The term operational amplifier itself induces one to ask, "what is this"? Most people in science and engineering know or have a good idea what amplification is and basically what it does. The "operational" part may be puzzling.

Firstly, to shorten the writing perhaps, operational amplifier becomes op amp. These devices were more or less introduced in the 1930's - with George Philbrick being recognized for pioneering work with the devices.

Second, we shall shortly see how op amps can be used to

- add signals
- subtract signals
- integrate signals
- differentiate signals (not recommended)
- multiply signals (not directly but in conjunction with other electronics)

It is because the device can perform those mathematical operations, that it was referred to (still is) as an operational amplifier.
In the text:

"Fundamentals of Electric Circuit";
Alexander H. Sadiku; 3rd edition,

The statements are made:
- measurements are a tool for understanding the physical world,
- instruments are tools for measurement,
- the operational amplifier is a building block for modern electronic instrumentation (one might add signal processing and control).

It behooves a person of science and engineering to have a basic understanding (at least from the standpoint of using as a device) how an operational amplifier functions.

**Physical Structure & History**

In the 1940-1960 time frame the operational amplifier was constructed with vacuum tubes. The circuitry was encapsulated in the base that held the vacuum tubes (usually 2 and 12AU7).
A picture of the GAP/IR model 2-24 op amp can be seen on my web site
http://www.ece.utk.edu/ngeor
look under ECE 302 Spring Operational Amplifier (power point). These devices
were expensive; ranging from $50 to $70 per unit. They were the backbone of the
analog computer. Basically, the analog computer was used to solve differential equations
that represented the models of physical systems. The vacuum tube op amp has
deen replaced by solid state semiconductor (chip) op amps. The price for a garden
variety op amp (μA741) is less than one dollar. At the same time analog
computers (which can be readily constructed from low-cost solid state op amps) have
all but faded into the past. Most
simulation today is performed using
digital computers with software packages
such as MATLAB/Simulink. We continue
now on looking at the physical properties
of the modern day op amp.
Basic Electric Circuits

Operational Amplifiers

The Philbrick Operational Amplifier.

From "Operational Amplifier", by Tony van Roon: http://www.uoguelph.ca/~antoon/gadgets/741/741.html
Perhaps the workhorse of op amps is the low cost LM741. One can purchase these with from one to four op-amps on a DIP (dual in-line pack). The "half chip" op amp is very popular, selling for under $0.4 per unit. A pin connection for the device is shown in Figure 18.1.

![Diagram of LM741 pin layout]

Figure 18.1: Pin layout for the LM 741 op amp.

The physical size is on the order of 0.25" by (3/8)", top view, by (3/16)" deep.

In simple connections it is most often not necessary to use the "balance" input. The no connection pin is there simply because of manufacturing format of an 8 pin chip.
The presentation of the op amp as shown in Figure 18.1 is non-descript as a device. The conventional display (drawing) is shown in Figure 18.2.

![Diagram of an op amp](image)

Connect to V+

Connect to System Ground

Connect to V-

**Figure 18.2:** Showing Hook-Up of The LM1741 Op Amp.

It is not uncommon for the first-time student to neglect (or not realize) the supply voltage must externally be connected to the device. Most op amps have an upper/lower voltage rating (this case ±15 V), often called the rail-to-rail voltage, but will operate quite successfully on lower voltages (±9 V is shown above).
Basic Electric Circuits

Operational Amplifiers

The op amp is built using VLSI techniques. The circuit diagram of an LM 741 from National Semiconductor is shown below.

\[ V_{\text{in}}(-) \]
\[ V_{\text{in}}(+) \]
\[ V_{\text{+}} \]
\[ V_{\text{o}} \]
\[ V_{\text{-}} \]

Figure 8.1: Internal circuitry of LM741.

Taken from National Semiconductor data sheet as shown on the web.
We will see later that an op amp does not include pin numbers and voltage supply pin identification when the op amp is connected to external circuitry. When this happens, one must keep in mind that the current for the device comes from the external power supply and input signals as shown in Figure 18.3.

![Diagram](image)

Figure 18.3: Reminder. Circuit showing external currents for an op amp.

