wlg

Nodal & Mesh Analysis

In Nodal analysis we encounter 4 cases to analize.

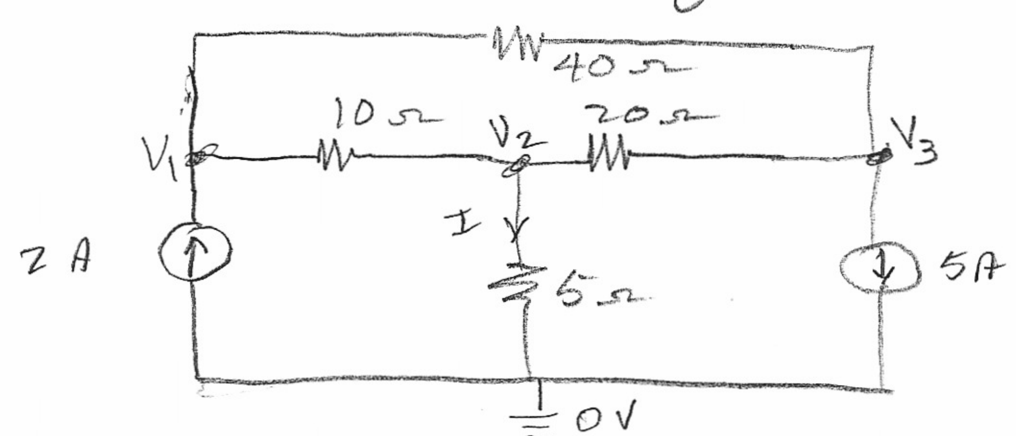
- ckt with all current sources (independent) plus resistors
- ckt with mixture of current sources and voltage sources (both independent) plus resistors
- ckt with independent voltage and current sources but with super node; plus resistance
- ckt with mixture of independent and dependent voltage and current sources plus resistance.

An example for each case will be presented below.

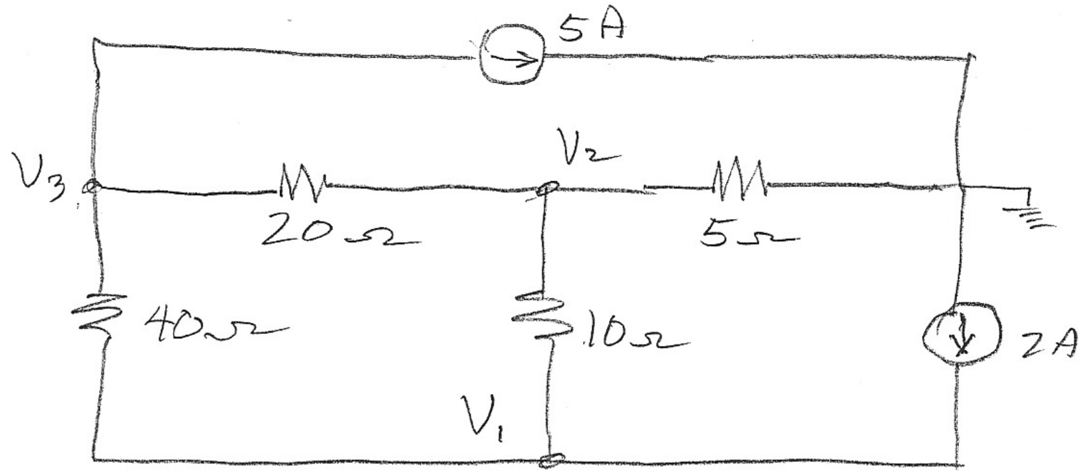
Example 5.1

Nodal analysis: All independent current sources plus resistors.

Consider the following ckt.



Regarding reference nodes.
We can redraw the circuit of Example 5.1.



We could just as well use V_1 as ground and relable the remaining nodes.

