Stopping Back and Looking at Things

Current:
\[ i = \frac{\Delta q}{\Delta t} \rightarrow \text{coulombs/second} \]

\[ i = \frac{\partial q}{\partial t} \]

Knows ket and how to use voltage

\[ V = \frac{\Delta W}{\Delta q} \rightarrow \text{Joules/coulomb} \]

\[ V = \frac{\partial W}{\partial q} \]

Power

\[ P(t) = \frac{\partial W}{\partial t} \]

\[ W = \int_{a}^{b} P \, dt \quad \text{(This is how we pay)} \]

\[ \text{Watt} \cdot \text{Hours} \]

KWH about $6\$

\[ P = \frac{\partial W}{\partial t} = \frac{\partial W}{\partial q} \cdot \frac{\partial q}{\partial t} \]
\[ P = VI \]

We will come back to this shortly.

**Ohm's Law**

\[ V = IR \]

\[ \frac{i}{R} = \frac{V}{R} \]

\[ V = iR \]

\[ V = -iR \]

**Now go back to Power**

\[ P = VI \]

(\( \text{let } V = iR \))

\[ P = iRi = i^2R \]

\[ P = i^2R \]

\[ P = VI \]

(\( \text{let } i = \frac{V}{R} \))

\[ P = \frac{V^2}{R} \]

\[ P = \frac{V^2}{R} \]

So:

\[ P = VI = i^2R = \frac{V^2}{R} \]

Know how to use...
The so-called passive sign convention for power.

Consider any electric circuit

```
20V 20Ω
  |   40Ω
  |   |
  |   |
  |   |
20V 10Ω
  |   5V
  |   |
  |   |
  |   |
10V
not
```

We are not quite ready to solve this circuit right now but what we can say is

\[ \sum P_{\text{supplied}} = \sum P_{\text{absorbed}} \quad (1) \]

or

\[ \sum P_{\text{supplied}} = 0 \quad (2) \]

\[ \sum P_{\text{absorbed}} = 0 \quad (3) \]

(2) and (3) must accommodate both \( P_{\text{supplied}} \) and \( P_{\text{absorbed}} \).

How is this done? Answer, by how we define each one.
Power supplied

Given a device and polarity assuming as below

\[ P_{sup} = +VI \]

If either \( V \) or \( I \) is reverse

\[ P_{sup} = -VI \]

Power absorbed

Same scenario:

\[ P_{abs} = +VI \]

\[ P_{abs} = -VI \]
Example

Solve for \( V_1, V_2 \) below.

Find \( P_{eq} \) by each source

Find \( P_{abs} \) by \( R_1, R_2 \)

\[ +V_1 - E \rightarrow + \]
\[ R_1 = 25 \Omega \]
\[ 20 V \]

\[ -V_2 \rightarrow \]
\[ R_2 = 5 \Omega \]

\[ 50 V \]

\[ \sum \text{drops} = 0, \text{ blank at } "a", \text{ go cw.} \]

\[ -50 + V_1 + 20 - V_2 = 0 \]

\[ \text{kVL} \]

\[ \text{but Ohm's law} \]

\[ V_1 = 25I \]

\[ V_2 = -5I \]

\[ 25I - (-5I) = 50 - 20 \]

\[ 30I = 30 \]

\[ I = 1 \text{ Amp} \]
\[ P_{\text{up}} = 50I = 60 \text{W} \]

\[ P_{\text{up}} = -20I = -20 \text{W} \]

\[ P_{\text{obs}} = I^2 R = 25 \text{W} \]

\[ P_{\text{obs}} = 1^2 \times 5 = 5 \text{W} \]

\[ \Rightarrow P_{\text{up}} = \sum P_{\text{obs}} \]

\[ 50 - 20 = 5 + 25 \quad \text{check} \]

\[ R \quad \text{ohms} \]

\[ G, \text{ conductance} = \frac{1}{R}, \text{ Siemen} (S) \]

Consider the circuit diagram with resistors and voltage source.
Default: Sign Convention.
Assume $V_2$ is + at the terminal where  $I$ enters, we have

$$-V + I(R_1 + I(R_2 + I(R_3) = 0$$

$$\frac{V}{I} = R_1 + R_2 + R_3$$

\[ \text{Since } V, \text{ have } I \]

$$\frac{V}{E} = R_{eq} = R_1 + R_2 + R_3$$

Resistors in series add.

Voltage Division Rule

\[ I = \frac{V_2}{R_1 + R_2} \]

$$V_{R_1} = IR_1 = \frac{V_2 R_1}{R_1 + R_2}$$
The rule in words

\[ V_{121} \text{ is equal to source voltage, } V_s, \text{ time } R_1 \text{ divided by } R_1 + R_2. \]

**Example**

\[ V_q = \frac{12 \times 9}{9 + 6} = \frac{12 \times 9}{15} = 7.2 \text{ V} \]

\[ V_6 = \frac{12 \times 6}{9 + 6} = 4.8 \]

\[ 12 = V_6 + V_q = 7.2 + 4.8 = 12 \text{ V} \]

You should observe the following:
Find \( V_{16} \) using Voltage Division.

\[
\begin{align*}
V_{16} &= \frac{50 \times 16}{25} = 32 \text{ V} \\
V_4 &= \frac{50 \times 4}{25} = 8 \text{ V}
\end{align*}
\]

