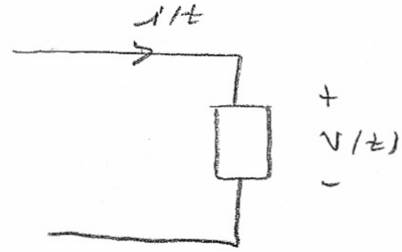
wlg. AC Power Lesson 14

Instantaneous power is expressed by

$$p(t) = v(t)\,i(t)$$




where we assume power absorbed using the passive sign convention.

Let

$$v(t) = V_m \cos(\omega t + \theta_V)\ ;\ i(t) = I_m \cos(\omega t + \theta_I)$$

giving

$$p(t) = V_m I_m \cos(\omega t + \theta_V)\cos(\omega t + \theta_I) \quad (14.1)$$

using the trig identity

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

gives

$$p(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta_V + \theta_I) + \cos(\theta_V - \theta_I)\right] \quad (14.2)$$

Notice we have a constant term

$$\frac{V_m I_m}{2}\cos(\theta_V - \theta_I)$$

and a time varying term

$$\frac{V_m I_m}{2}\cos(2\omega t + \theta_V + \theta_I)$$

and this second term is twice the frequency of either the voltage or current.

A typical plot of Eq (14.2) is shown in Figure 14.1 14.2

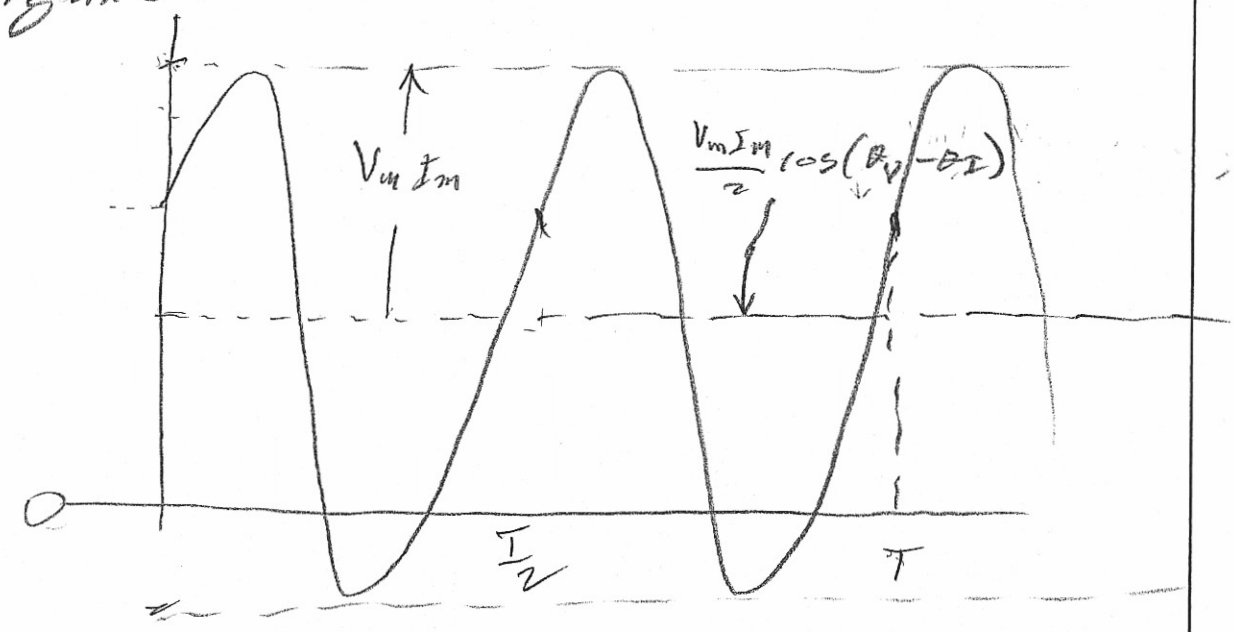



Figure 14.1: Plot of p(t)

This is nice to know but we are interested in average power.