Obviously, from this diagram:

\[ i_o = i_a + i_b + i_4 + i_- \]  \hspace{1cm} (18.1)

In our earlier study of circuits, we have seen dependent and independent sources. The model (typical model) of the op amp relies on dependent sources.
A model of the op-amp that gives reasonably good results is shown in Figure 18.4.

![Diagram of op-amp model]

Figure 18.4: A basic model of the op-amp.

We note that a dependent voltage-controlled voltage source is present at the output. The output voltage is given by

\[ V_o = A (V_b - V_a) \]  \hspace{1cm} (18.2)

\( A \) is the open-loop gain (no external feedback) of the op-amp.

Table 18.1 gives some typical values of op-amp parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Ideal Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Loop Gain, ( A )</td>
<td>( 10^5 - 10^8 )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Input Resistance, ( R_i )</td>
<td>( 10^4 - 10^5 , \Omega )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>Output Resistance, ( R_o )</td>
<td>( 10 - 100 , \Omega )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Supply Voltage</td>
<td>( 15 - 20 , V )</td>
<td>( )</td>
</tr>
</tbody>
</table>

Table 18.1: Typical Op-Amp Parameter Values.
There are limitations on how much current the op amp can output and how much the voltage can swing. The typical output swing is judged by the approximate sketch shown in Figure 18.5.

![Figure 18.5: Sketch showing linear range of operation for an op amp](image)

The device can successfully operate over \( \pm V_{\text{high}} \), which is the range of the supply voltage applied at pins 7 and 4 on the 741. This range, \(-V_{\text{cc}}\) to \(V_{\text{cc}}\), is sometimes called the rail to rail voltage.

**Analysis**

We now consider a typical application of the op amp.
A normal way of showing external circuitry connected to an op amp is shown in Fig. 18.5a. Using the model of Figure 18.5, produces the circuit of Figure 18.6b.

(a) A popular op amp configuration.

1b) Circuit of Figure 18.6a but using full model of the op amp.

Figure 18.6

We desire to find the transfer function $V_o/V_i$. An important note: We carry out the analysis of op amp circuits using nodal analysis. A great deal of this has to do with us adopting a simpler model later in which case we would have difficulty using, say mesh analysis.
At node $V_1$:

$$\frac{V_1 - V_3}{10k} + \frac{V_1}{2m} + \frac{V_1 - V_0}{40k} = 0$$

or

$$200 V_1 - 200 V_3 + V_1 + 50 V_1 - 50 V_0 = 0$$

$$200 V_3 = 251 V_1 - 50 V_0$$

Or

$$V_1 = 0.7968 V_3 + 0.1992 V_0$$  \hspace{1cm} (18.3)$$

At $V_0$:

$$\frac{V_0 - V_1}{40k} + \frac{V_0 - A V_1}{50} = 0$$

or

$$\frac{V_0 - V_1}{40k} + \frac{V_0 - A V_1}{50} = 0$$

Using $A = 200,000$ (17.41 typical)

$$V_0 - V_1 + 800 V_0 - 160 \times 10^6 V_1 = 0$$  \hspace{1cm} (18.4)$$

Using (18.3) in (18.4)

$$V_0 - 0.7968 V_3 - 0.1992 V_0 + 800 V_0$$

$$- 160 \times 10^6 (0.7968 V_3 + 0.1992 V_0) = 0$$

or

$$V_0 - 0.7968 V_3 - 1992 V_0 + 800 V_0$$

$$- 127.488 \times 10^6 V_3 - 31.872 \times 10^6 V_0 = 0$$

essentially

$$V_0 = -\frac{127.488 \times 10^6}{31.872 \times 10^6} V_3 = -3.9998796 V_3$$

or

$$\frac{V_0}{V_3} = -3.9998796$$

(very close to 4)
A Simplior Model

We have seen that

\[ |V_0| < V_c \] (supply)

which means

\[ A(V_b - V_a) < V_c \]

or

\[ (V_b - V_a) < \frac{V_c}{A} \]

but \( V_c \) is very finite, say \( 15V \); "A" runs on the order of \( 1 \times 10^6 \) or more

or

\( (V_b - V_a) \approx 15 \) micro volts

so we make the approximation

\[ V_b = V_a \]

An implication of this is that

\[ i_b = -i_a = \frac{V_b - V_a}{R_{in}} \]

\( V_b - V_a \) is very, very small; \( R_{in} \) is very large, so we say

\[ i_b = -i_a = 0 \]

This sets the stage for our simplified model. We make the following assumptions.
(1) A gain is infinity
(2) $R_i$, input resistance, is infinity
(3) $R_o$, output resistance, is zero
(4) $i_a = i_b = 0$
(5) $V_a = V_b$

Some of this is portrayed in Figure 18.7.

