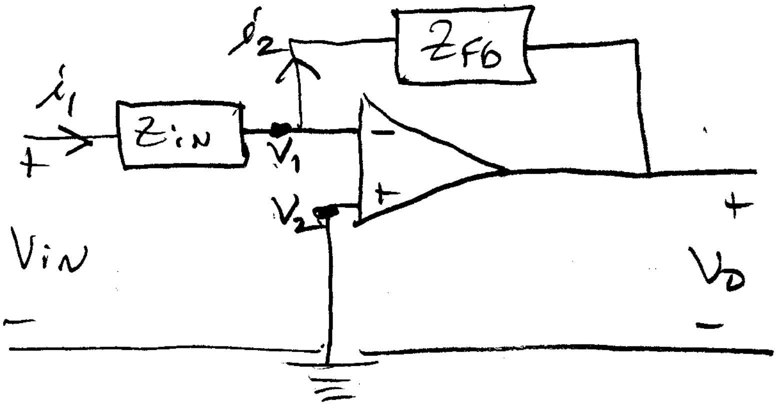
wlg

Active Filters

We consider active filters from the standpoint of using op-amps. As a quick review we recall the following:

Inverting Op-Amp



Assumptions

$V_1 = V_2$

$R_i = \infty$

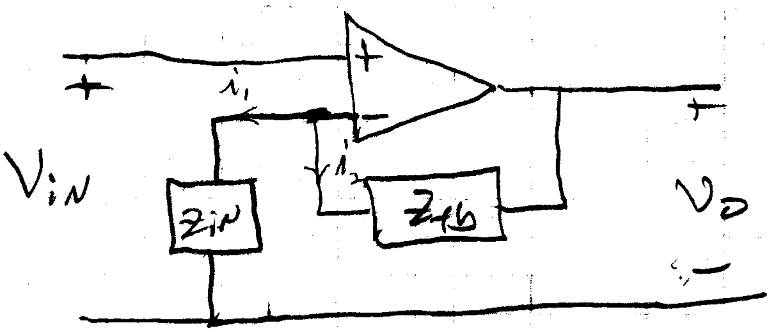
$R_o = 0$

$i_1 = i_2$

$$\frac{V_{in}}{Z_{in}} = -\frac{V_o}{Z_{fb}} \Rightarrow \boxed{\frac{V_o}{V_{in}} = -\frac{Z_{fb}}{Z_{in}}}$$

Non-Inverting Op-Amp

$i_1 + i_2 = 0$



$i_1 = \dfrac{V_{in}}{Z_{in}}$

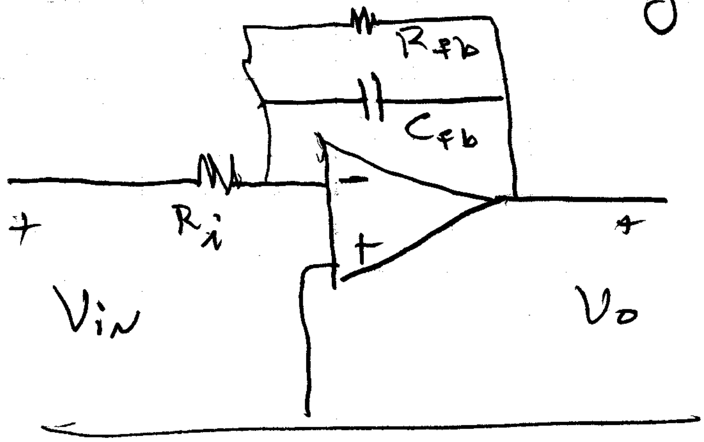
$i_2 = \dfrac{V_{in} - V_o}{Z_{fb}}$

so: $\dfrac{V_{in} - V_o}{Z_{fb}} = -\dfrac{V_{in}}{Z_{in}} \rightarrow V_{in}\left[\dfrac{Z_{in} + Z_{fb}}{Z_{in}\cancel{Z_{fb}}}\right] = \dfrac{V_o}{\cancel{Z_{fb}}}$

$$\boxed{\frac{V_o}{V_{in}} = \left[1 + \frac{Z_{fb}}{Z_{in}}\right]}$$

First Order Low Pass Filter

Consider the following circuit.