Average power is determined by

$$P = \frac{1}{T}\int_0^T p(t)\,dt \qquad (14.3)$$

If we place the p(t) of Eq (14.1) into (14.3) we get

$$\boxed{P = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I)} \qquad (14.4)$$

This is an important equation.

with

$$\hat{V} = V_m \angle \theta_V, \quad \hat{I} = I_m \angle \theta_I$$

we can write

$$\frac{1}{2} \hat{V} I^* = \frac{1}{2} V_m I_m \angle \theta_V - \theta_I$$

$$= \frac{1}{2} V_m I_m [\cos(\theta_V - \theta_I) + j \sin(\theta_V - \theta_I)]$$

Note, the first term (real part) give $P$. Thus

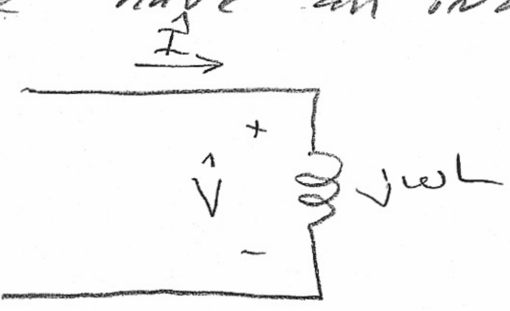
$$P = Re\left[\frac{\hat{V} I^*}{2}\right] = \frac{V_m I_m}{2} \cos(\theta_V - \theta_I) \quad (14.5)$$

If we are determing average power for a resistor, $(\theta_V - \theta_I) = 0$, so

$$P = \frac{V_m I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R} \quad (14.6)$$

we use $V_m = R I_m$ here.

When we have an inductor




$$\hat{V} = V_m \angle \theta_V = j\omega L I_m \angle \theta_I = \omega L I_m \angle \theta_I + 90°$$

so

$$\theta_V = \theta_I + 90°$$

In the equation

$$P = \frac{V_m I_m}{2} \cos(\theta_V - \theta_I)$$

we substitute $\theta_V = \theta_I + 90°$
we get

$$P_L = \frac{V_m I_m}{2} \cos 90 = 0$$

In a similar manner we find

$$P_C = 0$$

In words, the average power absorbed by either a capacitor or resistor is zero. In some respects this makes our work easier. Unless someone tells you to (like your teacher) there is no need to show this in a circuit.

Example 14.1

Find (a) the power absorbed by each passive element and (b) the power generated (supplied) by the source in the circuit of Figure 14.2.

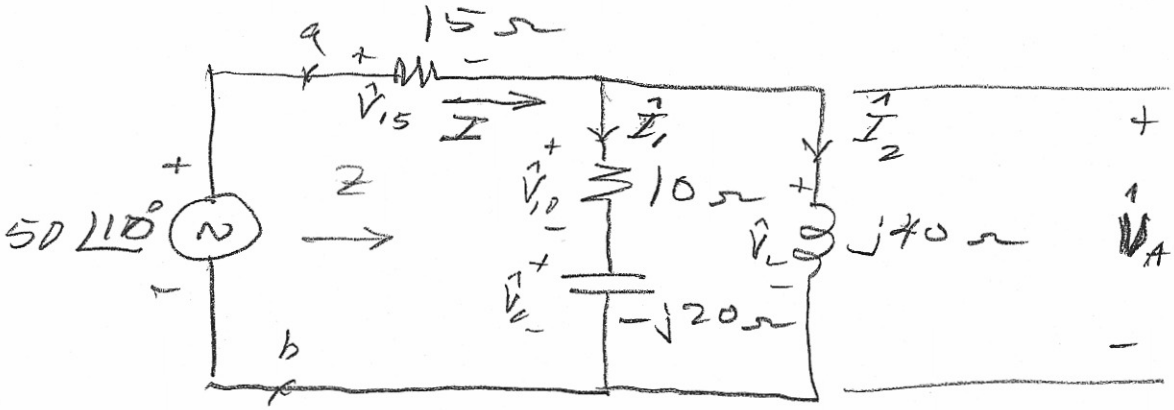



Figure 14.2: Circuit for Example 14.1

$$Z = 15 + (j40) \parallel (10 - j20)$$

$$= 15 + \frac{(j40)(10 - j20)}{10 + j20}$$

$$Z = 47 - j24 = 52.77\angle -27.05\ \Omega$$

$$\vec{I} = \frac{50\angle 110}{52.77\angle -27.05} = 0.948\angle 37.1^\circ\ A$$

