wlg

Lesson 13

AC Circuits
Solution of Circuits

The impedance of the inductor and capacitor depends on the frequency of the sinusoidal signal applied to a network. This dependence is shown by the relationship:

$$X_L = j\omega L$$

$$X_C = \frac{-j}{\omega C}$$

This is illustrated in the following example.

Example 13.1

Consider the circuit shown in Figure 13.1

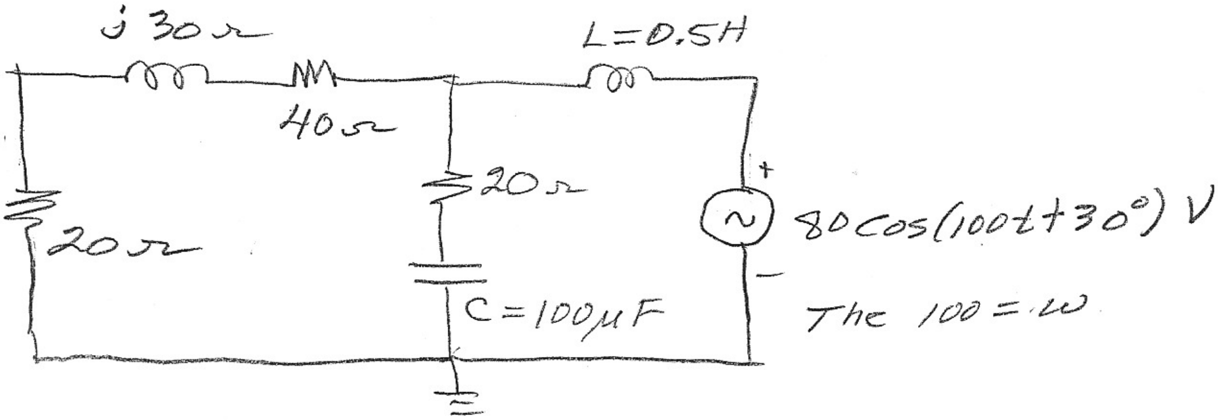


Figure 13.1; Circuit for Example 13.1.

We first put the circuit in phasor form.

$L = .5H \Rightarrow j\omega L = j\,100 \times .5\,\Omega = j\,50\,\Omega$

$C = 100\mu F \Rightarrow \frac{-j}{\omega C} = \frac{-j}{100 \times 100 \times 10^{-6}} = -j100\ \Omega$

$80\cos(100t - 30^\circ) \Rightarrow 80\angle -30^\circ\ V$

The "phasor" circuit is shown in Figure 13.2.

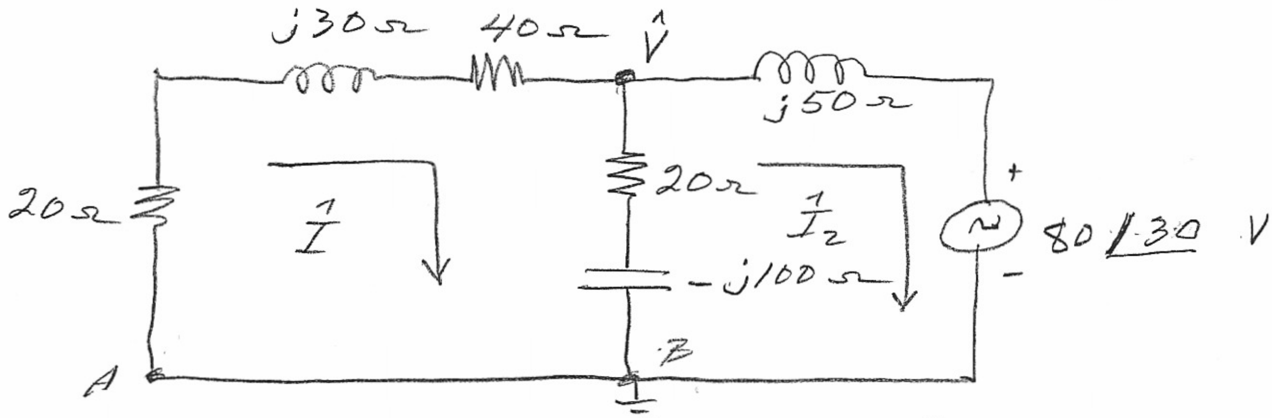


Figure 13.2: Circuit in "Phasor" form.

We can apply any of the techniques we learned in DC circuits to solving the above circuit. The major difference, technique wise, is that we are dealing with complex number rather than all real numbers.