---

Figure 18.7: Simplified Op Amp Model.

Two important things to remember:

- $i_a = i_b = 0$ (No current enters the non-inverting and inverting terminals.)

- $V_a$ follows the voltage of $V_b$.

This greatly simplifies the analysis of op amps. These approximations work very good in most all cases.
Return and analyze the earlier example

\[ \frac{V_a - V_s}{R_1} = \frac{V_o - V_a}{R_f} \]  \hspace{1cm} (18.8)

Since \( V_a = V_b \) and \( V_b \) is tied to ground (0 volts) then \( V_a = 0 \). Then (18.8) becomes

\[ -\frac{V_s}{R_1} = \frac{V_o}{R_f} \]

or

\[ \frac{V_o}{V_s} = -\frac{R_f}{R_1} \]  \hspace{1cm} (18.9)

which is sort of a famous equation for an op amp. Using \( V_o = 10 \) \( V \), we get

\[ \frac{V_o}{V_s} = -4 \]

As compared to

\[ \frac{V_o}{V_s} = -3.99999996 \]
Gain Configuration

\[ V_0 = -\frac{R_f}{R_1} V_{in} \]  \hspace{1cm} (18.10)

Figure 18.8: A voltage controlled voltage

Summing Amplifier

\[ V_0 = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3\right) \]

Figure 18.9: A summing op amp configuration. It is easy to show

\[ V_0 = -\left[V_1 + V_2 + V_3\right] \frac{R_f}{R_1} \]  \hspace{1cm} (18.11)

making \( R_1 = R_2 = R_3 \) gives
**Non-Inverting Configuration.**

In all cases up to now the output polarity is the negative of the input signal.

Consider the configuration shown in Figure 18.10.

\[ v_0 = \left[1 + \frac{R_2}{R_1}\right] V_{in} \]

![Diagram of a non-inverting op amp circuit](image)

**Figure 18.10:** A non-inverting op amp.

Now the voltage at 1 = voltage at 2 = voltage at A. So we can write the following nodal equation at A.

\[ \frac{V_{in} - V_0}{R_2} + \frac{V_{in}}{R_1} = 0 \]

or

\[ R_1 V_{in} - R_1 V_0 + R_2 V_{in} = 0 \]

\[ R_1 V_0 = (R_1 + R_2) V_{in} \]


\[ V_o = \frac{[R_1 + R_2]V_{in}}{R_1} \]

or

\[ V_o = \left[1 + \frac{R_2}{R_1}\right]V_{in} \]

Famous equation

Voltage Controlled Current Source

Considered the op amp configuration shown in Figure 18.11.

Figure 18.11: A voltage controlled current source configuration.
We write nodal equations at \( D = D(2) \).

At \( D \)
\[
\frac{V_x}{R_2} = \frac{V_o - V_x}{R_1} \quad \text{(18.13)}
\]

At \( 2 \)
\[
\frac{V_x - V_{in}}{R_2} + \frac{V_x - V_0}{R_1} + \dot{I}_D = 0 \quad \text{(18.14)}
\]

From (18.13)
\[
R_2 V_x = R_2 V_0 - R_2 V_x
\]
or
\[
V_0 = \frac{(R_1 + R_2) V_x}{R_2} \quad \text{(18.15)}
\]

From (18.14), getting a common denominator,
\[
R_1 V_x - R_1 V_{in} + R_2 V_x - R_2 V_0 + R_2 \dot{I}_D = 0 \quad \text{(18.16)}
\]