Using current division

$$\vec{I}_1 = \frac{\vec{I} \times j40}{10 - j20 + j40} = \frac{(0.948\angle 37.1)(j40)}{10 + j20}$$

$$\vec{I}_1 = 1.696\angle 63.67\ \ A$$

$$\vec{I}_2 = \frac{(0.948\angle 37.1)(10 - j20)}{10 + j20} = 0.948\angle -89.8$$

For a check

$$\vec{I}_1 + \vec{I}_2 = 1.696\angle 63.67 + 0.948\angle -89.8$$

$$= 0.9477\angle 37.13\ A \quad \text{checks}$$

$\vec{V}_A = 50\angle 10 - 15 \times \vec{I}$

$\vec{V}_A = 50\angle 10 - 15 \times (0.948\angle 37.1)$

$\vec{V}_A = 37.9\angle 0.158 \ V$

$\vec{V}_{10} = \dfrac{\vec{V}_A \times 10}{10 - j20} = \dfrac{(37.9\angle .158) \times 10}{10 - j20}$

$\vec{V}_{10} = 16.95\angle 63.6^\circ \ V$

$\vec{V}_c = \dfrac{\vec{V}_A(-j20)}{10 - j20} = 33.9\angle -26.4 \ V$

Now calculate the power.

$P_{15} = \dfrac{|\vec{I}|^2 \times 15}{2} = 6.74 \ W$

$P_{10} = \dfrac{|I_1|^2 \times 10}{2} = 14.38 \ W$

$P_{supplied} = \dfrac{50 \times 0.948}{2} \cos(10 - 37.1)$

$P_{sup} = 21.1 \ W$

For a check

$P_{15} + P_{10} = 6.74 + 14.38 = 21.1$ checks

$P_{inductor} = |\vec{V}_A| \times |\vec{I}_2| \cos(\theta_{V_A} - \theta_{I_2})$

$= 37.9 \times 0.948 \cos(-158 - (-89.8))$

$P_{inductor} = 0$ for all practical purposes

$P_{capacitor} = |\vec{V}_c||\vec{I}_1| \cos(\theta_{V_c} - \theta_{I_1})$

$= 33.9 \times 1.696 \cos(-26.4 - 63.67)$

$P_{capacitor} = 0$ for all practical purposes

QED

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## Maximum Power Transfer

We have a similar situtation here as we did for DC circuits. Consider that a circuit to the left of terminals a-b has been reduced to it's Thevenin equivalent. We want to know what impedance we connect to the load for maximum power transfer. We start with the configuration shown in Figure 14.3

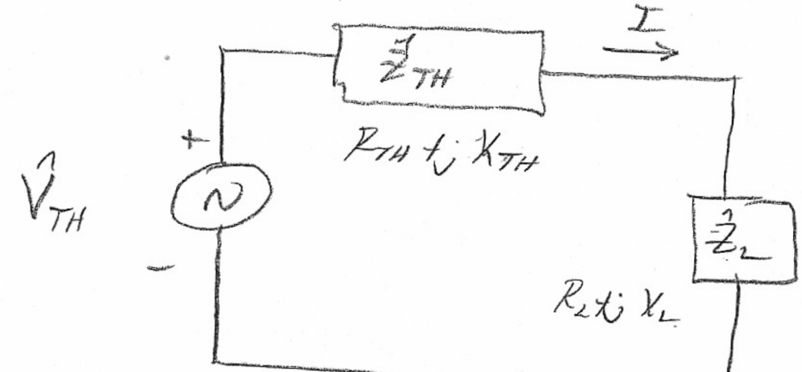



Figure 14.3: Setup for max pwr.

$$\vec{I} = \frac{\vec{V}_{TH}}{Z_{TH} + Z_L} = \frac{\vec{V}_{TH}}{(R_{TH}+R_L) + j(X_{TH}+X_L)}$$

$$P = \frac{1}{2}|\vec{I}|^2 R = \frac{|V_{TH}|^2 R}{2\left[(R_{TH}+R_L)^2 + (X_{TH}+X_L)^2\right]}$$