Suppose the solutions for $\hat{I}_1$ and $\hat{I}_2$ are desired. We can use mesh analysis to find these.

Mesh Analysis For A.C. Circuits:

We start at point A in Figure 13.2 and go clockwise use Σ drops $= 0$

$$20\hat{I}_1 + j30\hat{I}_1 + 40\hat{I}_1 + (\hat{I}_1 - \hat{I}_2)(20 - j100) = 0$$

OR

$$\boxed{(80 - j70)\hat{I}_1 + (-20 + j100)I_2 = 0} \qquad (13.1)$$

Next we start at "B" and go clockwise around mesh 2 using $\sum$ drops $= 0$. This gives the following equation

$$(\hat{I}_2 - \hat{I}_1)(20 - j100) + j50\hat{I}_2 + 80\angle{-30} = 0$$

OR

$$\boxed{(-20 + j100)\hat{I}_1 + (20 - j50)\hat{I}_2 = -80\angle{-30}} \qquad (13.2)$$

Express (13.1) and (13.2) in matrix form

$$\overset{\hat{Z}}{\begin{bmatrix} (80 - j70) & (-20 + j100) \\ (-20 + j100) & (20 - j50) \end{bmatrix}} \overset{\hat{I}}{\begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix}} = \overset{\hat{V}}{\begin{bmatrix} 0 \\ -80\angle 30 \end{bmatrix}} \quad -69.28 - j40$$

Express the solutions for $\hat{I}_1$ and $\hat{I}_2$ in both rectangular form

$$\hat{I}_1 = -0.817 + j0.647 \text{ A} = 1.042\angle 141.6^\circ \text{ A}$$

$$\hat{I}_2 = -1.09 + j0.0167 \text{ A} = 1.09\angle 179.1 \text{ A}$$

You can also solve the above as a matrix with MATLAB as

$$\hat{I} = \hat{Z}^{-1}\hat{V}$$

I do not know anyway in MATLAB to express polar for. So one would need to write $-80\angle 30$ as $-69.28 - j40$. The above equation was solved with MATLAB as shown below.

```
% Solving the matrix equation that results
% from analysis of an AC circuit.
%     (80-j20)I1 + (-20+j100)I2 = 0
%     (-20+j100)I1 + (20-j50)I2 =-69.28 - j40
% wlg: office computer: Oct 19,'06: program: AC_solution.m

Z = [80-j*70  -20+j*100; -20+j*100 20-j*50];
V = [0; -69.28-j*40];

I = inv(Z)*V

>>
>>
>> AC_solution

I =

  -0.8171 + 0.6473i
  -1.0865 + 0.0167i

>>
```

The solution for $\hat{V}$ (see Figure 13.2) can be obtained by

$$\hat{V} = (20 - j100)(\hat{I} - \hat{I}_2)$$

We find, using

$$\hat{V} = (20 - j100) * ((1.042\angle 141.6) - (1.09\angle 179.1))$$

$$\hat{V} = 68.48 - j14.72 \text{ V} = 70.04\angle -12.13 \text{ V}$$

We will use nodal analysis to check the answer.

Using Σi leaving node $\hat{V}$ in Figure 13.2 gives,

$$\frac{\hat{V}}{60+j30} + \frac{\hat{V}}{20-j100} + \frac{\hat{V} - 80\angle{+30}}{j50} = 0$$

This is a tedious problem to solve. There is no "easy" way. We use

$$(.0133-j0.0067)\hat{V} + (0.0019+j0.0096)\hat{V}$$
$$-j0.02\hat{V} - 0.8 + j1.386 = 0$$

$$(0.0152-j0.0171)\hat{V} = .8 - j1.386$$

$$\hat{V} = 68.51 - j14.11 \quad V$$

Close to previous answer but there is a difference in the decimal part. Probably because I did not take it out to as enough decimal places.

Another problem will be presented to be sure we get the hang of things.

It might be added that generally it is easier to solve AC circuits using mesh analysis. There can be exceptions.

Example 13.2

Use mesh analysis to find $\vec{I}_1$ and $\vec{I}_2$ in the following circuit. Also find $i_1(t)$.

