 4 \qquad (1)$$

solve for $x(t)$: Assume $x(0) = 0$

Several ways we can do this. One is by Laplace

$$\mathcal{L}\left[\frac{dx(t)}{dt} + 4x(t)\right] = \mathcal{L}[4]$$

$$sX(s) - \cancelto{0}{x(0)} + 4X(s) = \frac{4}{s}$$

$$[s+4]X(s) = \frac{4}{s}$$

$$X(s) = \frac{4}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

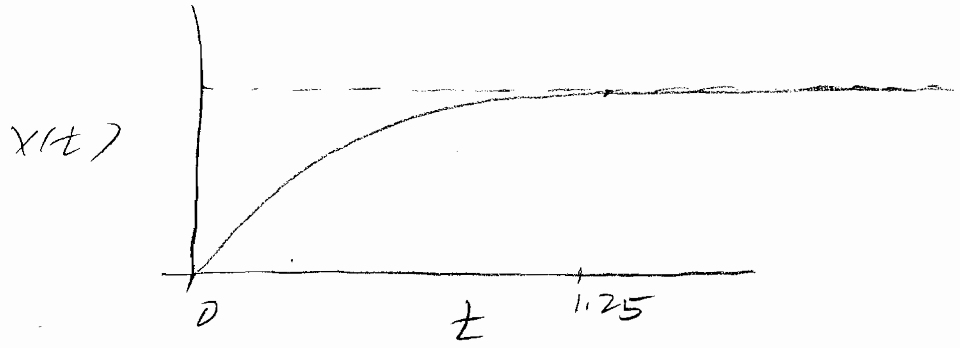
$$A = 1; \quad B = -1$$

$$X(s) = \frac{1}{s} - \frac{1}{s+4}$$

$$\mathcal{L}^{-1}[X(s)] = X(t) = \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+4}\right]$$

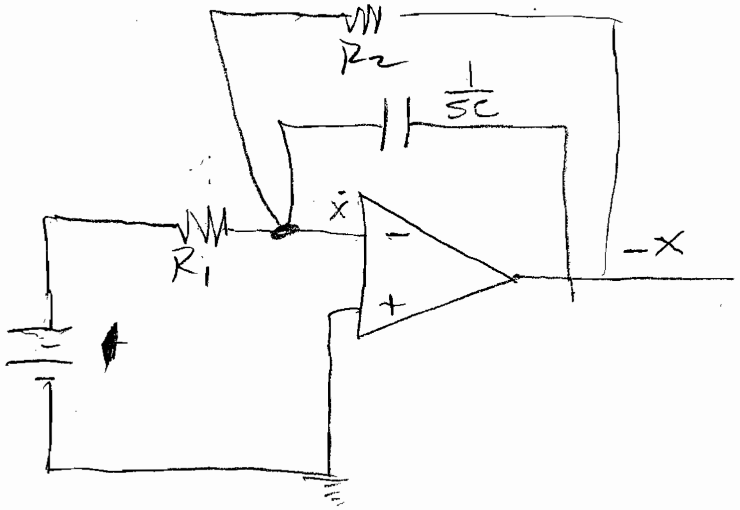
$$X(t) = 1 - e^{-4t}$$

τ = time constant = 0.25 sec.



Go back to (1)

$$\dot{x} = 4 - 4x \longrightarrow x = \int [4 - 4x]\,dt$$



$$\int \dot{x} \Rightarrow -x = \left[\frac{4}{R_1 C s} - \frac{x}{R_2 C s}\right] = \boxed{\frac{1}{s}}\left[\frac{1}{R_1 C} - \frac{1}{R_2 C}\right]$$