Perform

$$\frac{\partial P}{\partial X_L}, \quad \frac{\partial P}{\partial R_R}$$

and we find

$$X_L = -X_{TH}$$

$$R_L = R_{TH}$$

so

$$\boxed{Z_L = Z_{TH}^*} \qquad (14.7)$$

$$\boxed{P = \frac{|V_{TH}|}{8 R_{TH}}}$$

(power delivered to $R_L$)

(14.8)

If the load is restricted to be a resistor,

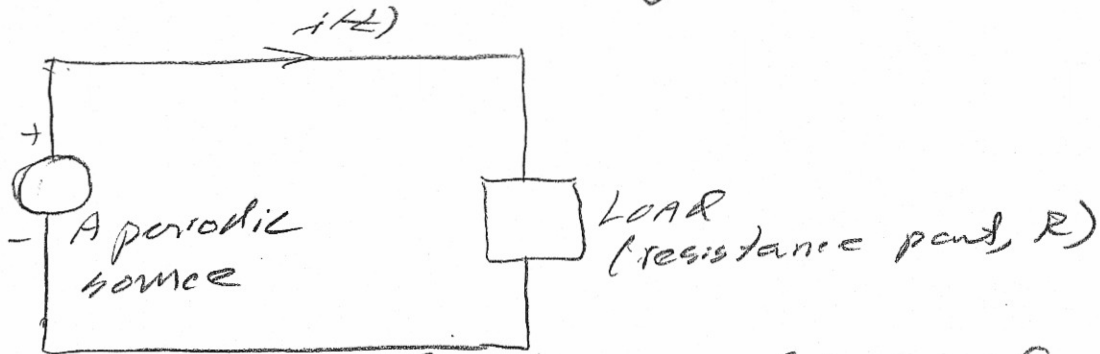
$$\boxed{R_L = |Z_{TH}| = \sqrt{R_{TH}^2 + X_{TH}^2}} \quad 14.9$$

I don't feel an example is necessary here. The work is straightforward

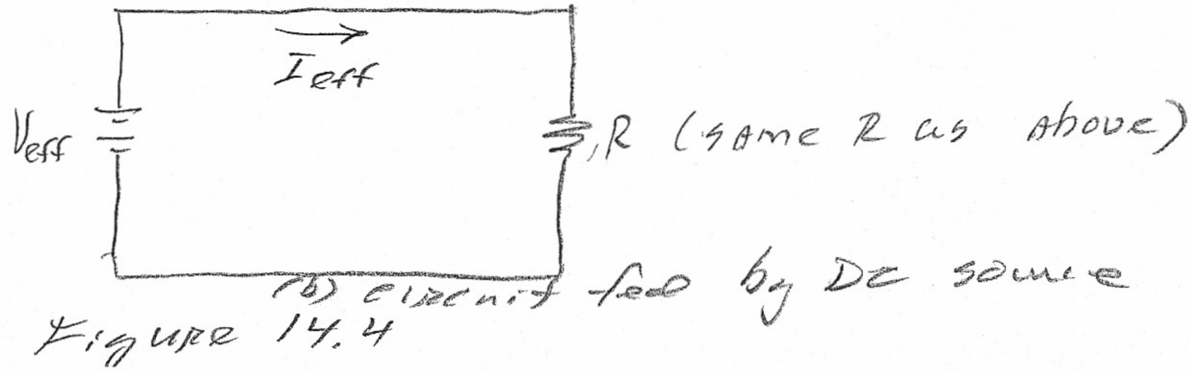
## Effective or RMS Values

The power industry deals with RMS (root mean square) values of voltage and current.

We consider the following:




(a) circuit fed by periodic signal




(b) circuit fed by DC source

Figure 14.4

The effective value of a periodic current is a dc value ($I_{eff}$) that delivers the same average power to a resistor as the periodic current.