In this case we have 3 node voltages as indicated. We wish to solve for V_1, V_2, V_3 and I

At V_1

$$\Sigma i \text{ leaving} = 0$$

$$\frac{V_1 - V_2}{10} + \frac{V_1 - V_3}{40} = 2$$

or

$$4V_1 - 4V_2 + V_1 - V_3 = 80$$

giving

$$\boxed{5V_1 - 4V_2 - V_3 = 80} \qquad (5.1)$$

At V_2

$$\Sigma i \text{ leaving} = 0$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{5} + \frac{V_2 - V_3}{20} = 0$$

$$2V_2 - 2V_1 + 4V_2 + V_2 - V_3 = 0$$

$$\boxed{-2V_1 + 7V_2 - V_3 = 0} \qquad (5.2)$$

At V_3

$$\Sigma i \text{ leaving} = 0$$

$$\frac{V_3 - V_1}{40} + \frac{V_3 - V_2}{20} = -5$$

$$V_3 - V_1 + 2V_3 - 2V_2 = -200$$

or

$$\boxed{-V_1 - 2V_2 + 3V_3 = -200} \qquad (5.3)$$

Express (5.1), (5.2) and (5.3) in matrix form:

$$\begin{bmatrix} 5 & -4 & -1 \\ -2 & 7 & -1 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ -200 \end{bmatrix}$$

Solve to find

$V_1 = -12.14V \qquad V_2 = -15V \qquad V_3 = -80.71V$

$I = \frac{V_2}{5} = -3A$

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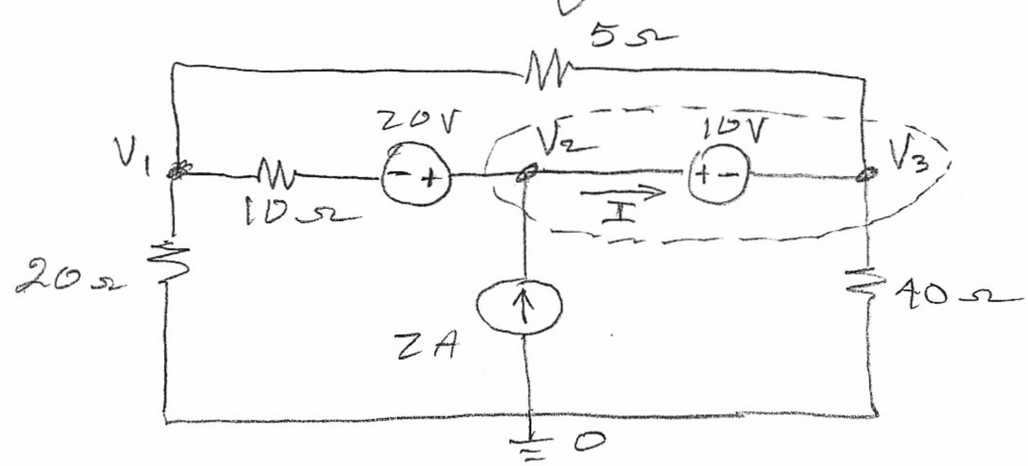
Example 5.2

Nodal analysis with supernode present.

Given the following ckt,



Find V_1, V_2, V_3 and I.

Method 1

At V_1

$$\frac{V_1}{20} + \frac{V_1 - V_3}{5} + \frac{V_1 + 20 - V_2}{10} = 0 \quad (5.4)$$

clear (5.4) gives

$$V_1 + 4V_1 - 4V_3 + 2V_1 + 40 - 2V_2 = 0$$

OR

$$\boxed{7V_1 - 2V_2 - 4V_3 = -40} \quad (5.5)$$

When we have a voltage source between two node (nothing else) we can form a super node and write KCL for branches leaving the supernode. Thus;

At the supernode

$$\frac{V_2 - 20 - V_1}{10} - 2 + \frac{V_3 - V_1}{5} + \frac{V_3}{40} = 0$$

OR

$$4V_2 - 80 - 4V_1 - 80 + 8V_3 - 8V_1 + V_3 = 0$$

$$\boxed{-12V_1 + 4V_2 + 9V_3 = 160} \quad (5.6)$$

Constraint Equation

The following equation involving the super node must hold;

$$V_2 - 10 - V_3 = 0$$

OR

$$\boxed{0V_1 + V_2 - V_3 = 10} \quad (5.7)$$

We put (5.5), (5.6) and (5.7) in matrix form and solve.

$$\begin{bmatrix} 7 & -2 & -4 \\ -12 & 4 & 9 \\ 10 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -40 \\ 160 \\ 10 \end{bmatrix}$$

$\boxed{V_1 = 24.21\ V}$ $\boxed{V_2 = 41.58V}$ $\boxed{V_3 = 31.58V}$

To find I:

At node V_2 we have, us KCL,

$$\frac{V_2 - 20 - V_1}{10} - 2 + I = 0$$

$$\frac{41.58 - 20 - 24.21}{10} - 2 + I = 0$$

OR $\boxed{I = .263 + 2 = 2.26\ A}$

Method 2

Write the following equations, directly.

