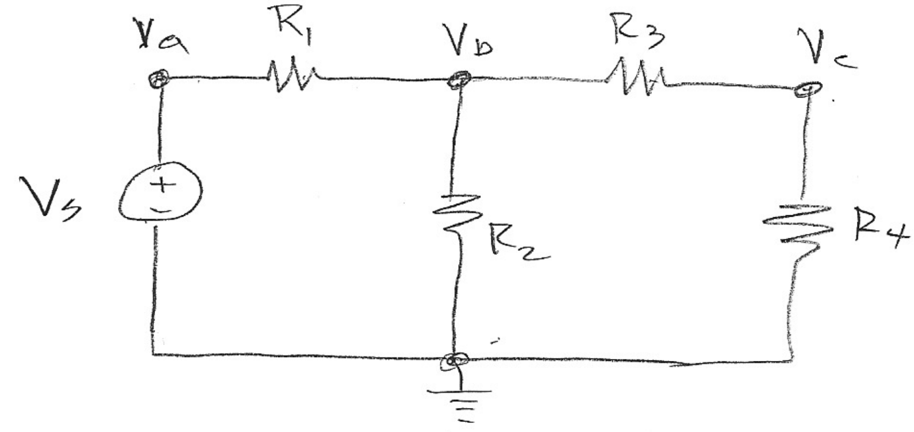
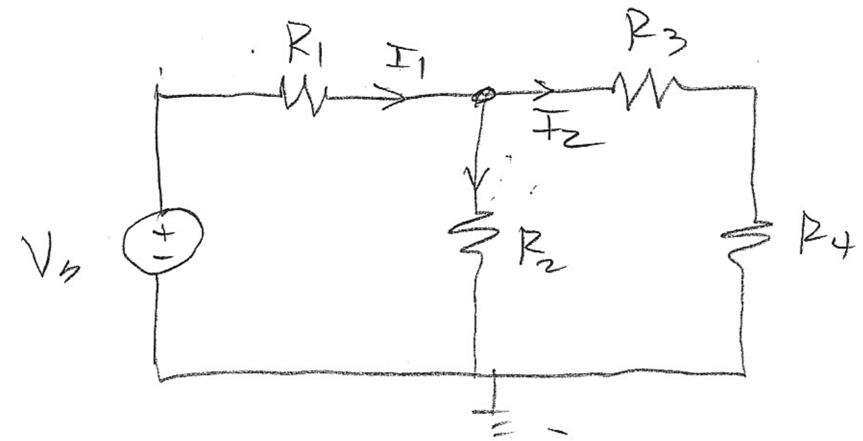
Ch. 3 Fall 06

What needs to be solved in this circuit?



If we know V_a, V_b, V_c we can solve for everything else.



If we know I_1 and I_2 we can solve for everything else.

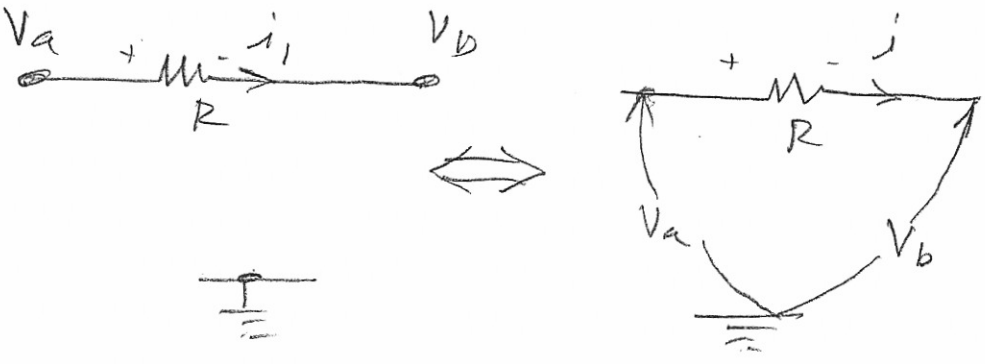
In this chapter we will learn how to solve for the above, plus some other techniques will be presented to aid us with circuit analysis.

The Node Voltage Method

A given circuit has N nodes. We designate one of these nodes as the reference node. The reference node is usually selected to be the node with the most branches connected to it. We assume the reference node has 0 volts. This leaves N-1 nodes in the ckt.

