Wg ECE 301 Lecture 8

Advanced Thevenin Circuits
Maximum Power Transfer

The presentation regarding Thevenin & Norton's theorems have dealt with circuits that contain independent sources only. We now look at circuits that contain both dependent and independent sources.

One is not likely to encounter circuits with dependent sources except in electronics. Particularly in the equivalent circuits of transistors. For completness we consider the presence of both independent and dependent sources.

Also, these notes will include a brief development and presentation of maximum power transfer.

Thevenin with Independent and Dependent Sources

Consider the circuit shown in Figure 8.1 We see a dependent current source of $2V_x$ and an independent voltage source of 30V.

We would like to find the Thevenin circuit looking into terminals a-b.

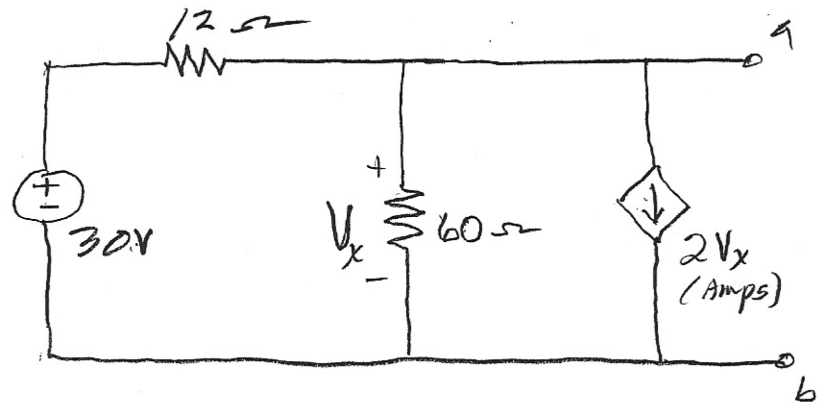


Figure 8.1: Circuit with both dependent and independent sources.

We would like to look into terminals a-b and replace the circuit with a standard Thevenin circuit as shown in Figure 8.2.

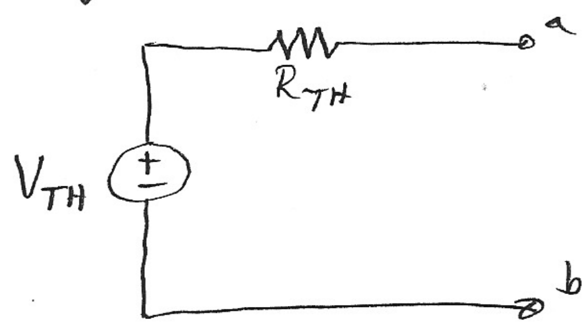


Figure 8.2: Standard Thevenin Equivalent circuit form.

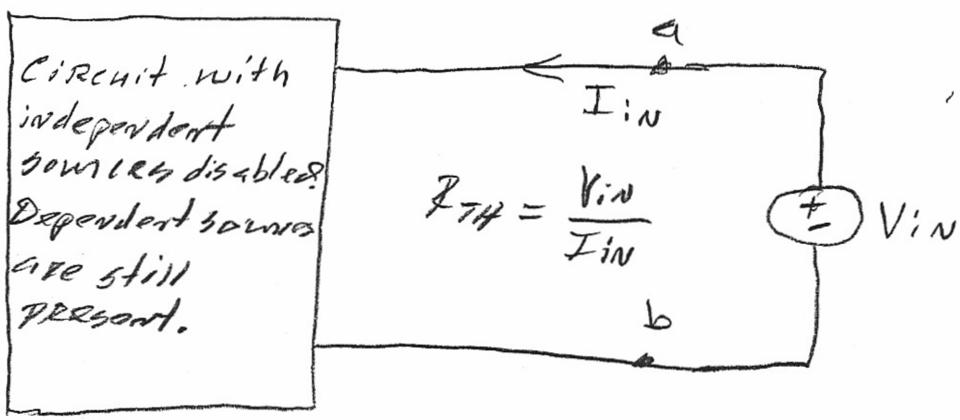
The concept and techniques of finding $V_{OS} = V_{TH}$ and $I_{SC} = I_{NORTON}$ carry directly over to circuits with dependent sources. In fact, we have already analyzed such circuits. The difference then is how we go about finding R_{TH}.