At V_1

(same as before; gives) (5.5)

$$\boxed{7V_1 - 2V_2 - 4V_3 + 0I = -40} \quad (5.8)$$

At V_2

$$\frac{V_2 - 20 - V_1}{10} - 2 + I = 0$$

$$V_2 - 20 - V_1 - 20 + 10I = 0$$

$$\boxed{-V_1 + V_2 + 0V_3 + 10I = 40} \quad (5.9)$$

At V_3 (method 2)

$$\frac{V_3 - V_1}{5} + \frac{V_3}{40} - I = 0$$

$$8V_3 - 8V_1 + V_3 - 40I = 0$$

$$\boxed{-8V_1 + 0V_2 + 9V_3 - 40I = 0} \quad (5.10)$$

Constraint Eq.

$$V_2 - 10 - V_3 = 0$$

$$\boxed{0V_1 + V_2 - V_3 + 0I = 10} \quad (5.11)$$

This gives

$$\begin{bmatrix} 7, & -2, & -4, & 0 \\ -1, & 1, & 0, & 10 \\ -8, & 0, & 9, & -40 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I \end{bmatrix} = \begin{bmatrix} -40 \\ 40 \\ 0 \\ 10 \end{bmatrix}$$

solving;

$\boxed{V_1 = 24.21 \text{ V}}$, $\boxed{V_2 = 41.58 \text{ V}}$, $\boxed{V_3 = 31.57 \text{ V}}$

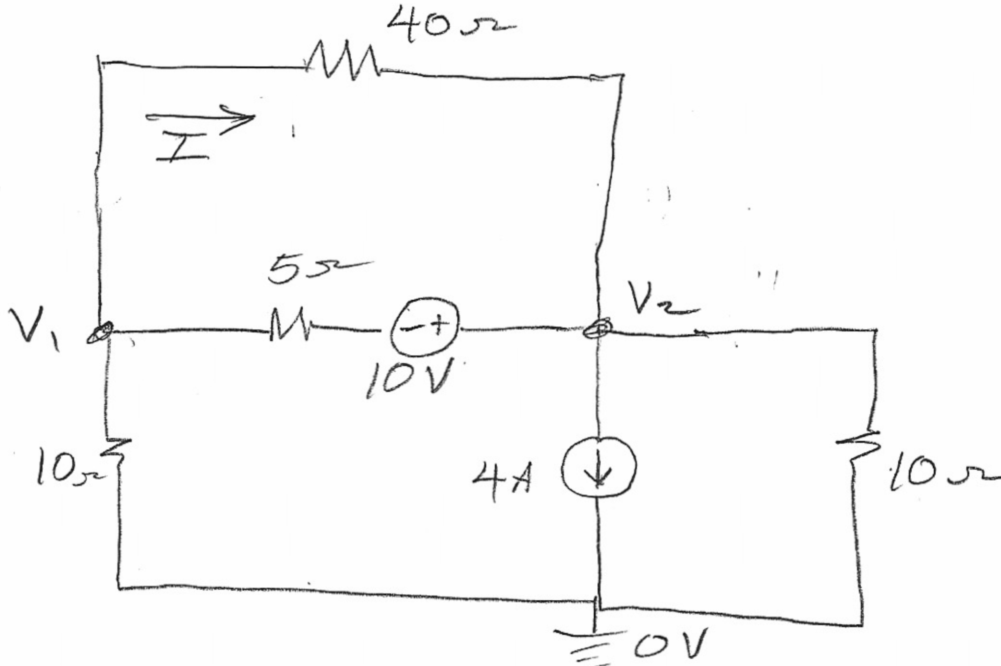
$\boxed{I = 2.263 \text{ A}}$

check

Example 5.3

Nodal: With independent Voltage and current source mix plus resistors.

Given the following ckt:



Use nodal analysis to find V_1, V_2 and I.