We write KCL at each of the N-1 nodes in terms of voltages that have been assigned. Each node is assigned a unique voltage.

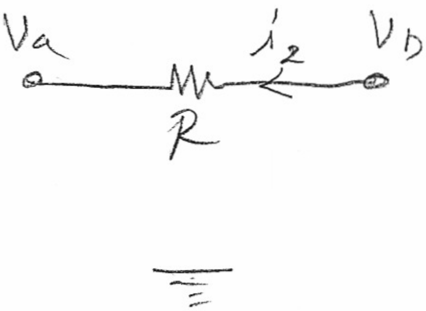
Consider the following



We have

$$-V_a + i_1 R + V_b = 0$$

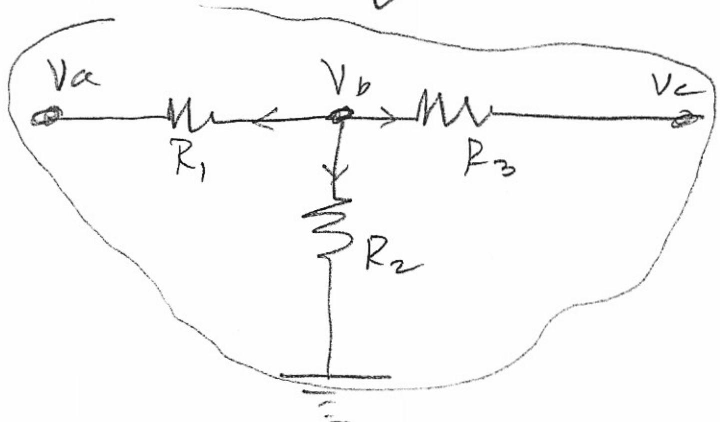
$$i_1 = \frac{V_a - V_b}{R}$$



Similarly,

$$i_2 = \frac{V_b - V_a}{R} \qquad i_1 = -i_2$$

In a more general sense

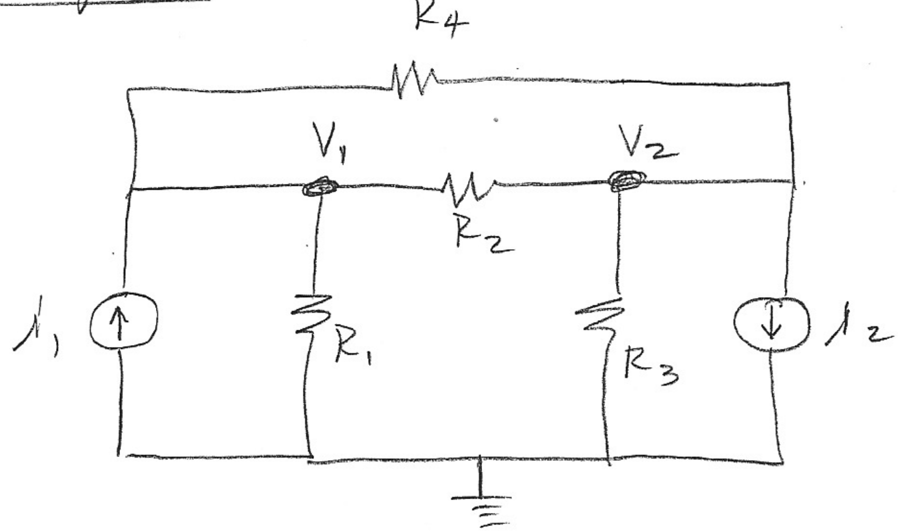


Suppose we apply KCL at node V_b and write the Σi. leaving V_b equal 0.