$\boxed{\text{Method 1 For Finding } R_{TH}}$

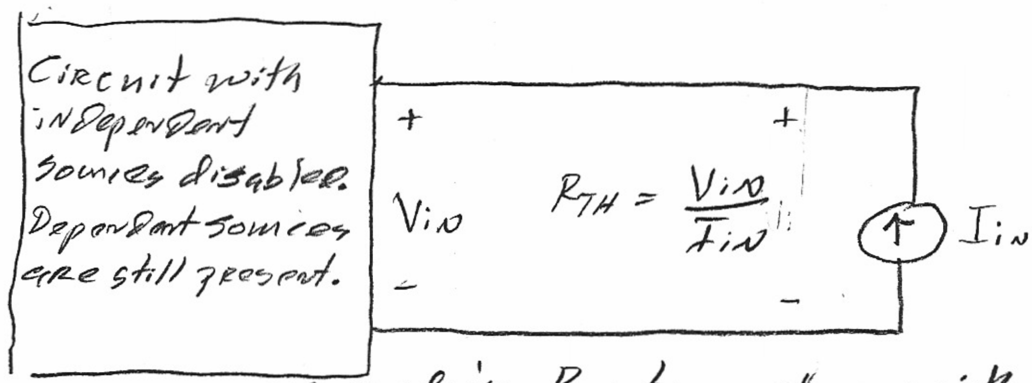
When we have all independent sources, one way of finding R_{TH} is to "look" into terminals a-b and disable the source and then find $R_{eq} = R_{ab} = R_{TH}$. When both independent and dependent sources are present we still look into terminals a-b but disable all independent sources. However, we do not disable the dependent sources. We are left with resistors and dependent sources. We have not previously encountered this in finding R_{TH}. In order to find R_{TH} we must energize the circuit at terminals a-b with a voltage, V_{in}, and find the resulting current, I_{in}. Then find

$$R_{TH} = \frac{V_{in}}{I_{in}} \qquad (8.1)$$

Alternately, we can energize the circuit with a current source, I_{in}, and find the resulting voltage. Again we find R_{TH} by V_{in}/I_{in}. This is shown in Figures 8.3a and 8.3b.



(a) Finding R_{TH} by exciting the circuit with a voltage.



(b) Finding R_{TH} by exciting with I_{in} source.

Figure 8.3: On Finding R_{TH}.

Let us apply this to the circuit of Figure 8.1 and first find R_{TH}. We will then find $V_{OS} = V_{TH}$ to complete the Thevenin circuit.

Example 8.1(a)

We disable the independent sources of Figure 8.1 and attach a 1 volt source. (It is not necessary for the source to be 1 volt: it could be any value, 1V, 10V, 60V: In fact it could be simply V_{IN})

We find the resulting current, I_{IN}. Then

$$R_{TH} = \frac{V_{IN}}{I_{IN}}$$

Lets see how this works when we apply $V_{IN} = 1V$

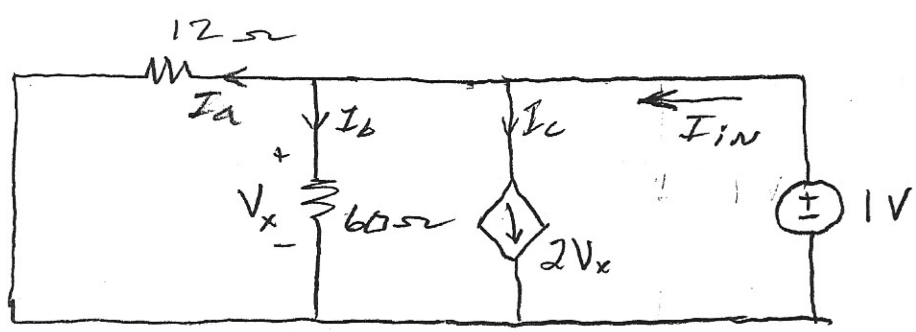


Figure 8.1a: Circuit for finding R_{TH}

We can use any circuit theory we have learned for finding I_{IN}.