At V_1 $\quad \sum i$ leaving $= 0$

$$\frac{V_1}{10} + \frac{V_1 - V_2}{40} + \frac{V_1 + 10 - V_2}{5} = 0$$

$$4V_1 + V_1 - V_2 + 8V_1 + 80 - 8V_2 = 0$$

$$\boxed{13V_1 - 9V_2 = -80} \qquad (5.12)$$

At V_2 $\quad \Sigma i$ leaving $= 0$

$$\frac{V_2 - V_1}{40} + \frac{V_2 - 10 - V_1}{5} + \frac{V_2}{10} + 4 = 0$$

$$V_2 - V_1 + 8V_2 - 80 - 8V_1 + 4V_2 + 160 = 0$$

$$\boxed{-9V_1 + 13V_2 = -80} \qquad (5,13)$$

In matrix form

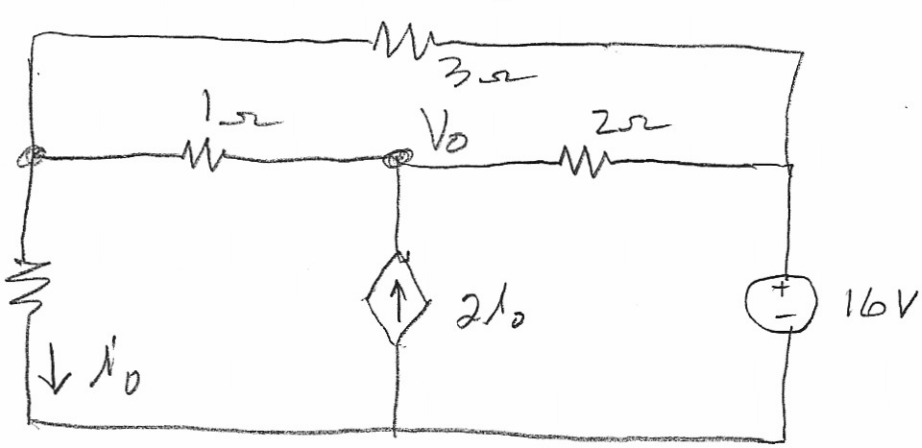
$$\begin{bmatrix} 13 & -9 \\ -9 & 13 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -80 \\ -80 \end{bmatrix}$$

$V_1 = -20V \qquad V_2 = -20V$

$$I = \frac{V_1 - V_2}{40} = \frac{-20+20}{40} = 0$$

Example 5.4

Nodal with a dependent source
Given the following circuit



Find V_o and I_o:

At V_1

Σi leaving $= 0$

$$\frac{V_1}{2} + \frac{V_1 - V_o}{1} + \frac{V_1 - 16}{3} = 0$$

$$3V_1 + 6V_1 - 6V_o + 2V_1 - 32 = 0$$

$$\boxed{11V_1 - 6V_o = 32} \qquad (5.14)$$

At V_o

Σi leaving $= 0$

$$\frac{V_o - V_1}{1} + \frac{V_o - 16}{2} - 2i_o = 0 \qquad (5.15)$$

but $i_o = \dfrac{V_1}{2}$. Substitute this into (5.15), so

$$\frac{V_o - V_1}{1} + \frac{V_o - 16}{2} - 2\left(\frac{V_1}{2}\right) = 0$$

$$2V_o - 2V_1 + V_o - 16 - 2V_1 = 0$$

$$\boxed{-4V_1 + 3V_o = 16} \qquad (5.16)$$

$$\begin{bmatrix} 11 & -6 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_o \end{bmatrix} = \begin{bmatrix} 32 \\ 16 \end{bmatrix}$$