$$Z_{fb} = R_{fb} \parallel \frac{1}{sC_{fb}} = \frac{\frac{R_{fb}}{sC_{fb}}}{R_{fb} + \frac{1}{sC_{fb}}} = \frac{R_{fb}}{1+sR_{fb}C_{fb}}$$

$$\frac{V_o}{V_{in}} = -\frac{Z_{fb}}{Z_{in}} = -\frac{R_{fb}}{R_i\left[1+sR_{fb}C_{fb}\right]}\Bigg|_{s=j\omega} = -\left[\frac{R_{fb}}{R_i}\left(\frac{1}{1+j\omega R_{fb}C_{fb}}\right)\right]$$

This is a low pass filter.

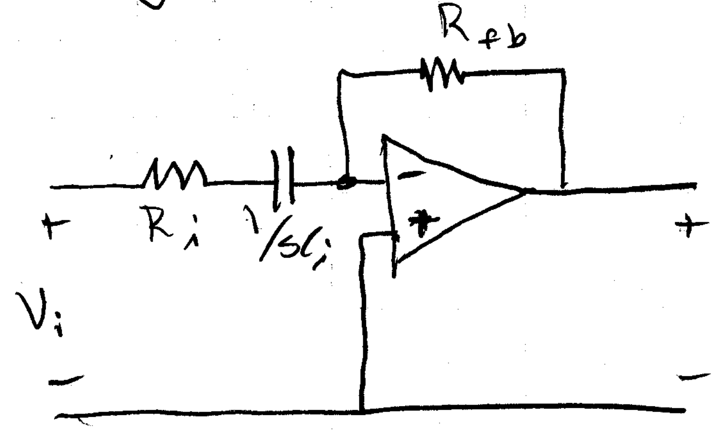
The -3 dB breakpoint is at

$$\omega = \frac{1}{R_{fb}C_{fb}}.$$

This filter has a dc gain of $-\frac{R_{fb}}{R_i}$ which could have an absolute value > 1 (if $R_{fb} > R_i$).

First Order High Pass Filter:

As shown below, the following op-amp configuration.