We see from this circuit that

$$I_a = \frac{1V}{12} \; ; \; I_b = \frac{1V}{60} \; ; \; I_c = 2V_x = 2A$$

so $$I_{IN} = I_a + I_b + I_c = \frac{1}{12} + \frac{1}{60} + 2$$

$$I_{IN} = 2.1A$$

$\therefore$ $$R_{TH} = \frac{V_{IN}}{I_{IN}} = \frac{1}{2.1} = 0.4762\,\Omega$$

We find V_{TH} from the circuit of Figure 8.1b

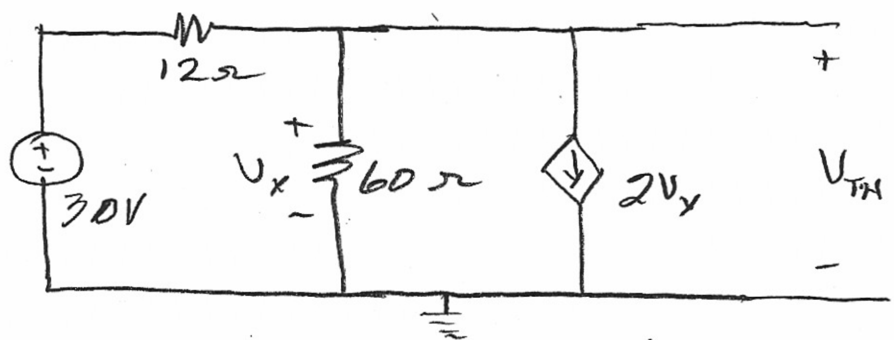


Figure 8.1b: Circuit for finding open circuit voltage.

We can use any circuit technique we like for finding $V_{TH} = V_x$. Nodal Analysis looks like a natural for this case.

We have,

$$\frac{V_x - 30}{12} + \frac{V_x}{60} + 2V_x = 0$$

$$5V_x - 150 + V_x + 120V_x = 0$$

$$V_x = 1.19\ \text{V}$$

The Thevenin circuit is as shown in Figure 8.4.

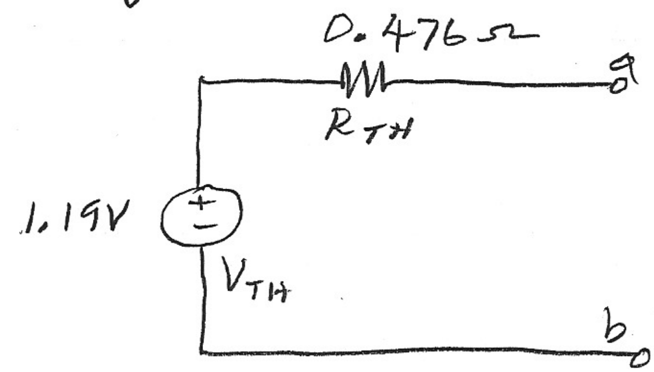


Figure 8.4: Thevenin circuit for Example 8.1.

Method 2 For Finding R_{TH}

In this method we leave the circuit "live" and find

$$R_{TH} = \frac{V_{TH}}{I_N} = \frac{V_{OC}}{I_{SC}}$$

For the current example we already know $V_{OC} = V_{TH}$. We need to find I_{SC}. We use the circuit of Figure 8.1c to do this.