Substitute (18.15) into (18.16)
\[
(R_1 + R_2) V_x - R_1 V_{in} - R_2 V_0 + R_2 \dot{I}_D = 0
\]
or
\[
R_1 R_2 \dot{I}_D = R_1 V_{in}
\]

\[
\dot{I}_D = \frac{V_{in}}{R_2} \quad \text{(18.17)}
\]

so the output current is controlled by the input voltage.
Buffer Amplifier

We often need a "buffer" between two stages of a process so that the final stage, when connected to the initial stage, does not load down the initial stage. Consider the oversimplified following case:

\[
\frac{V_o}{V_{in}} = \frac{75k}{25k + 75k} = 0.75
\]

Suppose we now have the case,
With \( \frac{75k \parallel 50k}{125} = \frac{75 \times 50}{125} = 30k \)

Therefore,

\[
\frac{V_o}{V} = \frac{30k}{30k + 25k} = 0.545
\]

We see the 50k load has loaded down the first stage and changed \( V_o \). We can avoid this by placing a buffer between the two stages. The buffer, as an operational amplifier, is shown in Figure 18.12.

![Buffer Amplifier Diagram](image)

\( V_o = V_{in} \)

**Figure 18.12: Buffer amplifier.**

The input impedance is very high (\( > 1 \text{ M\Omega} \)), the output impedance (looking back in the op-amp) is low (normally \( < 50 \Omega \)).
with the buffer amplifier inserted

\[ \text{Figure 16.13: Two stages separated by a buffer amplifier.} \]

The ratio of \( V_0 / V_{\text{in}} \) should now return to 0.75. You are encouraged to try this on your own to see if it works.

A Difference Amplifier

Suppose you have two signals \( V_1 \) and \( V_2 \) and you would like to subtract \( V_2 \) from \( V_1 \). Consider the configuration of Fig 18.14

\[ \text{Figure 18.14: A difference amplifier.} \]
At the upper $V_x$ in Fig 18.14

\[ \frac{V_x - V_2}{R_1} + \frac{V_x - V_0}{R_2} = 0 \]

or

\[ V_x \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_2}{R_1} + \frac{V_0}{R_2} \quad (18.15) \]

At the lower $V_x$ of Figure 18.14

\[ \frac{V_x - V_1}{R_1} + \frac{V_x}{R_2} = 0 \]

or

\[ V_x \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V_1}{R_1} \quad (18.14) \]

Equate (18.18) to (18.14)

\[ \frac{V_1}{R_1} = \frac{V_2}{R_1} + \frac{V_0}{R_2} \]

\[ \frac{V_0}{R_2} = \frac{V_1}{R_1} - \frac{V_2}{R_1} \]

or

\[ V_0 = \frac{R_2}{R_1} \left[ V_1 - V_2 \right] \quad (18.20) \]

One can control gain by a selection of the resistors $R_1$ and $R_2$. 
Example 1

You are given the following op amp configuration. Find the voltage $V_o$.

![Circuit Diagram]

Figure 18.15: Op Amp Circuit for Example 1.

At $V_x$

\[ \frac{V_x - 5}{10k} + \frac{V_x}{20k} = 0 \]

or

\[ 2V_x - 10 + V_x = 0 \]

\[ 3V_x = 10 \Rightarrow V_x = \frac{10}{3} \quad (18.21) \]

At $V_y$

\[ \frac{V_y - 5}{4k} + \frac{V_y - V_o}{10k} = 0 \]

\[ 5V_y - 25 + 2V_y - 2V_o = 0 \]

\[ 7V_y = 2V_o + 25 \]

\[ V_y = \frac{2V_o + 25}{7} \quad (18.22) \]
\[
\begin{align*}
70 & = 6V_0 + 75 \\
6V_0 & = 70 - 75 \\
V_0 & = \frac{-5}{6}
\end{align*}
\]

**Example 2**

Find \( V_0 \) for the following op-amp circuit.

\[\frac{V_x - 6}{10K} + \frac{V_x}{20K} = 0\]
\[2V_x - 12 + V_x = 0\]
\[3V_x = 12 \quad \Rightarrow \quad V_x = 4V = V_1\]
\[V_D = \frac{V_1 \times 5K}{5K + 15K} = \frac{4}{4} = 1V\]
\[V_0 = 1V\]
Example 3

You are given the op amp circuit as shown in Figure 18.16. What value of $R_f$ will give an output of $V_o = 5 - 4V_a$?