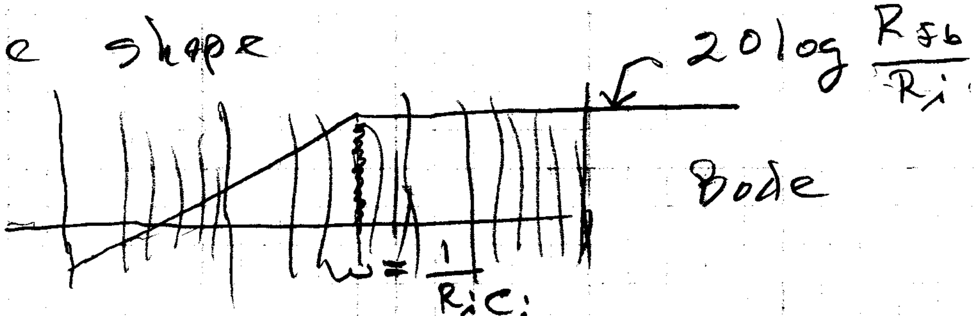


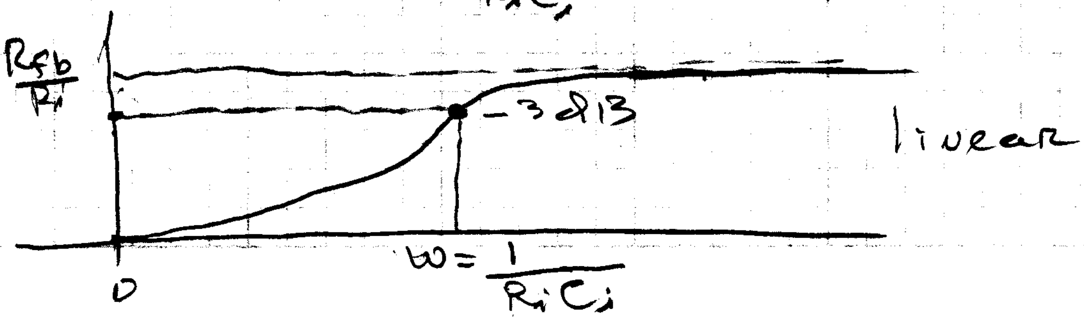
$$\frac{V_o(s)}{V_i(s)} = -\left[\frac{R_{fb}}{R_i + \frac{1}{sC_i}}\right] = \frac{-sR_{fb}C_i}{1 + sR_iC_i}$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\left[\frac{j\omega R_{fb}C_i}{1 + j\omega R_iC_i}\right]$$

The dc gain of this filter is zero.