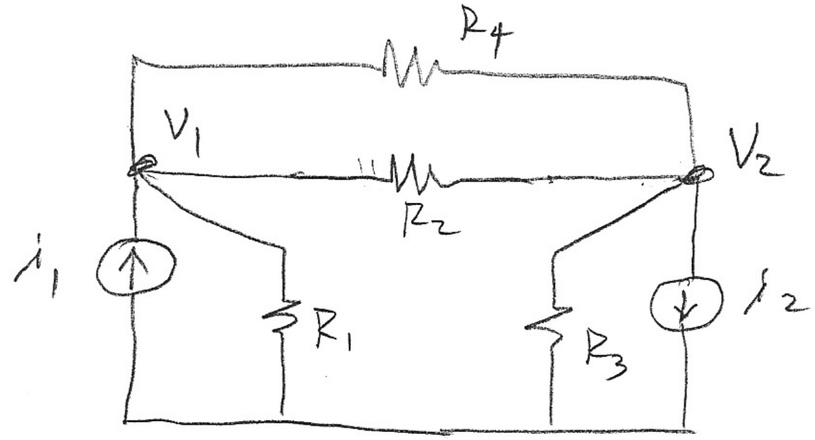
$$\frac{V_b - V_a}{R_1} + \frac{V_b}{R_2} + \frac{V_b - V_c}{R_3} = 0$$

We illustrate by example (First with only current sources)

Example 1



Same As



AA V_1

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_4} + \frac{V_1 - V_2}{R_2} = i_1$$

AA V_2

$$\frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_1}{R_4} = -I_2$$

This becomes, at V_1

$$G_1 V_1 + G_4(V_1 - V_2) + G_2(V_1 - V_2) = i_1$$

OR

$$\boxed{(G_1 + G_2 + G_4)V_1 - (G_2 + G_4)V_2 = i_1}$$

At V_2

$$G_3 V_2 + G_2(V_2 - V_1) + G_4(V_2 - V_1) = -i_2$$

$$-(G_2 + G_4)V_1 + (G_2 + G_3 + G_4)V_2 = -i_2$$

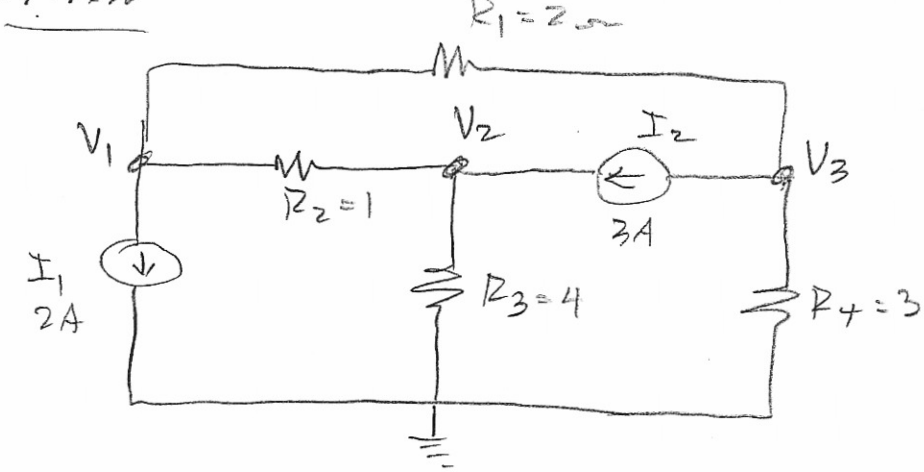
OR

$$\begin{bmatrix} G_1 + G_2 + G_4, & -(G_2 + G_4) \\ -(G_2 + G_4), & (G_2 + G_3 + G_4) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -i_2 \end{bmatrix}$$

Given the resistance and current sources, we can solve for V_1 and V_2. Knowing V_1 and V_2 we can then solve for all other voltages and currents.

Example 3.5 (text) p 89

Given



At V_1

$$\frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_1} = -I_1$$

$$G_2(V_1 - V_2) + G_1(V_1 - V_3) = -I_1$$

$$\boxed{(G_1 + G_2)V_1 - G_2 V_2 - G_1 V_3 = -I_1}$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + 0V_3 = I_2$$

$$\boxed{-G_2 V_1 + (G_2 + G_3)V_2 + 0 V_3 = I_2}$$

$$\frac{V_3 - V_1}{R_1} + \frac{V_3}{R_4} = -I_2$$

$$\boxed{-G_1 V_1 + 0 V_2 + (G_1 + G_4)V_3 = -I_2}$$

$$\begin{bmatrix} G_1+G_2, & -G_2, & -G_1 \\ -G_2, & G_2+G_3, & 0 \\ -G_1 & 0 & G_1+G_4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -I_1 \\ I_2 \\ -I_2 \end{bmatrix}$$