At $V_b$

$$V_b = \frac{5 \times 2k}{2k + 18k} = 1V$$

At "a"

$$\frac{1 - V_a}{5k} = \frac{V_o - 1}{R_f}$$

$$V_o - 1 = \frac{R_f(1 - V_a)}{5k}$$

$$V_o = \frac{R_f(1 - V_a)}{5k} + 1$$
\[ Rf \frac{(1 - V_a)}{5k} + 1 = 5 - 4V_a \]

\[ Rf \frac{(1 - V_a)}{5k} = 4 \left( 1 - V_a \right) \]

\[ Rf (1 - V_a) = 4 \times 5k (1 - V_a) \]

\[ Rf = 20k \Omega \]

Example 4:

You are given the op amp circuit of Figure 18.17. Determine \( V_o \).

\[ \frac{1 - 3}{4k} = \frac{V_o - 1}{10k} \]

\[ (-0.5)10 + 1 = V_o \]

\[ V_o = -4 \text{ V} \]
Example 6

Determine the output current $i_x$ for the following op amp circuit.

$$V_a = 0.5 \text{ V}$$

but

$$V_a = 0.5 = \frac{V_x \times 12K}{(2+3)K} = \frac{4}{5} V_x$$

$$V_x = \frac{2.5}{4}$$

$$i_x = \frac{V_x}{10K} = 0.0625 \text{ mA}$$

Example 6

Voltage Follower.

You are given the circuit of Figure 18.18.
Figure 18.18: Circuit for Example 6.

The voltage $V_2$ follows $V_1$. Since

$$V_1 = \frac{15 \times 20k}{20k + 20k} = 2.5V$$

$$V_2 = V_1 = 2.5V$$

$$V_0 = \frac{V_2 \times 10k}{10k + 10k} = \frac{V_2}{2}$$

$$V_0 = 1.25V$$

Example 7

For the op amp circuit of Figure 18.19, find $I_x$ and $I_y$.

Figure 18.19: Circuit for Example 7.
\[ V_a = \frac{V_0 \times 8K}{8K + 2K} = 0.8V_0 \]  \hspace{1cm} (1)

At \( a \):

\[ I_x = \frac{0.8V_0 - V_0}{10K} = -0.02V_0 \text{ mA} \]

At \( a' \):

\[ \frac{8V_0 - 0.5}{5K} = \frac{V_0 - 0.8V_0}{10K} \]

\[ 1.6V_0 - 1 = 0.2V_0 \]

\[ 1.4V_0 = 1 \]

\[ V_0 = \frac{1}{1.4} V \]

\[ I_x = -\frac{0.02}{1.4} \approx -0.0142.8 \text{ mA} \]

\[ I_y = \frac{V_0 - 0.8V_0}{10K} + \frac{V_0}{10K} \]

\[ = \frac{0.2V_0 + V_0}{10K} = \frac{1.2V_0}{10K} = \frac{(1.2)(\frac{1}{1.4})}{10K} \]

\[ I_y = 0.0857 \text{ mA} \]
Example 8

Find the voltage gain for the following op amp circuit.

![Circuit Diagram]

**Figure 18.20: Circuit for Example 8.**

At "a"

\[
\frac{V_0 - V_s}{R_1} = \frac{V_0 - 0}{R_f}
\]  

(these currents, then \( R_2 = 0 \))

\[
\frac{V_0}{V_f} = -\frac{R_f}{R_1}
\]

In earlier times, one of the most important roles of the op amp was its use as an integrator, that is, the ability to electrically perform the function of mathematically integrating a signal. This was the backbone of the analog computer. Whereas analog computers
have expired, the operation of integration is still important in signal processing. We now look at the process of integration with the op-amp.

**Example 9**

Consider the op-amp circuit shown in Figure 18.21

![Op-amp circuit diagram](image)

**Figure 18.21**: Op-amp circuit used as an integrator.

From the diagram we see that

\[ i_1 = i_2 \quad \text{(no current enters the inverting terminal)} \]

or

\[ 0 - \frac{V_{in}(t)}{R} = C \frac{dV_o(t)}{dt} \]

and

\[ \frac{dV_o(t)}{dt} = -\frac{V_{in}(t)}{RC} \quad (18.23) \]

We separate variables and integrate:
\[
\int_{t=0}^{t=T} V(t) \, dt = -\frac{1}{RC} \int_{t=0}^{t=T} V(t) \, dt
\]

\[
V_0(t) - V_0(0) = -\frac{1}{RC} \int_{t=0}^{t=t} V(t) \, dt \quad (18.24)
\]

Initial condition, \( V_0(0) \), can be applied to the op amp by applying an initial voltage to the capacitor.