Referring to Figure 14.4 (a)

$$P = \frac{1}{T}\int_0^T i^2 R\,dt \qquad (14.10)$$

Referring to Figure 14.4 b

$$P = I_{eff}^2 R \qquad (14.11)$$

Equating (14.10) to (14.11)

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^t i^2\,dt} \qquad (14.11)$$

We could go through the same argument for voltage and arrive at

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad (14.12)$$

The concept of RMS has been extended to include any periodic signal (function), say $x(t)$.

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^t x^2(t)\,dt}$$

Example 14.2

Find the RMS value of the voltage

$$v(t) = V_m \cos \omega t$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2 \omega t\, dt}$$

$$\cos^2 x = \frac{1}{2}\left[1 + \cos 2x\right]$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2T}\left[1 + \cos 2\omega t\right] dt}$$

so $$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}} \qquad (14.13)$$

Very important result.

It follows that the rms value of a sinusoidal current is

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad (14.14)$$

If you apply this to the fundamental equation for find average power, AC circuit —

$$P = \frac{V_m I_m}{2} \cos(\theta_V - \theta_I)$$

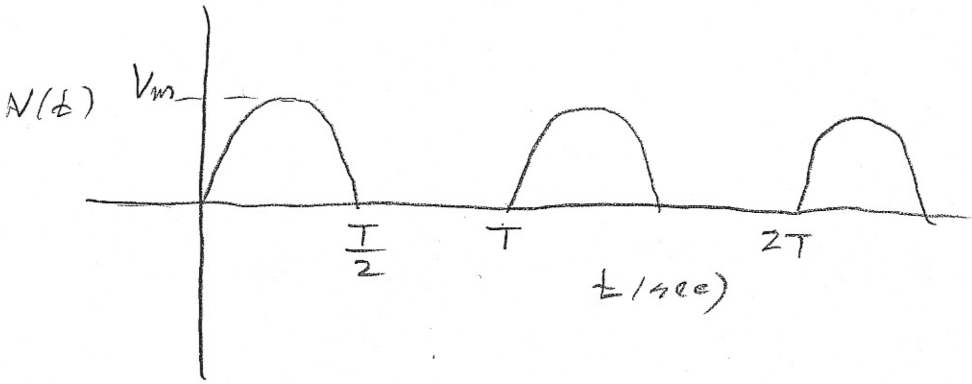
$$P = \frac{\sqrt{2}\, V_{rms} \sqrt{2}\, I_{rms}}{2} \cos(\theta_V - \theta_I)$$

$$\boxed{P = V_{rms} I_{rms} \cos(\theta_V - \theta_I)} \qquad (14.15)$$

In any given circuit you need to check to see if the sources are expressed in maximum values or rms values. Use accordingly.
In practice we measure and use rms values.

## Example 14.3

Find the RMS value of the following waveform.




$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} V_m^2 \sin^2 \omega t \, dt}$$

$$\sin^2 x = \frac{1}{2}\left[1 - \cos 2x\right]$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2T}\left[\frac{T}{2}\right]}$$

integral $\cos 2\omega t$ over 0 to T is 0

$$V_{RMS} = \frac{V_m}{2}$$

In finding RMS of non conventional signal one needs to be careful with setting up the equations that describe the function (goes back to analytical geometry).

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## Apparent Power and Power Factor

We define apparent power as

$$\boxed{S = V_{rms} I_{rms}} = \frac{V_m I_m}{2} \qquad (14.16)$$

We use the word "apparent" because of the linkage to DC circuit where $P = VI$. The units of apparent power are volt·amps or just VA.

We define power factor as

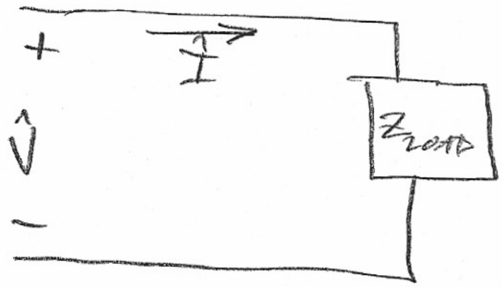
$$p.f. = \frac{P}{S} = \cos(\theta_V - \theta_I) \qquad (14.17)$$

Note $P = S\cos(\theta_V - \theta_I)$

In power lingo, one refers to $\theta_V - \theta_I$ as the power factor angle.