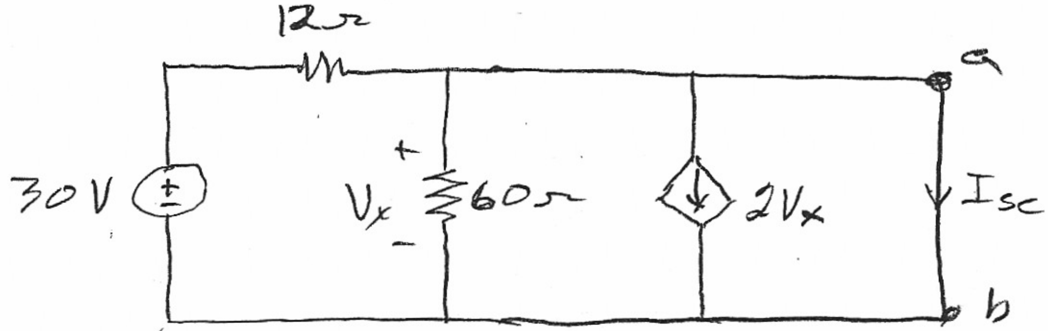


Figure 8.1c; Example 8.1; Circuit for finding I_{sc}.

We notice that shorting a-b also makes $V_x = 0$. If $V_x = 0$, the current source $2V_x = 0$. This means the circuit of Figure 8.1c becomes as shown in Figure 8.1d.

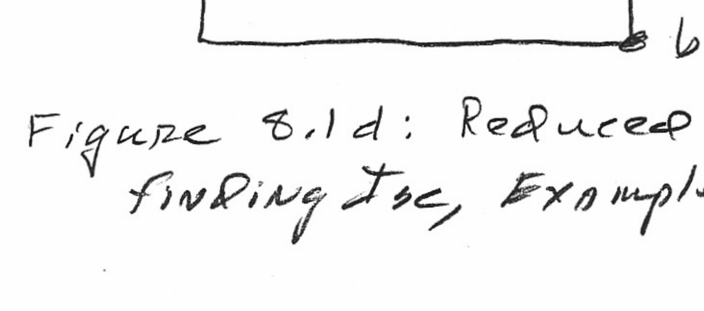


Figure 8.1d: Reduced circuit for finding I_{sc}, Example 8.1

We find from Figure 8.1d that

$$I_{sc} = \frac{30}{12} = 2.5A$$

As a "back door" check, we can use V_{TH}, R_{TH} to find I_{sc}. That is

$$I_N = I_{sc} = \frac{V_{os}}{R_{TH}} = \frac{V_{TH}}{R_{TH}} = \frac{1.19}{0.476}$$

$$I_N = I_{sc} = 2.5A$$

The Norton equivalent circuit is shown in Figure 8.5

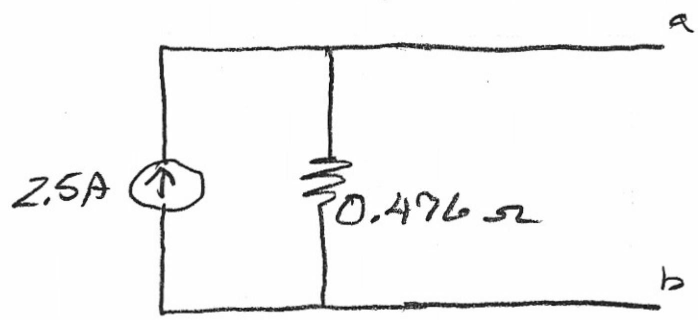


Figure 8.5: Norton equivalent circuit for Example 8.1

MAXIMUM POWER TRANSFER

Consider the circuit configuration of Figure 8.6.