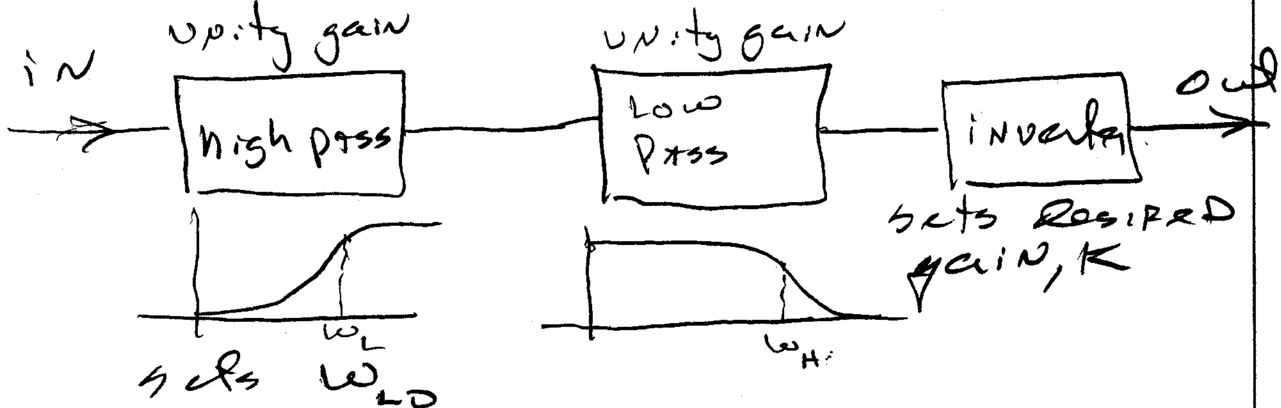
Bode shape



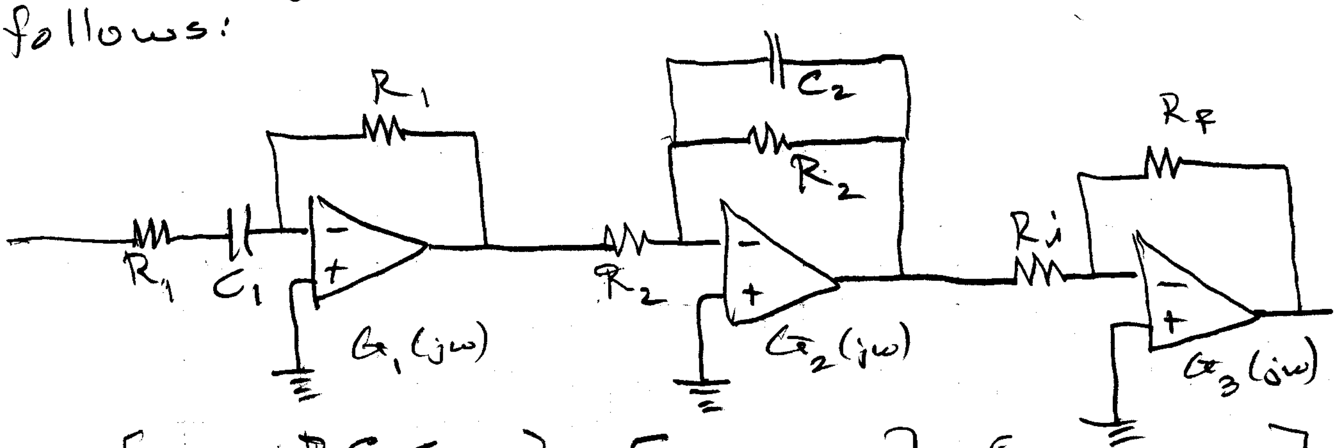


Bandpass Filter

An easy way to understand a method for designing a bandpass filter is to think of it as follows,

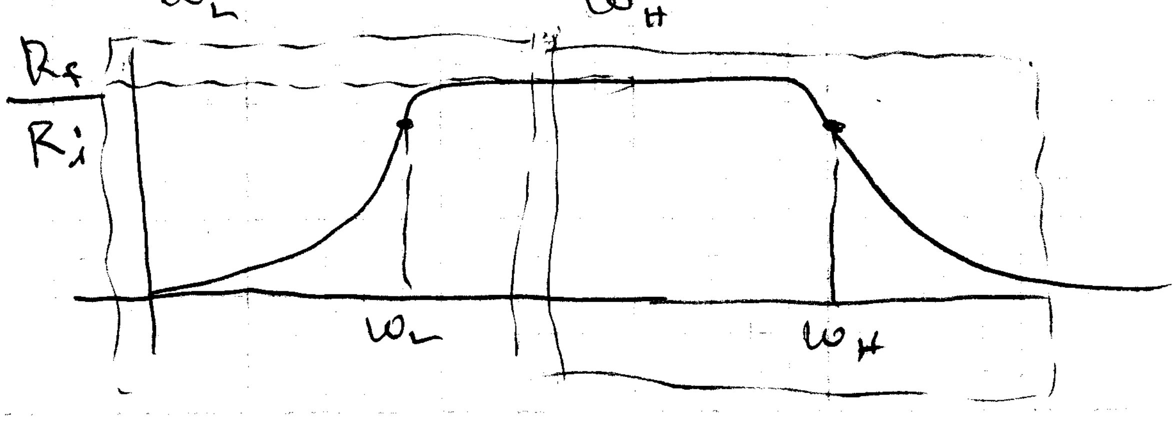


This diagram can be constructed as follows:



$$\left[-\frac{R_1C_1s}{1+R_1C_1s}\right] \quad \left[-\frac{1}{1+R_2C_2s}\right] \quad \left[-\frac{R_f}{R_i}\right]$$