The power factor angle is equal to the angle of the load impedance.

This is clear by considering




$$\hat{Z} = \frac{\hat{V}}{\hat{I}} = \frac{V_m \angle \theta_V}{I_m \angle \theta_I} = \frac{V_m}{I_m} \angle \theta_V - \theta_I$$

A good comment to remember is

$$\boxed{P = S \times p.f. = \text{Apparent power} \times \text{power factor}}$$

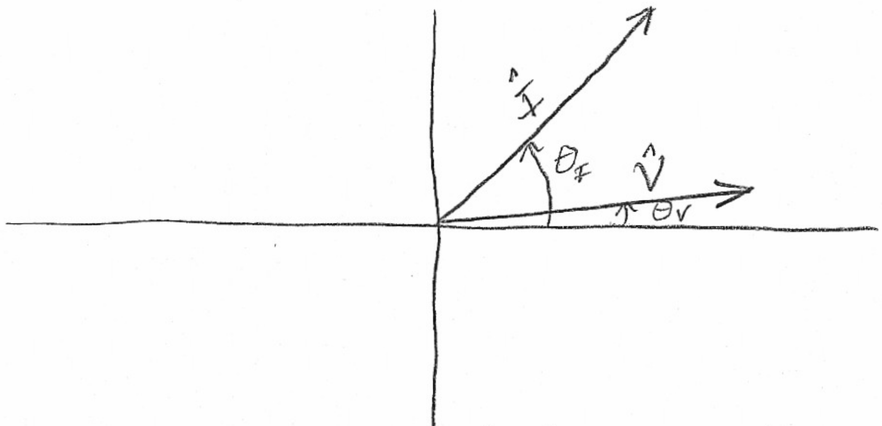
The p.f. ranges from $0 \le p.f. \le 1$ because this is the nature of $\cos(\theta_V - \theta_I)$.

<u>We define</u>

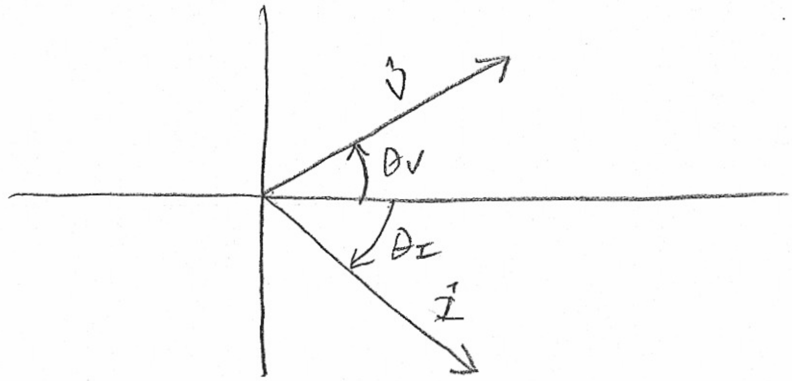
<u>Leading p.f.</u> when the current leads the voltage