$V_1 = 21.33V$ $\quad V_o = 33.78V$

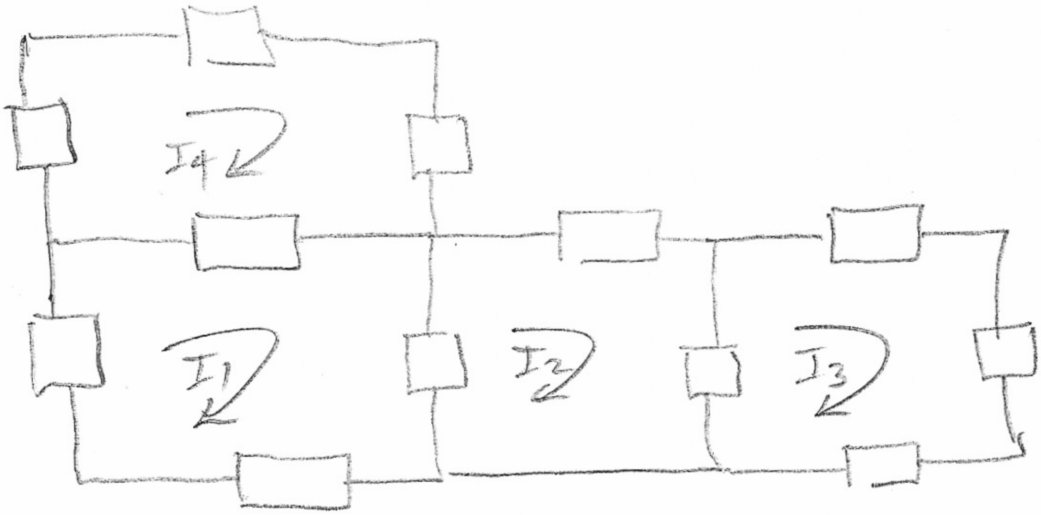
$I_o = \frac{V_1}{2} = 10.67A$

We now look at mesh analysis

In mesh analysis there will be 3 cases and they will be explained by examples.

The basic thought of mesh analysis stems from the following.

Consider a circuit with a structure as shown. We assign a clockwise mesh current to each window. We then write a KVL equation around each window. We solve these equations for the mesh currents.



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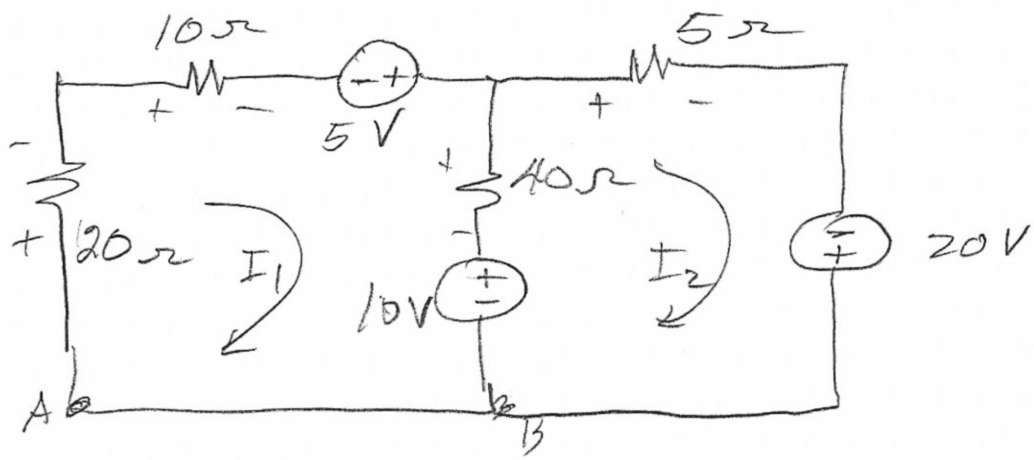
We illustrate by example

<u>Example 5.5</u>

Mesh with all voltage source (independent) plus resistors.



Start at A, go CW use Σ drops = 0 around <u>mesh 1</u>.