$$\frac{1}{R_1 C} = \frac{1}{R_2 C} = 4$$

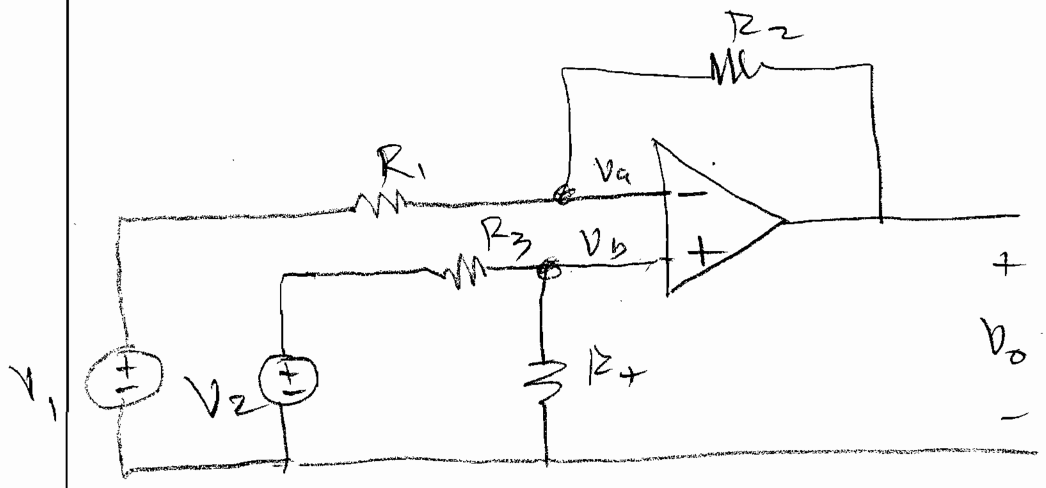
$$R_2 = \frac{1}{4C} = \frac{.25}{C} \qquad C = 1\,\mu F$$

$$R_2 = .25 \times 10^6 = 250K$$

The Difference Amplifier.

Use when there is a need to amplify the difference between two signals. It amplifies the difference between two signals but rejects any signals common to the two inputs.

Consider



At Va

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_o}{R_2} = 0 \qquad (1)$$

At Vb

$$V_b = \frac{V_2 R_4}{R_3 + R_4} \qquad (2)$$

Arrange (1) and (2) in the form of $\underbrace{V_a,\ V_b,\ V_o}_{\text{variables output}}$; $\underset{\text{input}}{V_1,\ V_2}$

From (1)

$$R_2 V_a - R_2 V_1 + R_1 V_a - R_1 V_o = 0$$

$$(R_1 + R_2) V_a + 0 V_b - R_1 V_o = R_2 V_1$$

so

$$\boxed{\left(\frac{R_1 + R_2}{R_2}\right) V_a + 0 V_b - \frac{R_1}{R_2} V_o = V_1}$$

From (2)

$$(R_3 + R_4) V_b = R_4 V_2$$

$$\boxed{0 V_a + \left(\frac{R_3 + R_4}{R_4}\right) V_b + 0 V_o = V_2}$$

Then $V_a = V_b$ or

$$\boxed{V_a - V_b + 0 V_o = 0}$$

$$\underset{A}{\begin{bmatrix} (R_1+R_2)/R_2, & 0 & -R_1/R_2 \\ 0, & (R_3+R_4)/R_4, & 0 \\ 1, & -1, & 0 \end{bmatrix}} \underset{V}{\begin{bmatrix} V_a \\ V_b \\ V_o \end{bmatrix}} = \underset{B}{\begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix}}$$

$$V = A^{-1} B$$

The program (MATLAB) is given on the following pages. We find

$$V_o = -\frac{R_2}{R_1}V_1 + \frac{(R_1+R_2)}{(R_3+R_4)}\frac{R_4}{R_1}V_2$$

If $V_1 = V_2$ we would like

$$V_o = 0$$

$$V_o = -\frac{R_2}{R_1}V_1 + \frac{\left(1+\frac{R_1}{R_2}\right)R_2\cancel{R_4}}{\left(1+\frac{R_3}{R_4}\right)R_1\cancel{R_4}}V_2$$

If $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Then

$$V_o = \frac{R_2}{R_1}\left[V_2 - V_1\right]$$

EKG