ω_L ω_H



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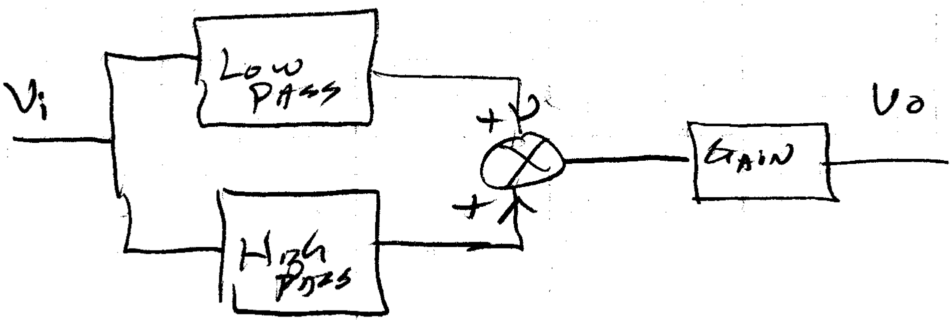
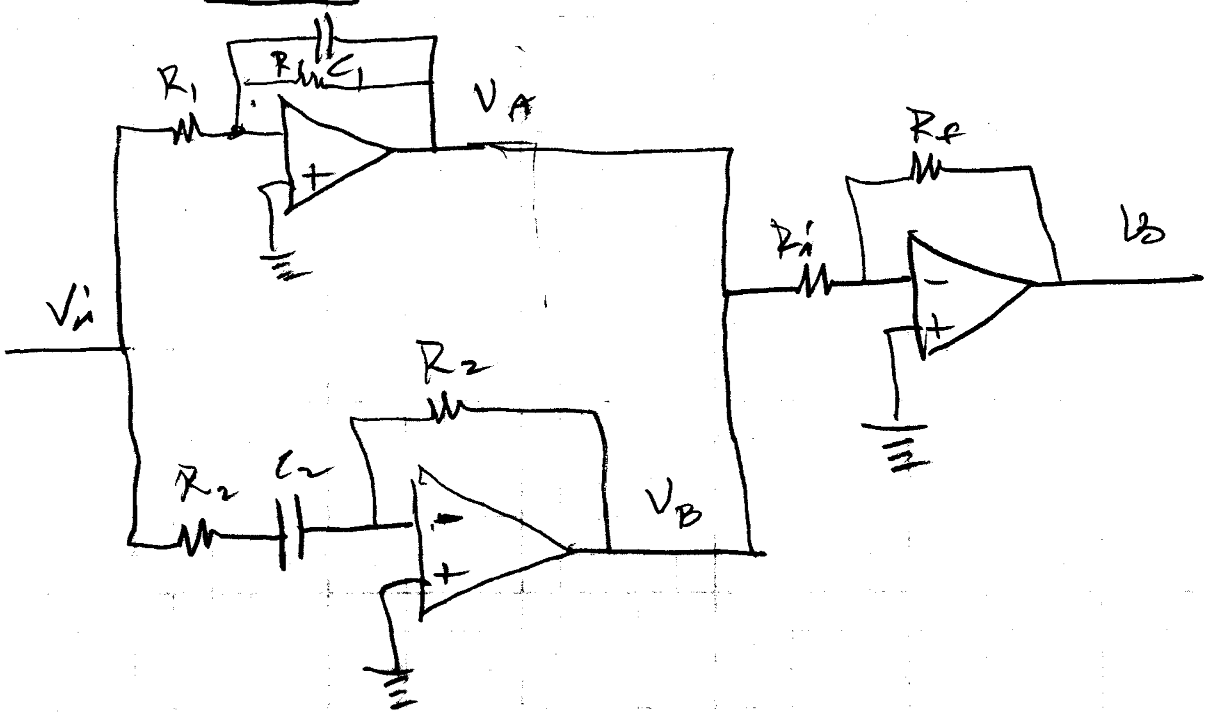
For this previous design we know
$$\omega_H = \frac{1}{R_2C_2}, \quad \omega_L = \frac{1}{R_1C_1}$$

The center frequency, ω_0, is related to ω_1 and ω_2 as for the series resonant circuit;
$$\omega_0 = \sqrt{\omega_L\omega_H}$$
and
$$B = \omega_H - \omega_L$$
and
$$Q = \frac{\omega_0}{B}$$
We can set the amplifier mid frequency gain:
We have $G(j\omega) = G_1(j\omega)G_2(j\omega)G_3(j\omega)$
so,
$$G(j\omega) = \frac{(-j\omega/\omega_L)\left(\frac{R_f}{R_i}\right)}{(1+j\omega/\omega_L)(1+j\omega/\omega_H)} = \frac{(-j\omega\omega_H)\left(\frac{R_f}{R_i}\right)}{(\omega_L+j\omega)(\omega_H+j\omega)}$$
$$G(j\omega) = \frac{-j\omega\omega_H\left(\frac{R_f}{R_i}\right)}{(\omega_L\omega_H + j\omega(\omega_L+\omega_H) - \omega^2}$$
At the center frequency, $\omega = \omega_0 = \sqrt{\omega_L\omega_H}$
$$G(j\omega_0) = \frac{R_f}{R_i}\frac{\omega_H}{\omega_L+\omega_H} = A_{\text{desired}}$$
What do you want the filter gain at ω_0 to be? This is $= A$ then solve for $\frac{R_f}{R_i}$