$$20I_1 + 10I_1 - 5 + 40(I_1 - I_2) + 10 = 0$$

OR

$$\boxed{70I_1 - 40I_2 = -5} \qquad (5.17)$$

Around Mesh 2

start at B; go cw using Σ rises = 0

$$-10 - 40(I_1 - I_2) + 5I_2 - 20 = 0$$

$$\boxed{-40I_1 + 45I_2 = 30} \qquad (5.14)$$

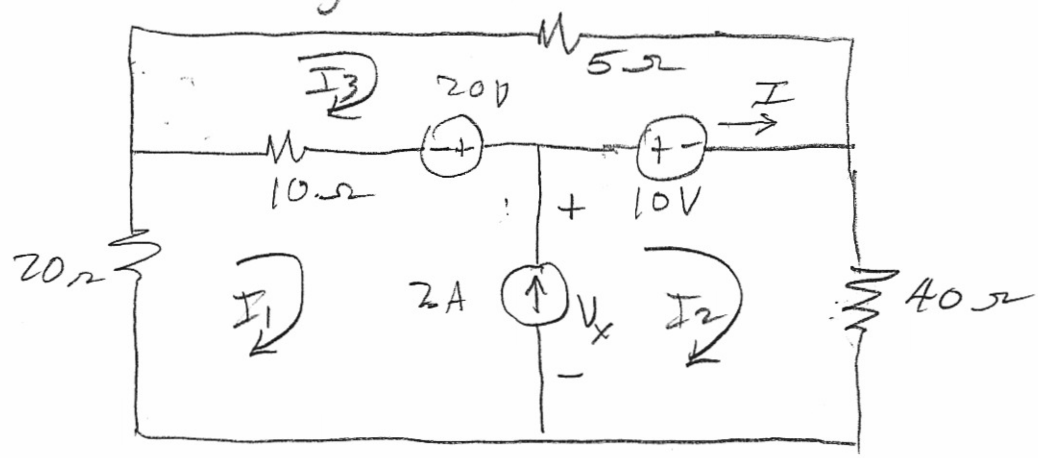
$$\begin{bmatrix} 70 & -40 \\ -40 & 45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -5 \\ 30 \end{bmatrix}$$

$I_1 = 0.629\,A$; $I_2 = 1.23\,A$

Example 5.6

Mesh analysis with a mix of independent voltage and current sources, and resistors.

This example will be the same as Example 5.2 worked by nodal analysis.



Method I

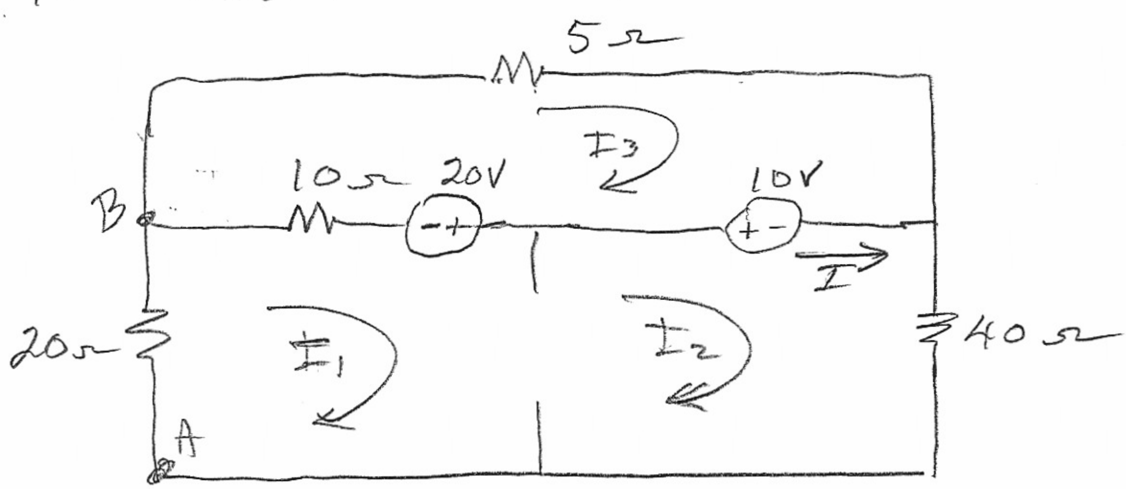
The procedure is to assign mesh currents as shown but "omit" the current source but note the constraint equation. In this case the constraint equation is

$$I_2 - I_1 = 2A$$

OR

$$\boxed{-I_1 + I_2 + 0I_3 = 2} \qquad (5.15)$$

The "modified" circuit is



We still retain the identity of I_1, I_2, and I_3. In the above ckt we have 2 closed paths. We write KVL around these paths. This gives 2 equations.

We also have the constraint equation (5.15). This gives 3 equations and 3 unknowns.