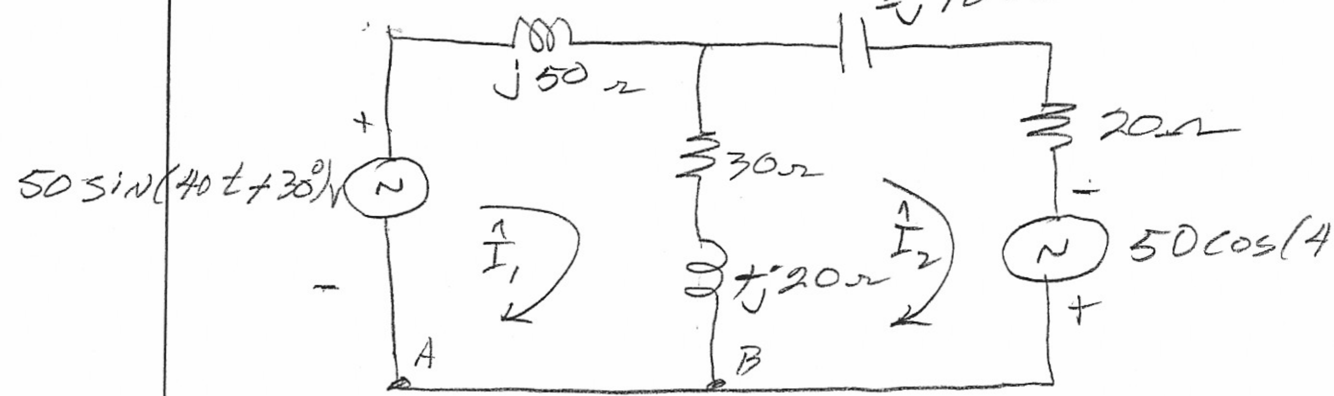


Figure 13.3: Circuit for Example 13.2.

We must change the sources from time domain to phasor form. We use cosine reference so we must change the sine source to a cosine source.

$$50\sin(40t+30^\circ) = 50\cos(40t-60)$$
$$\rightarrow 50\angle{-60^\circ}\,V$$

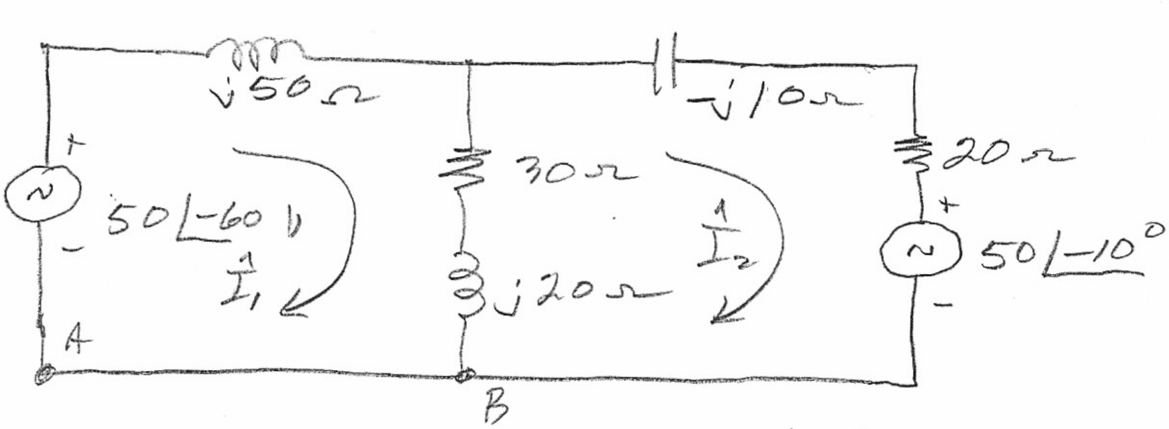


Figure 13.4: Phasor circuit for example 13.2.

Start at "A" go CW using Σ drops = 0.

$$-50\angle{-60} + j50\vec{I}_1 + (30+j20)(\vec{I}_1 - \vec{I}_2) = 0$$

OR

$$\boxed{(30+j70)\vec{I}_1 + (-30-j20)\vec{I}_2 = 50\angle{-60^\circ}}$$

Start at "B" go CW use Σ drops = 0

$$(30+j20)(\vec{I}_2 - \vec{I}_1) + (20-j10)\vec{I}_2 + 50\angle{-10} = 0$$

OR

$$\boxed{(-30-j20)\vec{I}_1 + (50+j10)\vec{I}_2 = -50\angle{-10}}$$

OR

$$\begin{bmatrix} (30+j70) & (-30-j20) \\ (-30-j20) & (50+j10) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 50\angle{-60} \\ -50\angle{-10} \end{bmatrix}$$

$$\vec{I}_1 = -1 - j0.128 \text{ A} \qquad = 1.008\angle{-172.7} \text{ A}$$

$$\vec{I}_2 = -1.533 + j0.0035 \text{ A} = 1.53\angle{179.89} \text{ A}$$

To get $i_1(t)$, $i_2(t)$ we go back to the cosine reference, Thus

$$i_1(t) = 1.008\cos(40t - 172.7^\circ) \text{ A}$$

$$i_2(t) = 1.53\cos(40t + 179.89) \text{ A}$$

When we have current sources we follow the same rules (techniques) we used with DC circuits. We illustrate this with an example.