Lagging p.f. when the current lags the voltage

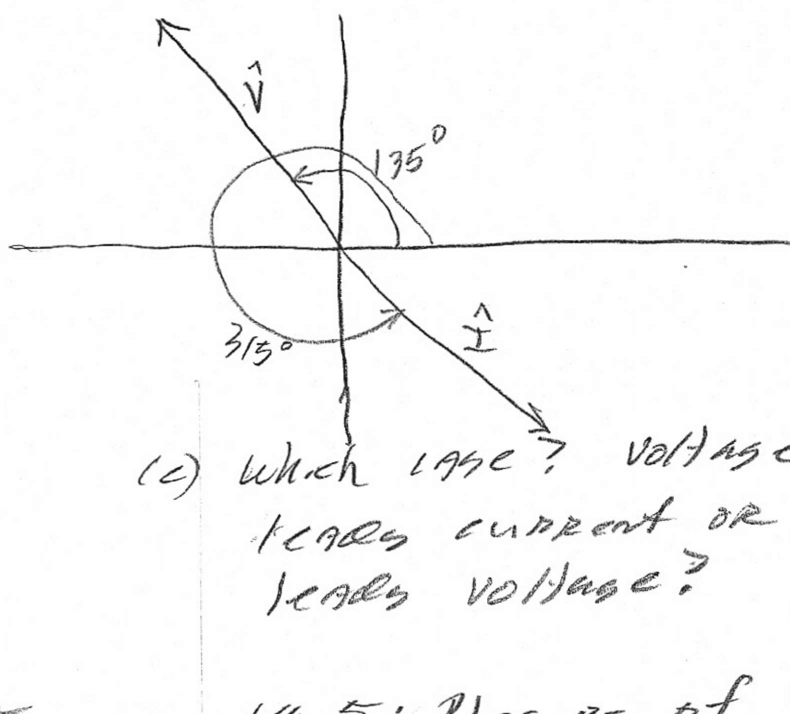
Graphically, the current leads the voltage as shown in Figure 14.5 (a). The current lags the voltage in Figure 14.5 b.




(a) Current lead voltage




(b) Voltage leads current.




(c) Which case? Voltage leads current or current leads voltage?

Figure 14.5: Phasors of Voltage and current.

Since the p.f. angle is the angle of the load impedance, in extreme cases, the voltage can lead the current by 90° (lagging p.f.) or voltage can lag the current by 90° (leading p.f.) This is true because for a load with a pure inductor has an angle of +90°. A load with a pure capacitor has impedance angle of −90°. All other loads must fall between +90° and −90°. That is, for circuits with R, L, and C, the angle of Z must be such that

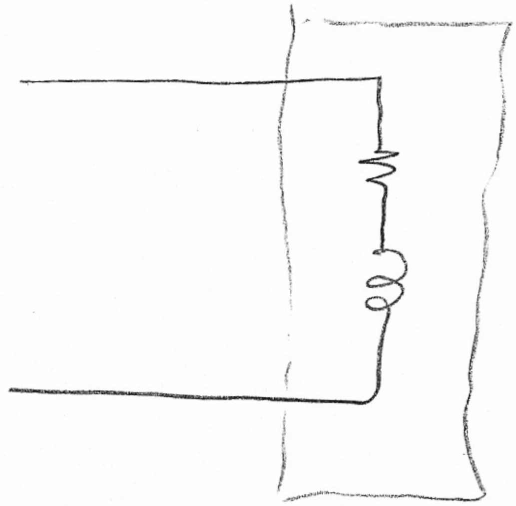
$$-90^\circ \leq \angle Z \leq 90^\circ$$

So Z is restricted to the first and forth quadrant. $\hat{V}$ and $\hat{I}$ of a load (element) can never be more than 90° apart. In summary,

if $\angle Z$ is negative, $\hat{I}$ leads $\hat{V}$
leading p.f.

if $\angle Z$ is positive, I lags V
lagging p.f.

Considder the following;



inductive load

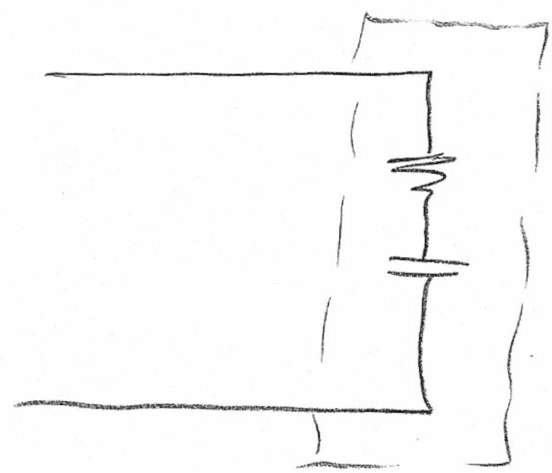
$Z = R + j\omega L$

$Z$ is in the 1st quadrant,

Voltage leads the current

"ELI"

voltage before current for an inductive load



capactive load

$Z = R - \frac{j}{\omega C}$

$Z$ in 4th quadrant,

Current leads the voltage.

"ICE"

current leads the voltage for a capactive load

ELI the ICE man