Bandstop Filter





$$H(s) = \left[\frac{1}{1+sR_1C_1} + \frac{sR_2C_2}{1+sR_2C_2}\right]\frac{R_f}{R_i}$$

$$= \left[\frac{1}{1+s/\omega_1} + \frac{s/\omega_2}{1+s/\omega_2}\right]\frac{R_f}{R_i}$$

$$= \left[\frac{1+s/\omega_2 + s/\omega_2 + s^2/\omega_1\omega_2}{(1+s/\omega_1)(1+s/\omega_2)}\right]\frac{R_f}{R_i}$$

$$= \left[\frac{1 + 2s/\omega_2 + s^2/\omega_1\omega_2}{1 + s(1/\omega_1 + 1/\omega_2) + s^2/\omega_1\omega_2}\right]\frac{R_f}{R_i}$$

Let $s \to j\omega$

$$H(j\omega) = \frac{R_f}{R_i}\left[\frac{1 + j2\omega\left(\frac{1}{\omega_2}\right) + (j\omega)^2/\omega_1\omega_2}{1 + j\omega\left(1/\omega_1 + 1/\omega_2\right) + (j\omega)^2/\omega_1\omega_2}\right]$$

$$H(0) = \frac{R_f}{R_i}$$

$$H(\infty) = \frac{R_f}{R_i}$$

$$H(j\omega_0) = \frac{R_f}{R_i}\left[\frac{1 + j2\omega_0/\omega_2 - \omega_0^2/\omega_1\omega_2}{1 + j\omega_0\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) - \omega_0^2/\omega_1\omega_2}\right]$$

$$\omega_0^2 = \omega_1\omega_2$$

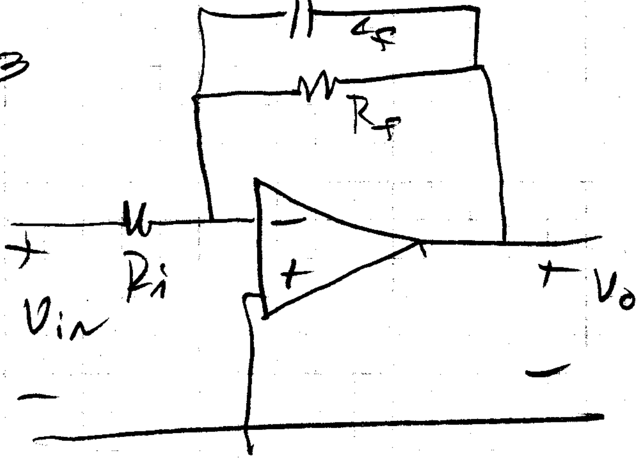
$$H(j\omega_0) = \frac{R_f}{R_i}\left[\frac{\cancel{j}2\cancel{\omega_0}/\omega_2}{\cancel{j}\cancel{\omega_0}\left[\frac{1}{\omega_1} + \frac{1}{\omega_2}\right]}\right]$$

$$= \frac{R_f}{R_i}\left[\frac{2}{\cancel{\omega_2}\left(\frac{\omega_1 + \omega_2}{\omega_1\cancel{\omega_2}}\right)}\right]$$

$$H(j\omega_0) = \frac{R_f}{R_i}\left[\frac{2\omega_1}{\omega_1 + \omega_2}\right]$$