For the diagram of the ckt on page 13, start at A & go cw using Σ drops = 0.

$$20I_1 + 10(I_1 - I_3) - 20 + 10 + 40I_2 = 0$$

OR

$$\boxed{30I_1 + 40I_2 - 10I_3 = 10} \quad (5.16)$$

Around the next mesh, starting at B, going cw, using Σ drops = 0 gives

$$5I_3 - 10 + 20 - 10(I_1 - I_3) = 0$$

OR $$\boxed{-10I_1 + 0\,I_2 + 15I_3 = -10} \quad (5.17)$$

The constraint equation, (5.15)

$$\boxed{-I_1 + I_2 + 0\,I_3 = 2}$$

We put these in matrix form;

$$\begin{bmatrix} 30 & 40 & -10 \\ -10 & 0 & 15 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \\ 2 \end{bmatrix}$$

$I_1 = -1.21\,A$, $I_2 = 0.789\,A$, $I_3 = -1.47\,A$

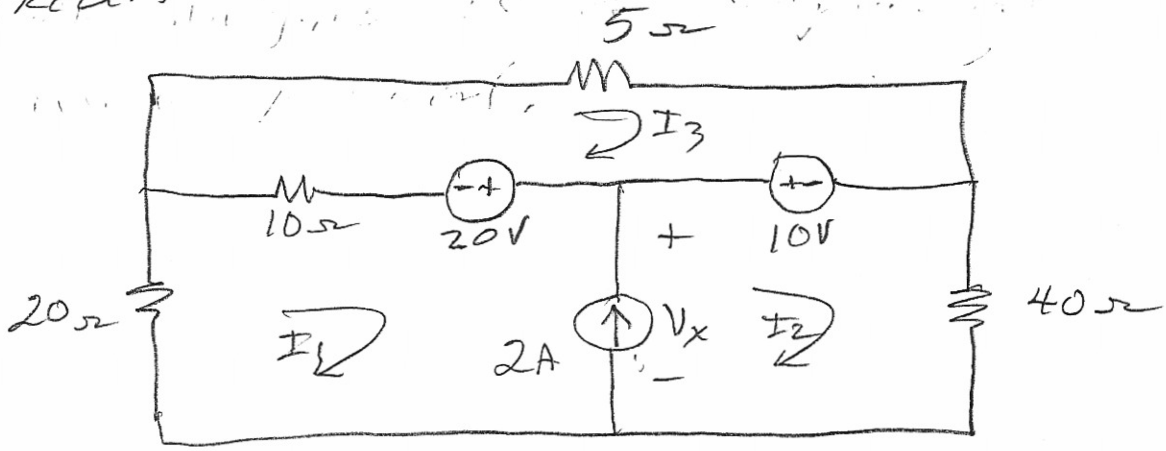
We see that

$$I = I_2 - I_3 = 2.26\,A$$

as we had in example 5.2.
QED

Method 2

We work the following circuit



We assign a voltage V_x across the 2A current source.

We have an equation for each mesh, giving 3 equations. We still have the constraint equation. This gives 4 equations from which we can solve I_1, I_2, I_3 and V_x.

From the ckt on the previous page we have

Mesh 1

$$20I_1 + 10(I_1 - I_3) - 20 + V_x = 0$$

$$\boxed{30I_1 + 0I_2 - 10I_3 + V_x = 20} \quad (5.18)$$

Mesh 2

$$-V_x + 10 + 40I_2 = 0$$

$$\boxed{0I_1 + 40I_2 + 0I_3 - V_x = -10} \quad (5.19)$$

Mesh 3

$$5I_3 - 10 + 20 - 10(I_1 - I_3) = 0$$

$$\boxed{-10I_1 + 0I_2 + 15I_3 + 0V_x = -10} \quad (5.20)$$

Constraint

$$\boxed{-I_1 + I_2 + 0I_3 + 0V_x = 2} \quad (5.21)$$

In matrix form

$$\begin{bmatrix} 30 & 0 & -10 & 1 \\ 0 & 40 & 0 & -1 \\ -10 & 0 & 15 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_x \end{bmatrix} = \begin{bmatrix} 20 \\ -10 \\ -10 \\ 2 \end{bmatrix}$$

$I_1 = -1.21\,A$, $I_2 = 0.789\,A$, $I_3 = -1.47\,A$

$V_x = 41.58\,V$

This checks with the solutions of Ex 5.2.

We can apply the same technique to previous Ex 5.4 but just work with the dependent source current as previous