Example 13.3

Compute $\vec{V}_0$ in the circuit of Figure 13.5 using mesh analysis.

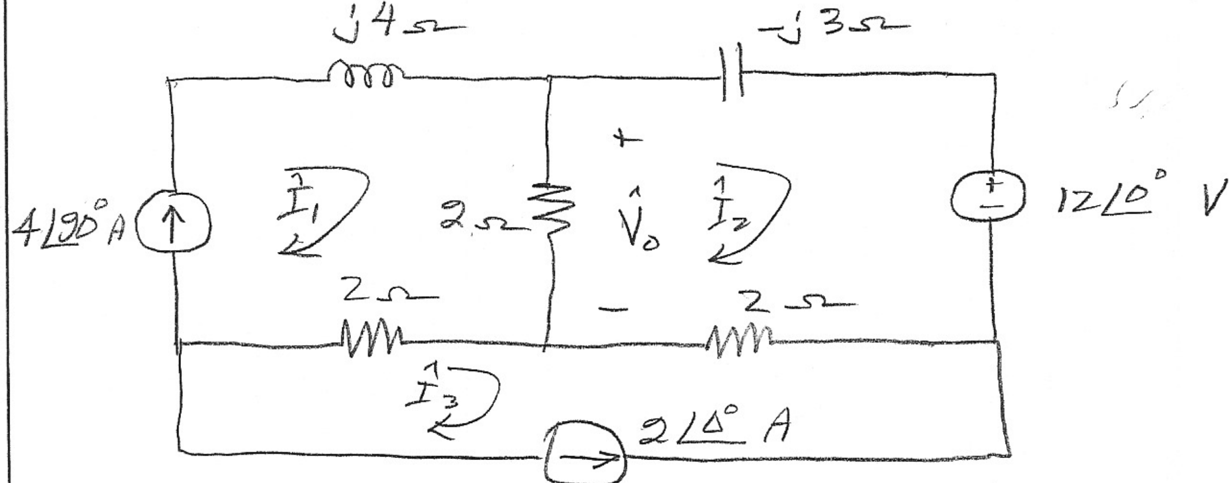


Figure 13.5: Circuit for Example 13.3.

We assign mesh currents as shown. We open the current sources, redraw the circuit.

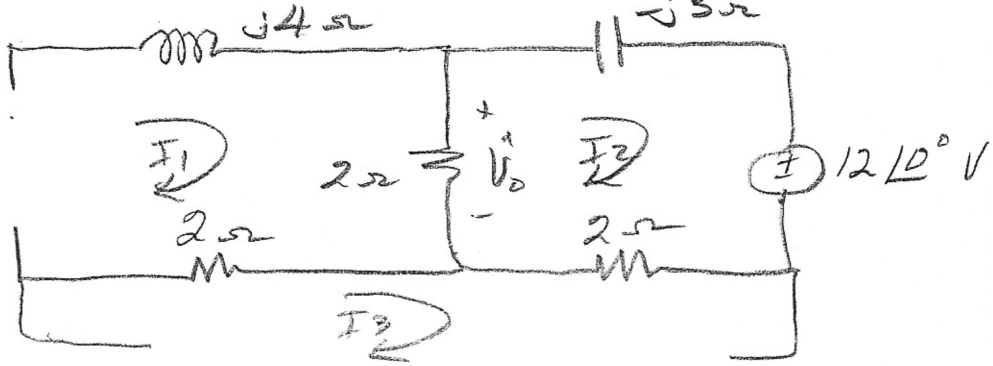


Figure 13.6: Modified Circuit, Ex 13.3

We have only one "closed" mesh to write KVL. It gives

$$2(\hat{I}_2 - \hat{I}_1) - j3\hat{I}_2 + 12 + 2(\hat{I}_2 - \hat{I}_3) = 0$$

or

$$\boxed{-2\hat{I}_1 + (4 - j3)\hat{I}_2 - 2I_3 = -12\angle 0}$$

We have a constraint equation

$$I_1 = 4\angle 90$$

or

$$\boxed{\hat{I}_1 + 0\hat{I}_2 + 0\hat{I}_3 = 4\angle 90^\circ}$$

We have another current constraint equation,

$$\hat{I}_3 = -2\angle 0^\circ$$

or

$$\boxed{0\hat{I}_1 + 0\hat{I}_2 + \hat{I}_3 = -2\angle 0}$$

In matrix form

$$\begin{bmatrix} -2 & (4-j3) & -2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_3 \end{bmatrix} = \begin{bmatrix} -12 \\ j4 \\ -2 \end{bmatrix}$$