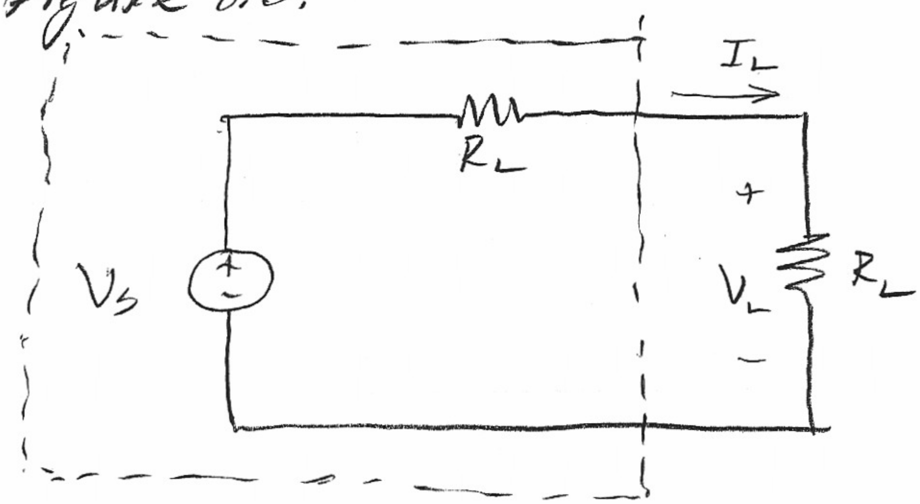


Figure 8.6: For consideration of maximum power transfer to R_L.

The circuit portion in the dashed lines could be the Thevenin circuit of a more complex dc circuit.

The task here is to find the load resistance R_L that will result in maximum power being delivered to R_L.

A little reflection shows that if $R_L = 0$, then $P_L = 0$. If $R_L \rightarrow \infty$, [an open circuit] then P_L is again 0. We can conclude from these end points that power to R_L would have a profile as shown in Figure 8.7

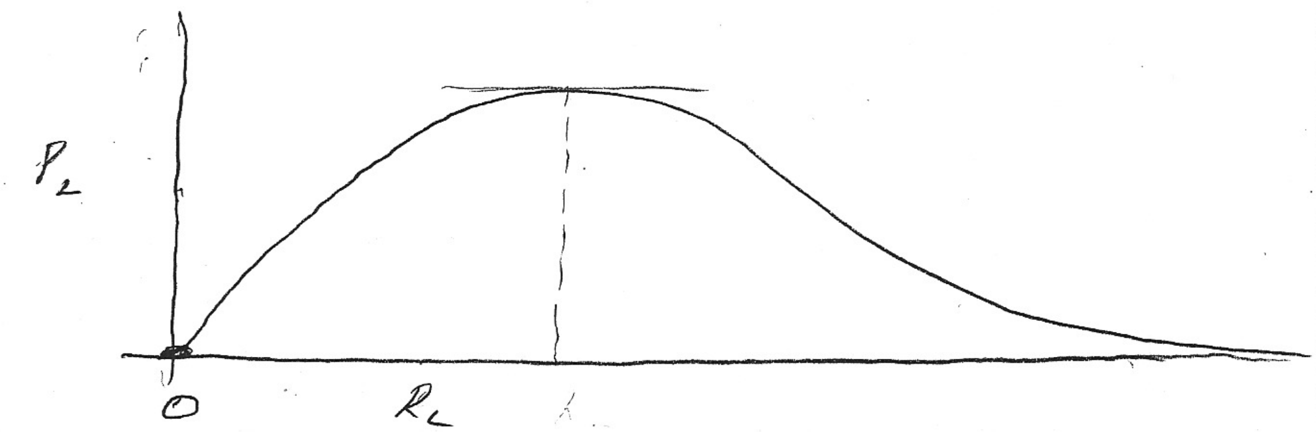


Figure 8.7: Profile of power absorbed by R_L, from the circuit of Figure 8.6.

We recall from calculus that if we want to find the "peak" of a curve (function) also called the maximum value, we perform the operation

$$\frac{d P_L(V_S, R_S, R_L)}{d R_L}$$

we set the derivative to zero and solve for R_L.

We need to find the equation for P_L. This is not difficult. We can use $P_L = I^2 R_L = \frac{V_L^2}{R_L} = V_L I_L$.