$G_1 + G_2 = 0.5 + 1 = 1.5\ S$

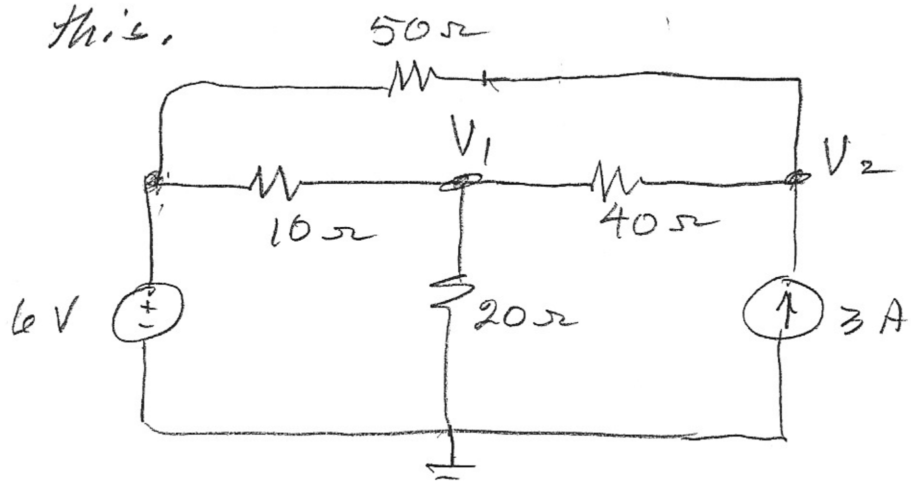
$G_2 + G_3 = 1 + .25 = 1.25\ S$

$G_1 + G_4 = 0.5 + .33 = .833\ S$

$$\begin{bmatrix} 1.5, & -1, & -0.5 \\ -1, & 1.25, & 0 \\ -.5, & 0 & .833 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -3 \end{bmatrix}$$

$V_1 = -3.5V, \quad V_2 = -0.402V, \quad V_3 = -5.7V$

We may also have voltage sources to contend with. We now address this.



At V_1

$$\frac{V_1-6}{10}+\frac{V_1}{20}+\frac{V_1-V_2}{40}=0$$

$$4V_1-24+2V_1+V_1-V_2=0$$

$$\boxed{7V_1-V_2=24}$$

At V_2

$$\frac{V_2-V_1}{40}+\frac{V_2-6}{5}=30$$

$$.25V_2-.25V_1+.2V_2-1.2=30$$

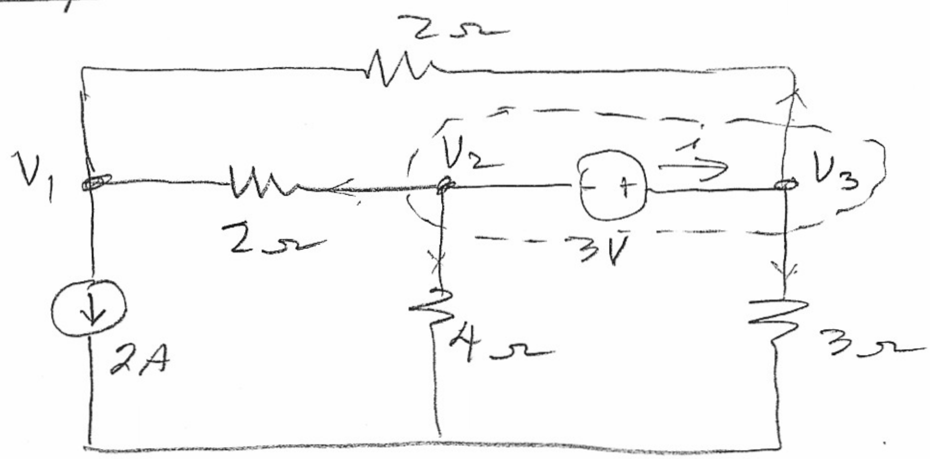
$$\boxed{-0.25V_1+0.45V_2=31.2}$$

$$\begin{bmatrix} 7 & -1 \\ -.25 & .45 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 24 \\ 31.2 \end{bmatrix}$$

$V_1=14.48\,V$, $V_2=77.38\,V$

Sometimes we have a voltage source between 2 nodes. We can treat this as a super node

Example 3.6



Find V_1, V_2 and V_3.