set $\frac{R_f}{R_i}$ as you want $H(j\omega_0)$

14.53



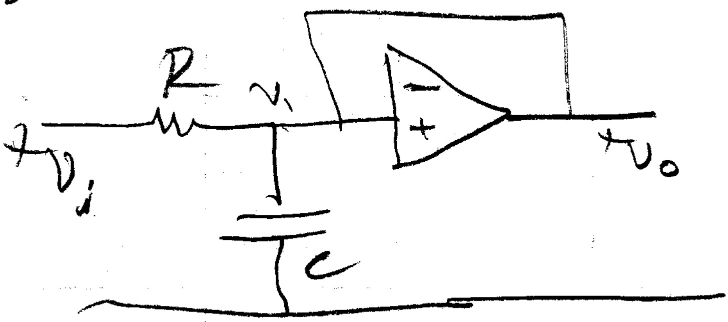
$$\frac{V_o}{V_{in}} = \frac{-\dfrac{R_f/sC_f}{(R_f + 1/sC_f)}}{R_i}$$

$$\frac{V_o}{V_{IN}} = -\frac{R_f}{R_i} \times \frac{1}{1 + sR_fC_F}$$

$$\frac{R_F}{R_i} = 0.5 \qquad \omega_c = \frac{1}{R_f C_f}$$

$$f_c = \frac{1}{2\pi R_f C_f}$$

14.48

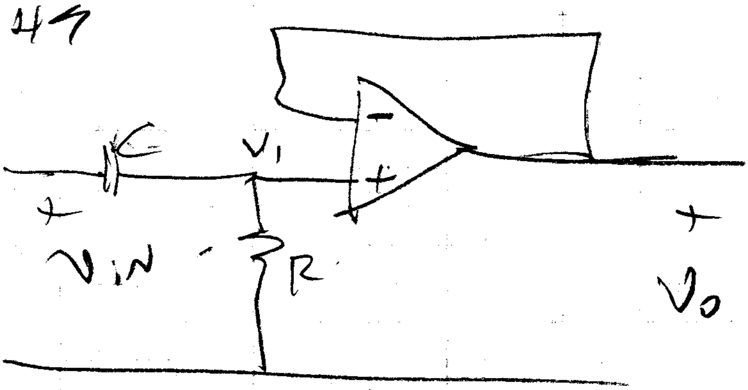


$$V_1 = \frac{(1/sC)\,V_i}{R + 1/sC} = \frac{V_i}{1+sRC}$$

$$v_1 = v_o$$

$$\frac{v_o}{v_i} = \frac{1}{1+sRC}$$

14.49



Low pass

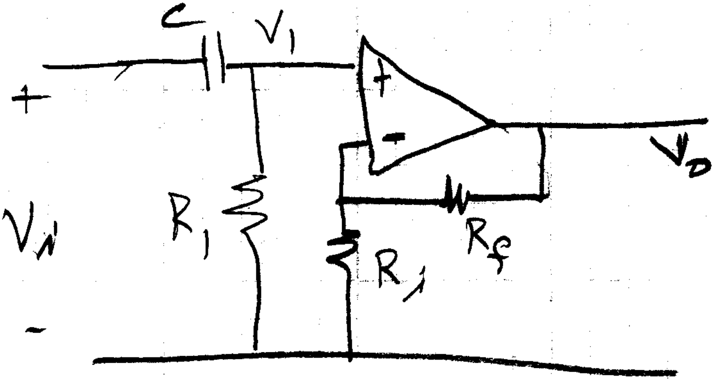
$$\frac{V_1}{V_{in}} = \frac{R}{R+\frac{1}{sC}} = \frac{sRC}{1+sRC}$$

$$v_1 = v_o$$

$$\frac{V_o}{V_{IN}} = \frac{sRC}{1+sRC}$$

High pass

14.51



$$V_1 = \frac{R_1}{R_1 + \frac{1}{sC}} = \frac{sR_1C \; V_i}{1 + sR_1C}$$

$$V_1 = V_o \times \frac{R_i}{R_i + R_f}$$

$$V_o \left[\frac{R_i}{R_i + R_f}\right] = \frac{sR_1C \; V_i}{1 + sR_1C}$$

$$\frac{V_o}{V_i} = \left[1 + \frac{R_f}{R_i}\right]\left[\frac{sR_1C}{1 + sR_1C}\right]$$