$I_1 = j4A$, $I_2 = (-3.52 - j0.64)\,A$

$\hat{I}_3 = -2A$

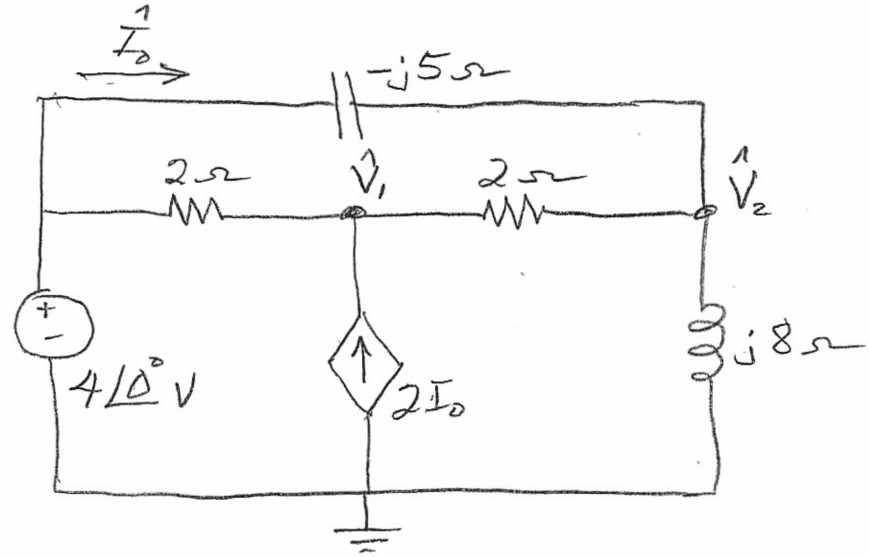
$$\vec{V}_o = 2(\vec{I}_1 - \vec{I}_2) = 2(j4 + 3.52 + j0.64)$$

$$\vec{V}_o = 2(3.52 + j4.64) = 7.04 + j9.28 = 11.65\angle 52.8^\circ \text{ V}$$

We handle dependent source with the same techniques as in DC circuits.

Example 13.4

Apply Nodal Analysis to find $\vec{I}_o$ in the following circuit (From Alexanderia, 3rd Ed, Problem 10.11 p 444)



At $\hat{V}_1$

$$\frac{\vec{V}_1 - 4}{2} + \frac{\vec{V}_1 - \vec{V}_2}{2} - 2\vec{I}_o = 0$$

but

$$\vec{I}_o = \frac{4 - \vec{V}_2}{-j5} = -j0.2\vec{V}_2 + j0.8$$

substitute into the previous equation

$$\hat{V}_1 - 0.5V_2 - 2I_o = 2$$

$$\hat{V}_1 - 0.5\hat{V}_2 - 2(-j0.2V_2 + j0.8) = 2$$

$$\boxed{\hat{V}_1 + (-0.5 + j0.4)\hat{V}_2 = 2 + j1.6}$$

At $\hat{V}_2$

$$\frac{\hat{V}_2 - \hat{V}_1}{2} + \frac{V_2}{j8} + \frac{V_2 - 4}{-j5} = 0$$

$$.5\hat{V}_2 - .5\hat{V}_1 - j0.125\hat{V}_2 + j0.2V_2 - j0.8 = 0$$

$$\boxed{-0.5\hat{V}_1 + (0.5 + j0.075)V_2 = j0.8}$$

$$\begin{bmatrix} 1 & (-0.5 + j0.4) \\ -0.5 & (.5 + j0.075) \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} 2 + j1.6 \\ j0.8 \end{bmatrix}$$

$$\hat{V}_1 = 4.86 + j0.054 \quad V$$

$$\hat{V}_2 = 4.9955 + j0.905 \quad V.$$

$$\hat{I}_o = \frac{4 - \hat{V}_2}{-j5} = \frac{4 - 4.9955 - j0.905}{-j5}$$

$$\hat{I}_o = 0.181 - j0.1991 \quad A$$