Let us use $P_L = V_L I_L$. We know

$$I_L = \frac{V_S}{R_L + R_S} \quad (8.2)$$

Therefore,

$$P_L(V_S, R_L, R_S) = \frac{V_S \times V_L}{(R_L + R_S)} \quad (8.3)$$

We use

$$V_L = I_L R_L \quad (8.4)$$

in (8.3)

This gives

$$P_L(V_S, R_L, R_S) = \frac{V_S I_L R_L}{(R_L + R_S)} \quad (8.5)$$

Then use (8.2) for I_L.

$$P_L(V_S, R_L, R_S) = \frac{V_S^2 R_L}{(R_L + R_S)^2} \quad (8.6)$$

We perform

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left(\frac{V_S^2 R_L}{(R_L + R_S)^2}\right) \quad (8.7)$$

$$\frac{dP_L}{dR_L} = \frac{(R_L+R_S)^2 V_S^2 - 2(R_L+R_S) V_S^2 R_L}{(R_L+R_S)^4} \qquad (8.8)$$

We set (8.8) to 0 and solve for R_L. V_S cancels out: We have

$$\frac{(R_L+R_S)^2 V_S^2 - 2(R_L+R_S) V_S^2 R_L}{(R_L+R_S)^4} = 0$$

or

$$(R_L + R_S) - 2R_L = 0$$

or

$$\boxed{R_L = R_S} \qquad (8.9)$$

This is easy to remember.

We use this in electronics and to a degree in stereo systems to match the load resistance to the source resistance. Most often a transformer is required to do this. We do not use this in power distribution systems. Why?

Example 8.2

You are given the circuit of Figure 8.7.

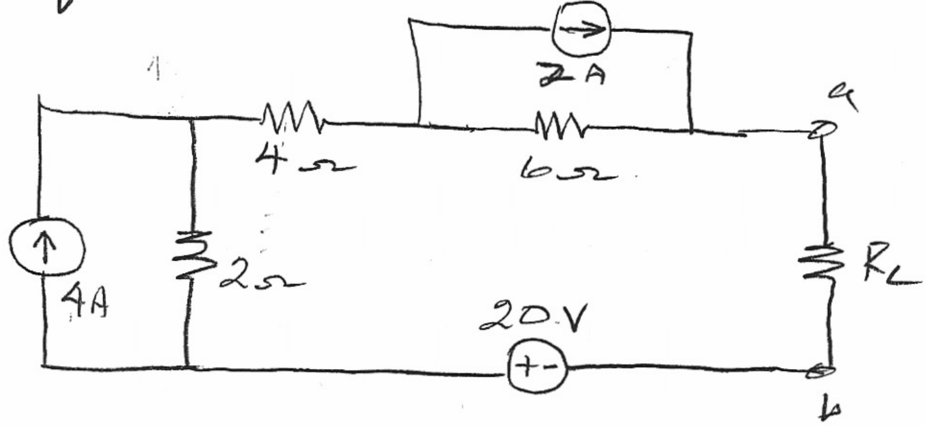


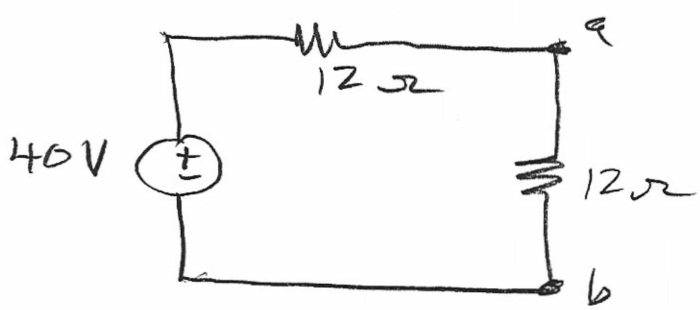
Figure 8.7: Circuit for example 8.2.

(a) Find R_L for maximum power transfer.

(b) Find the power P_L with R_L set for maximum power transfer.

Solution

On your own, find $V_{TH} = 40\,V$, $R_{TH} = 12\,\Omega$ so we know that $R_L = 12\,\Omega$. We have



We find that

$P_L = 33.33\,W$