This is handy. Saves time. Used very often in circuits.
\[ R_{\text{eq}} \text{ in Parallel} \]

\[ I = \frac{V}{R_1} \quad I_2 = \frac{V}{R_2} \]

\[ I = I_1 + I_2 \]

\[ I = V \left[ \frac{1}{R_1} + \frac{1}{R_2} \right] \quad I = V \left[ \frac{1}{R_{\text{eq}}} \right] \]

\[ \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (3) \]

\[ G_{\text{eq}} = G_1 + G_2 \quad (4) \]

From (3)

\[ \frac{1}{R_{\text{eq}}} = \frac{R_1 + R_2}{R_1 R_2} \]
\[ R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad (\text{Product divided by sum}) \]

Current splitting on current division

\[ I_1 = \frac{V}{R_1} = \frac{I R_{eq}}{R_1} = \frac{I R_1 R_2}{R_1 + R_2} \]

\[ I_1 = \frac{I \times R_2}{R_1 + R_2} \]

Similarly we can show

\[ I_2 = \frac{I \times R_1}{R_1 + R_2} \]

Words

\( I_1 \) equals incoming current multiplied by opposite resistor divided by the sum of the resistors.
A general expression any number of resistors,

\[ I = \frac{I_1 \times R_{eq}}{R_i} \]

**Example**

Find \( I_{12} \) below

\[ I_{12} = \frac{2A}{20} \]

\[ 30 \quad 2 \quad 20 \]

\[ 12 \quad 12 \]

Find \( R_{eq} \):

\[ \frac{30 \times 20}{20 + 20} = \frac{600}{50} = 12 \Omega \]

\[ R_{eq} = \frac{12 \times 12}{12 + 12} = 6 \Omega \]
\[ I_{12} = \frac{2 \times 20}{12} = \frac{40}{12} = \frac{10}{3} = 1 A \]

How much is \( I_{20} \), \( I_{30} \)?

\[ I_{20} = \frac{12V}{20} = \frac{6}{10} = 0.6 A \]

\[ I_{30} = \frac{12V}{30} = \frac{6}{15} = \frac{2}{5} = 0.4 A \]

\[ I_0 = I_{10} + I_{20} + I_{30} = 1 + 0.6 + 0.4 = 2.2 A \]

**Combination of Resistance**

First a comment:

The equivalent resistance of two resistors in parallel is always less than the smaller resistor.
Let $R_2 < R_1$

$$R_2 = kR_1, \quad 0 < k < 1$$

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{(R_1)(kR_1)}{R_1 + kR_1}$$

$$\frac{1}{R_{eq}} = \frac{kR_1}{1 + k} = \frac{R_1}{1 + \frac{1}{k}}$$

$$1 + \frac{1}{k} > 1, \quad \text{so}$$

$$R_{eq} < R_1 \quad (\text{smallest resistance})$$

What if $R_1 = R_2$

$$R_{eq} = \frac{R_1R_1}{R_1 + R_1} = \frac{R_1}{2}$$

$R_{eq}$ for 2 resistors in parallel of the same value is half the resistance.

[Diagram: Two 40Ω resistors in parallel with $\Rightarrow$ connecting them, resulting in 15Ω]
For the previous problem

Generally, start at the right and work left.

\[ \frac{5}{1.5} = 2.5 \]

\[ R_y \| (10 + 2.5) \| 20 \]

\[ R_y = \frac{12.5 \times 20}{12.5 + 20} \]

\[ R_y \approx 7.69 \Omega \]

Example 2.32

Given the following

\[ V \]

\[ V_1 = \frac{1}{10} V \]

\[ 12 \Omega \]

\[ R \]

Given: Power supply \( P = 40 \text{ mW} \)

\[ V_1 = \frac{V}{10} \]
\[ V_1 = 40 \times 10^{-3} \] (a)

\[ V_1 = \frac{V}{4} = 10 \times 10^{-3} I \] (b)

\[ V = 40 \times 10^{-3} I \]

Substitute \( V \) into (a)

\[ 40 \times 10^{-3} I^2 = 40 \times 10^{-3} \]

\[ I = 1 \times 10^{-3} = 1 \text{ mA} \]

\[ V = \frac{40 \times 10^{-3}}{1 \times 10^{-3}} = 40 \text{ V} \]

\[ V_1 = \frac{I}{4} = 10 \text{ V} \]

\[ V_1 = R I = 1 \times 10^{-3} R = 10 \]

\[ R = \frac{10}{1 \times 10^{-3}} = 10 \text{ k\Omega} \]

\[ R_1 = 10 \text{ k\Omega} \]
Example

Find $i_1$, $i_2$, $i_3$ in the circuit below.

Using KCL (Kirchhoff Current Law),

$-10 + 20i_1 + 40i_2 - 20 = 0$

$20i_1 + 40i_2 + 0i_3 = 30$

$20 - 40i_2 + 10i_3 + 40 = 0$

$0i_1 - 40i_2 + 10i_3 = -60$

$-i_1 + i_2 + i_3 = 0$

\[
\begin{bmatrix}
20 & 40 & 0 \\
0 & -40 & 10 \\
-1 & 1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2 \\
\bar{x}_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
30 \\
-60 \\
0 \\
\end{bmatrix}
\]
\[ \lambda_1 = -0.643 \quad \lambda_2 = 1.0714 \quad \lambda_3 = -1.7143 \]

\[ \begin{bmatrix} 140 & -40 \\ -40 & 50 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 240 \\ -60 \end{bmatrix} \]

\[ \lambda_1 = -0.643 \quad \lambda_2 = \lambda_3 = -1.7143 \]

ck.