We form a super node as shown.

At V_1

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{2} = -2$$

$$V_1 - V_2 + V_1 - V_3 = -4$$

$$\boxed{2V_1 - V_2 - V_3 = -4}$$

At the supernode:

(12) $$\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_3 - V_1}{2} + \frac{V_3}{3} = 0$$

$$6V_2 - 6V_1 + 3V_2 + 6V_3 - 6V_1 + 4V_3 = 0$$

$$\boxed{-12V_1 + 9V_2 + 10V_3 = 0}$$

The constraint equation

$$V_2 + 3 - V_3 = 0$$

or

$$\boxed{0V_1 + V_2 - V_3 = -3}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ -12 & 9 & 10 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ -3 \end{bmatrix}$$

$V_1 = -5.64V$, $V_2 = -5.14V$, $V_3 = -2.14V$

We can go ahead and use i as shown. We end up with 4 eq's 4 unknowns.

As before, at

Node V_1

$$\boxed{2V_1 - V_2 - V_3 + 0i = -4}$$

Node V_2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{4} + i = 0$$

$$\boxed{-2V_1 + 3V_2 + 0V_3 + 4i = 0}$$

AT V_3

$$\frac{V_3 - V_1}{2} + \frac{V_3}{3} - i = 0$$

$$3V_3 - 3V_1 + 2V_3 - 6i = 0$$

$$\boxed{-3V_1 + 0V_2 + 5V_3 - 6i = 0}$$

CONSTRAINT EQ

$$\boxed{0V_1 + V_2 - V_3 + 0i = -3}$$

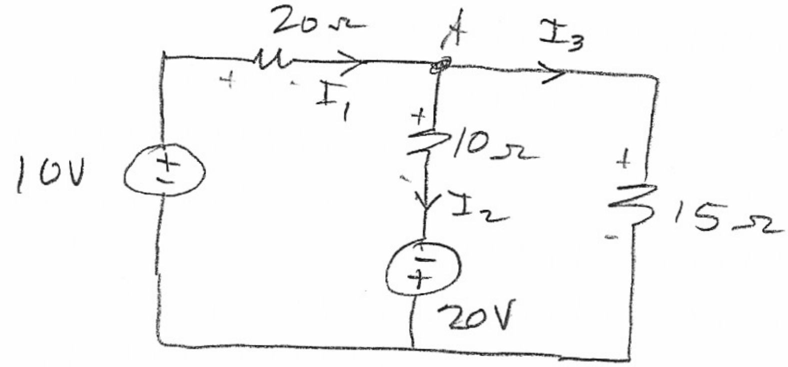
$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -2 & 3 & 0 & 4 \\ -3 & 0 & 5 & -6 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ i \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 0 \\ -3 \end{bmatrix}$$

$V_1 = -5.64V$, $V_2 = -5.14V$, $V_3 = -2.14V$, $i = 1.036A$

QED

Mesh Analysis

Mesh analysis is an outgrowth of branch analysis: Consider the following:



Find I_1, I_2, I_3: We have 3 unknowns, I_1, I_2, I_3. We must have 3 equations. We have 2 independent loop equations and KCL at node A.