Most often when we use op amps we use complex impedance rather than actually carrying out integration as above. For example

\[
\frac{jωL}{\text{Impedance}} \quad \Rightarrow \quad \frac{1}{\frac{1}{jωC}\text{Impedance}}
\]

\[
\frac{1}{\frac{1}{jωC}\text{Impedance}} \quad \Rightarrow \quad \frac{1}{\frac{1}{jωC}\text{Impedance}}
\]

And mathematically

\[
L \left[ \int f(t) \, dt \right] = \frac{F(s)}{s} \quad \text{with initial condition}
\]
We recall

\[ \frac{V_o(s)}{V_{in}(s)} = -\frac{Z_{fb}(s)}{Z_{in}(s)} \]

If \( Z_{fb}(s) \) corresponds to the complex impedance of a capacitor

\[ Z_{fb}(s) = \frac{1}{sC} \]

and \( Z_{in}(s) \) is the complex impedance of a resistor

\[ Z_{in}(s) = R \]

so

\[ V_o(s) = -\frac{V_{in}(s)}{RCs} \]

in the time domain

\[ v_o(t) = -\frac{1}{RC} \int v_{in}(t) \, dt \]

which is the result we got earlier.
Example 10

We saw earlier on page 14 how an op amp can be set-up to add input signals. This example illustrates how to set-up an op amp so it averages the input signals. Consider the circuit of Figure 18.22.

\[
\begin{align*}
V_1 & \quad V_2 \quad + \\
\frac{V_1 - V_0}{R_1} + \frac{V_2 - V_0}{R_2} + \frac{V_3 - V_0}{R_3} = 0 \\
\text{Or} \\
\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} &= V_0 \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] \\
\text{Or} \\
R_2R_3V_1 + R_1R_2V_2 + R_1R_3V_3 &= V_0 \left[ R_2R_3 + R_1R_2 + R_1R_3 \right]
\end{align*}
\]

Figure 18.22: Op Amp used for averaging.

The voltage at "a" is V_0 so we can write
OR

\[ V_0 = \frac{R_2 R_3 V_1 + R_1 R_3 V_2 + R_1 R_2 V_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} \]

IF \( R_1 = R_2 = R_3 = R \)

\[ V_0 = \frac{(V_1 + V_2 + V_3) R^2}{3 R^2} \]

OR

\[ V_0 = \frac{(V_1 + V_2 + V_3)}{3} \]
Closing on Op-Amps

Integrating with an op-amp.

Consider the following:

\[ i_A = \frac{V_{in}}{R_A} \]

\[ i_A = -C \frac{dV_o}{dt} \]

\[ \frac{dV_o}{dt} = -\frac{V_{in}}{R_A} \]

\[ \frac{dV_o}{dt} = -\frac{1}{R_A C} V_{in} \delta t \]

\[ V_o = \left[-\frac{1}{R_A C}\right] \int V_{in}(t) \delta t \]

The output is equal to the input multiplied by the scaling factor \[ \left[-\frac{1}{R_A C}\right] \]
Suppose you have

$$\frac{d^2x(t)}{dt^2} + 4x(t) = 4$$  \hspace{1cm} (1)

solve for \(x(t)\): Assume \(x(0) = 0\)

Several ways we can do this. One is by Laplace

$$L\left[ \frac{d^2x(t)}{dt^2} + 4x(t) \right] = L\left[ 4 \right]$$

$$5X(s) - X(0)^0 + 4X(s) = \frac{4}{s}$$

$$\left[5 + 4\right]X(s) = \frac{4}{s}$$

$$X(s) = \frac{4}{5(4+4)} = \frac{A}{s} + \frac{B}{s+4}$$

\(A = 1\); \(B = -1\)

$$x(s) = \frac{1}{5} - \frac{1}{s+4}$$

$$L^{-1}[x(s)] = x(t) = L^{-1}\left[ \frac{1}{5} \right] - L^{-1}\left[ \frac{1}{s+4} \right]$$

$$x(t) = 1 - e^{-4t}$$
\( T = \text{time constant} = 0.25 \text{ sec.} \)

\[ x(0) \]

\[ 0 \quad 1.25 \]

Go back to (1)

\[ x = 4 - 4x \quad \Rightarrow \quad x = \int [(4 - 4x) \cdot \delta(t)] \cdot dx \]

\[
\int x \Rightarrow -x = \left[ \frac{4}{R_1 C} s - \frac{x}{R_2 C s} \right] = \frac{1}{s} \left[ \frac{1}{R_1 C} - \frac{1}{R_2 C} \right]
\]

\[ \frac{1}{R_1 C} = \frac{1}{C_2 C} = 4 \quad \text{and} \quad C = \frac{1}{4} \text{ \mu F} \]

\[ R_2 = \frac{1}{4C} = \frac{\mu F}{C} \Rightarrow \]

\[ R_2 = 0.25 \times 10^6 = 250k \]
The Difference Amplifier.

Use when there is a need to amplify the difference between two signals. It amplifies the difference between two signals but rejects any signals common to the two inputs.

Consider

\[ \frac{V_a - V_1}{R_1} + \frac{V_a - V_0}{R_2} = 0 \quad (1) \]

At \( V_a \)

\[ \frac{V_a}{R_1} = \frac{V_2 R_4}{R_2} \quad (12) \]

At \( V_b \)

\[ V_b = \frac{V_2 R_4}{R_2 + R_4} \]
According to (1) and (2), we can write the form of
\[ \begin{align*}
V_a, & \quad V_b, & \quad V_0 \\
& \quad \left( \begin{array}{c}
\frac{1}{R_1} + \frac{1}{R_2} \\
\frac{1}{R_1 + R_2}
\end{array} \right) V_a + \begin{array}{c}
0 \\
\frac{R_1}{R_2}
\end{array} V_b - \begin{array}{c}
R_1 \\
R_2
\end{array} V_0 \\
& \quad V_1, \quad V_2
\end{align*} \]
From (1)
\[ R_2 V_a - R_2 V_1 + R_1 V_a - R_1 V_2 = 0 \]
\[ (R_1 + R_2) V_a + 0 V_b - R_1 V_0 = R_2 V_1 \]
so
\[ \left( \begin{array}{c}
\frac{R_1 + R_2}{R_2} \\
\frac{R_1}{R_2}
\end{array} \right) V_a + 0 V_b - \frac{R_1}{R_2} V_0 = V_1 \]
From (2)
\[ \left( \begin{array}{c}
R_3 + R_4 \\
R_3 + R_4
\end{array} \right) V_b = R_4 V_2 \]
\[ 0 V_a + \left( \frac{R_3 + R_4}{R_4} \right) V_b + 0 V_0 = V_2 \]
Then \[ V_a = V_b = 0 \]
\[ V_a - V_b + 0 V_0 = 0 \]
\[ \left[ \begin{array}{ccc}
\frac{R_1 + R_2}{R_2} & 0 & -\frac{R_1}{R_2} \\
0 & \frac{R_3 + R_4}{R_4} & 0 \\
1 & -1 & 0
\end{array} \right] \left[ \begin{array}{c}
V_a \\
V_b \\
V_0
\end{array} \right] = \left[ \begin{array}{c}
V_1 \\
V_2 \\
0
\end{array} \right] \]
\[ V = A^{-1} B \]
The program (MATLAB) is given on the following pages. We find

\[ V_0 = -\frac{R_2}{R_1} V_1 + \frac{(R_1 + R_2) R_4}{(R_3 + R_4) R_1} V_2 \]

If \( V_1 = V_2 \) we would like

\[ V_0 = 0 \]

\[ V_0 = -\frac{R_2}{R_1} V_1 + \frac{(1 + \frac{R_1}{R_2}) R_2 R_4}{(1 + \frac{R_3}{R_4}) R_1 R_4} V_2 \]

If \( \frac{R_1}{R_2} = \frac{R_3}{R_4} \)

Then

\[ V_0 = \frac{R_2}{R_1} \int \left( V_2 - V_1 \right) \]

\[ E = G \]