$$-10 + 20I_1 + 10I_2 - 20 = 0$$

$$\boxed{20I_1 + 10I_2 + 0I_3 = 30}$$

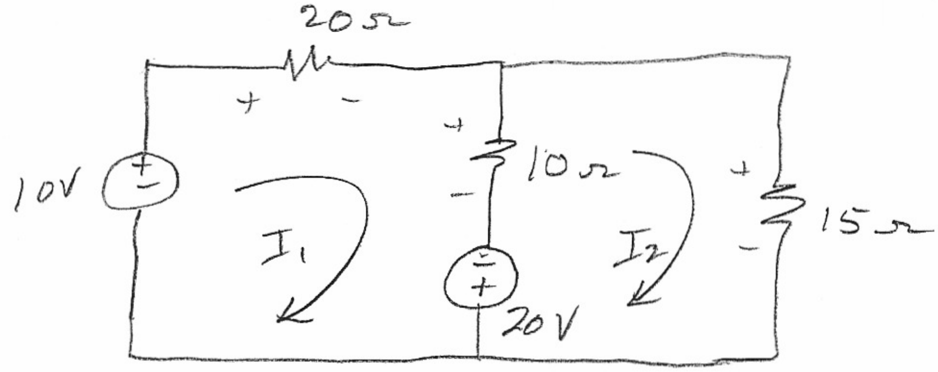
$$20 - 10I_2 + 15I_3 = 0$$

$$\boxed{0I_1 - 10I_2 + 15I_3 = -20}$$

$$\boxed{I_1 - I_2 - I_3 = 0}$$

$I_1 = 0.846A$, $I_2 = 1.31A$, $I_3 = -0.462$ A

Now Assign "mesh" currents.



$$-10 + 20I_1 + 10(I_1 - I_2) - 20 = 0$$

$$\boxed{30I_1 - 10I_2 = 30}$$

$$20 - 10(I_1 - I_2) + 15I_2 = 0$$

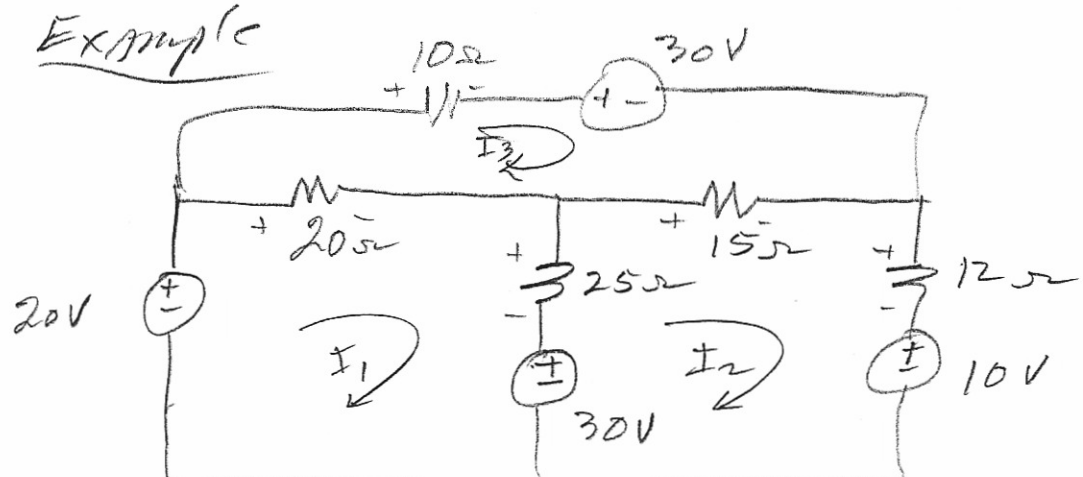
$$\boxed{-10I_1 + 25I_2 = -20}$$

$I_1 = 0.846A$, $I_2 = -0.462A$

Same Answers As before.

We work another example, more detail.

Example



$$-20 + 20(I_1 - I_3) + 25(I_1 - I_2) + 30 = 0$$

$$\boxed{25I_1 - 25I_2 - 20I_3 = -10}$$

$$-30 - 25(I_1 - I_2) + 15(I_2 - I_3) + 12I_2 + 10 = 0$$

$$\boxed{-25I_1 + 52I_2 - 15I_3 = 20}$$

$$10I_3 + 30 - 15(I_2 - I_3) - 20(I_1 - I_3) = 0$$

$$\boxed{-20I_1 - 15I_2 + 45I_3 = -30}$$

$$\begin{bmatrix} 25 & -25 & -20 \\ -25 & 52 & -15 \\ -20 & -15 & 45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 20 \\ -30 \end{bmatrix}$$

$I_1 = 3.18A$, $I_2 = 2.35A$, $I_3 = 1.53A$

When ever we have all voltage sources we can write the equations by inspection. 3.19. D3.13 for node analysis

Explain how to do this in class.